Game theory for best results in academics

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Abstract: Game Theory (GT) is a technique or tool for analyzing the problems for studying and evolving the strategies, for reaching out towards the rational decision processes of individual persons and their interactions in an environment of a group. Game Theory focuses on studying the different approaches, in which strategic interactions among different economic agents evolve outcomes with respect to the choices (or utility) of the agents, irrespective of whether the results of their efforts were intended. Thus, mostly GT is used for a study of different mathematical models of variations, uncertainties, conflicts and interactions among intelligent; rationally deciding subjects i.e. human beings.

In teaching learning process large number of students, faculties are involved. This results in large number of variables. Large number of students with

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unpredictable behaviors, number of faculties with different methods of teaching, makes it difficult to achieve the best academic results. So a study is undertaken and a game theory (GT) is adapted to evolve an academic system so that academic process becomes interesting for the participants that are the students and faculty.

The basic reason of choosing a GT is as it allows users to consider the different variations in the chosen model. The objective of this study is to evolve a composite optimum strategy for the faculty member in-order to achieve the best possible result for any fragmented group of students. The analysis is carried out at the level of some subject in which maximum use of mathematics is observed, such as economics and financial analysis and management.

Key words: game theory, strategy, risk, career, decision, higher education, fusion, check and balance

1. Introduction:

Game Theory (GT) is best suitable for the study and analysis of strategic and scientific decisions in complicated cases where large numbers of unpredictable variables such as many human beings are involved[01]. According to Ross[02], Game Theory focuses on the analysis of different ways with which various choices or preferences of the variables.



Using Game Theory study of the strategic interaction among possible economic variables may be performed, irrespective of outcomes and the effects of the persons involved[03]. Therefore in the discussion about the Game Theory, according to Myerson[04] generally it is said that GT is a study of mathematical models of different variables and collaboration between intelligent rationally deciding subjects. Game Theory avails a base for developing mathematical models for analyzing any situations where large numbers of variables are involved. Myerson[04] studied the interaction of individuals and the surroundings where one persons suspected action is modified due to the rational response of another individual or environment. G. D'Agostini[05] explored the possibilities of using Game Theory in academics.

Armstrong[06] showed that, using Game Theory, it is possible to predict the responses of individuals or the system for obtaining the solution within the rational algorithm. For achieving the predefined targets or goals Game Theory helps in deciding the strategy for real or virtual systems. Game Theory can be used where ever the results or behavior is not an outcome of predefined action as GT is much descriptive and normative. Thus Game Theory is used for finding best option where number of variables is involved, or where little complex is involved such as social, ethical or economical behavior.

As the educational processes are much complicated due to unknown variables such as psychological and pedagogical responses of the humans involved, thus for implementing mathematical models basic research of fundamental facts is necessary[07]. According to study presented by Ereneli[08], the results of same teaching methods for different students will be always different; therefore, many immaterial training cannot be quantitatively defined.

Using the Game Theory of generalized Nash games the state conjecture formulation of DTDGs was introduced by Abraham and Kulkarni[09]. Kulkarni and Abraham[10] also presented an approach based on generalized Nash Games and Shared Constraints for Discrete Time which transforms a DTDG into an equivalent generalized Nash game. This research study aims for using Game Theory, for predicting the responses of individuals i.e. students or the system for obtaining the solution within the rational algorithm.

2. Use of Games Theory in didactical process planning

Without doing the basic research on psychological and pedagogical responses of individual the perfect mathematical model in actual process of didactical practice is not possible. The outcome of teaching and training process varies from person to person. It depends mainly on students' ability and specifically on memory. A student with good memory will acquire the knowledge better and faster than the one who has poor memory. In the statistical analysis presented by Grațiela and Carmen[11] it is observed that the acquisition of knowledge depends 25 % on memory and 56% on the number of repetition of exercises. From this it may be concluded that the exercises need to be repeated numerous times till the students with poor memory will start remembering it. Although there are chances that due to repetition some students may lose the interest in memorizing the exercise and ultimate result may be poor. One alternative for memorizing the exercise is repetition, but the repetition needs to be optimized to have the best result. In order to get the desired result this optimum number of repetition must be considered. Excess number of repetition may lead to over learning and saturation while lesser number of repetitions will lead to under learning. From the perspective of Game Theory this is a conflicting situation which an academician has to resolve. He has to evolve such strategy so that all students undergoing the exercise should perform with their best possible abilities. This is a game situation between two sides, one is academician and another is students. In this game situation pedagogical results will vary with academician's abilities to choose right method to impart the knowledge and students' abilities to acquire and memorize the knowledge. For example, the academician uses purely Method- 01 e.g. only repeatedly reading the exercise loudly. In another trial, academician uses purely Method- 02 e.g. only repeatedly writing the exercise. The set of 10 problems of moderate level of difficulty were chosen regardless the memory level of students. The performance of student depends on the category in which that student belongs. For uniformity students with 60 to 70 % academic performance were chosen.

The experiment and analysis conducted over 100 students showed that the first type of students who are inclined towards memorizing the exercises through repetition acquires the knowledge 95%, while the



Table 1: Matrix of outcomes for Method No. 01

	Students	
	Method No. 01	Method No. 02
Academician- Method No. 01	100	75

second type of students who gets saturated due to monotones repetition acquires the knowledge 75%. This is represented in matrix of outcome as above.

If the academician decides to adopt method- 01 along with 10 problems of different type, then the first type of students who are inclined towards memorizing the exercises through repetition will show poor performance due to frequent change in problems. The first type of students will get confused and will find difficult to cope up with frequent change of problems. Table 02 shows the outcome of this exercise:

Table 2: Matrix of outcomes for Method No. 02

Γ		Students	
Ì		Method No. 01	Method No. 02
ſ	Academician- Method No. 02	65	100

Table 3: Matrix of combined outcomes for both Methods

	Students		
	Method No. 01	Method No. 02	
Academician-Method No. 01	100	75	
Academician-Method No. 02	65	100	

The matrix provided above reflects the conflicting circumstances which the academician has to handle due to unpredictability and variation in responses which varies from person to person. As the number of participating students may affect the results obtained by the academician, 45 students of first and 15 students of second category were considered. Thus the results of method no. 1 are calculates as below:

$$45 \times \left(\frac{100}{100}\right) + 15 \times \left(\frac{75}{100}\right) = 56.25 \tag{01}$$

of 60 students having good results,

Whereas the results of method no. 2 is,

$$45 \times \left(\frac{65}{100}\right) + 15 \times \left(\frac{100}{100}\right) = 44.25 \tag{02}$$

of 60 students having good results,

In the above situation method no. 01 is better as compared to method no. 02. However, if in the same class 15 are the students of category 1 and 45 are the students of category 2, then the whole analysis will change.

$$15 \times \left(\frac{100}{100}\right) + 45 \times \left(\frac{75}{100}\right) = 48.75 \tag{03}$$

$$15 \times \left(\frac{65}{100}\right) + 45 \times \left(\frac{100}{100}\right) = 54.75 \tag{04}$$

of 60 students having good results for method no. 2,

In above combination of students now method no. 2 is better than method no. 1. Thus depending on the proportion of type of students in the class, the method needs to be chosen.

3. Case study (Quantitative Techniques in Management, first year)

For more critical analysis, five teams of students studying MBA in the 'School of Management' of 'SAGE University' are considered. Students of 'School of Management' are chosen for analysis because this course is having heterogeneous mix of students with graduation in Commerce, Science and Engineering. Total 05 groups of students from first year were analyzed. The students were divided in two categories; graduates in commerce and graduates in science.

Table 4: Variation in academic strategy due to variation in number of students of Type 01,

Type 02 & Combined method

	Number of graduates in		Chosen method		Combined
	Commerce	Science	Method No.	Method No.	method
			01	02	
Team 01	100	30	110/130	90/130	109/130
Team 02	40	90	100/130	115/130	109/130
Team 03	70	60	116/130	80/130	109/130
Team 04	54	76	116/130	108/130	109/130
Team 05	78	52	101/130	75/130	109/130

Now the question arises, that can we find out the right combination of method no. 01 and method no. 02 so that the academician can obtain the best possible results for any class. Thus now we have to formulate the problem in such a way that the academician will obtain the best results for optimum combination of method no. 01 and method no. 02 irrespective of mix of type 1 and type 2 students.

There can be two strategies in this game situation;

- 1) Each side will have two strategies &
- 2) The loss happened to one side must be equal to the gain of the other side.

This will result that the total of gain and loss will be zero. As per the rules of Game Theory following combination will be the best for obtaining the right proportion of mixed strategy,

- i. The difference of the values in the first column and in the second column are taken
- ii. The results are expressed as absolute values written in reverse order as percentages in Figure No. 01 as below,

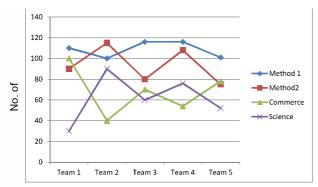


Fig. 1: No of students appeared & passed in the examination

For the optimum mixed strategy, the percentage ratio defines the frequency of application for the method no. 01 and method no. 02.

$$|100 - 75| = 25\tag{05}$$

$$|65 - 100| = 35 \tag{06}$$

$$35:25 = 7:5$$

Irrespective of mix of students of type 1 and type 2, this is the proportion with which we have to apply method no. 01 and method no. 02 in-order to obtain the best results. In reality this exercise will be repeated four times for both the method.

For this situation the academician's result for first type of students will be,

$$\frac{(7 \times 100) + (5 \times 65)}{7 + 5} = 85.42\% \tag{07}$$

For the second type of students,

$$\frac{(7 \times 75) + (5 \times 100)}{7 + 5} = 85.42\% \tag{08}$$

Irrespective of combination of students, in this case the results will be 85.42%.

In Team 04 (from Table- 04) there are 40% students of Type 01 and 60% students of Type 02, so the combination will be 4:6. The results of this situation can be tabulated for the academician's pure strategies (by multiplying to figures in columns by 0.4 and 0.6)

Table 5: The new results for academician's pure methods

	Students		
	Method No. 01	Method No. 02	
Academician Method No. 01	42	44	
Academician-Method No. 02	27	59	

Using the same mixed strategy developed for the academician earlier i.e. 7:5, for the students of Type 01, we will get the following results;

$$(7*42+5*27)/(7+5) = 35.75\%$$
 (09)

For the second type of students;

$$(7 \times 44 + 5 \times 59)/(7 + 5) = (308 + 295)/12 = 6 \ 0.3 = 50.25\% \ 83.65\% \ (10)$$

In the above case obtained result of 83.65% is the best possible result. The Method 01 of pure strategy resulted into 116/130= 89.23% good results and the Method 02 of pure strategy provided 108/130= 83% good results.

By adopting pure method, the academician is obtaining the 83.65% results; which may not be the best possible results for any combination of students. But it is also clear that for any worst possible combination of Type 01 and Type 02 students, the results will never go below 83.65%, till he adopts best method. Following are the results shown in Figure No. 02 using different methods,

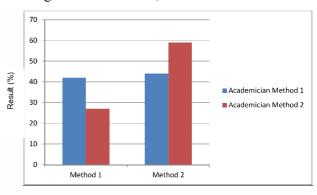


Fig. 2: Variation of results using Method 1 & Mathod2

Table 6: Advantage to academician while using different methods

	Chosen method		
	Method No. 01	Method No. 02	Combination of both methods
Team 01	84.62%	69.23%	83.65%
Team 02	77%	88.46%	83.65%
Team 03	89.23%	61.54%	83.65%
Team 04	89.23%	83.08%	83.65%
Team 05	77.70%	57.69%	83.65%

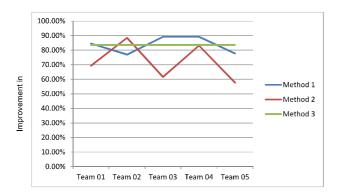


Fig. 3: Result improvement with combination of methods

For the first year of Master of Business Management (MBA), for the subject Quantitative Techniques in Management, to obtain the breakeven point the same mixed method is adopted. The results are presented as below,

Table 7: The result matrix for the subject Quantitative Techniques in Management

	Students		
	Method No. 01	Method No. 02	
Academician-Method No. 01	100	60	
Academician-Method No. 02	45	100	

As the students under consideration are post graduate students who are more focused and learned to effectively use their memory to remember the topics taught by the teacher, so we cannot adopt methods chosen for school students. Thus academician should evolve the best possible mixed method as it is difficult for these students to precisely classify as Type 01 and Type 02 students.

Table No. 06 highlights the advantages for the academicians by using the combination of method No. 01 & Method No. 02. The use of combined method results in improvement in performance of the students or the participants irrespective of their academic background. From Table No. 07, it is explained that, for a particular pattern of study adopted by the students the exclusive use of certain technique will be giving high results.

4. Conclusion

In teaching learning process choosing the right pedagogical method for obtaining the best solution is highly difficult as it involves large number of variables and multi facet circumstances. In addition to this planning of didactical activity is much demanding. For such complicated and conflicting situations a planning model is provided by Game Theory.

In this study after real situations are simplified on larger extent it is observed that the best possible mixing of strategies provides highly reasonable solutions. It helps academicians to plan the didactical process well in advance i.e. before knowing the learning abilities and basic knowledge levels of the participant group of students.

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Demographic Data of Students of 'School of Management' of 'SAGE University' CLASS:- MBA (FIRST YEAR)

Team No.	Team member with academic background	No of Students	GRADUATED IN
	<u>-</u>	TEAM 1	NO. 01
TEAM			COMMERCE
NO. 01	COMMERCE	72	i.e. Group of Students admitted to MBA after completing B.Com
TEAM NO. 01	SCIENCE	24	SCIENCE i.e. Group of Students admitted to MBA after completing B.Sc
		TEAM 1	NO. 02
TEAM NO. 02	COMMERCE	24	COMMERCE i.e. Group of Students admitted to MBA after completing B.Com
TEAM NO. 02	SCIENCE	72	i.e. Group of Students admitted to MBA after completing B.Sc
		TEAM 1	NO. 03
TEAM NO. 03	COMMERCE	48	COMMERCE i.e. Group of Students admitted to MBA after completing B.Com
TEAM NO. 03	SCIENCE	48	SCIENCE i.e. Group of Students admitted to MBA after completing B.Sc
		TEAM 1	NO. 04
TEAM NO. 04	COMMERCE	39	COMMERCE i.e. Group of Students admitted to MBA after completing B.Com
TEAM NO. 04	SCIENCE	57	SCIENCE i.e. Group of Students admitted to MBA after completing B.Sc
TEAM NO. 05			
TEAM NO. 05	COMMERCE	57	COMMERCE i.e. Group of Students admitted to MBA after completing B.Com
TEAM NO. 05	SCIENCE	39	SCIENCE i.e. Group of Students admitted to MBA after completing B.Sc

