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ORIGINAL ARTICLE

Integrating Interval value Intuitionistic Fuzzy Vague Set and COCOSO method with Friedman Test for Handling multi criteria Decision Making

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Abstract

Objectives: To evaluate and prioritize the authentication technique for artifact theft, it is implemented and compared in sculptures of the Chola Dynasty relocated to the homeland of Tamil Nadu. **Methods:** This study integrates the COCOSO ranking method and interval-value intuitionistic fuzzy vague sets in multi-criteria decision-making (MCDM). To obtain the rank quality from the alternative rank order, the Friedman test is processed. The nuanced analysis of interval value intuitionistic fuzzy vague sets provides qualitative decision stability and a realistic framework for solving real-world problems. **Findings**: The international issue of fraudulent practices in ancient architecture presents significant challenges in case resolution through risk assessment. The unified technique enables the resolution to solve reliable cases, that is, to be applied to the Lord Shiva Nataraja Sculpture theft of Chola bronze from the Peruvudaiyar Kovil temple, compromising the historical value of artifacts. This finding provides a clear ranking procedure accomplished through the Friedman test using the Statisty software. This result provides both a qualitative and prioritized investigation framework technique and leaves the historical fact unchanged. **Novelty:** An integrated COCOSO method for weight aggregation for decision-making using interval-valued intuitionistic fuzzy vague sets in Multi-Criteria Decision Making (MCDM). The Friedman test was used to rank quality.

Keywords: COCOSO method; Interval value Intuitionistic fuzzy vague set (IVIFVS); Score function; Fuzzy entropy; Friedman test; lindo software

1 Introduction

Awareness of fraud is prevalent in all areas, including social media and finance. It has become extremely difficult to maintain one's own business unless appropriate documentation is preserved. Some outcome technologies, including MCDM, can be used to determine the most appropriate and valid options for lawyers in these situations.

For instance, we could use this technology to address the issue of whether we can achieve the same outcome. In order to do this, we employ the Interval Value Intuitionistic Fuzzy Vague Set (IVIFVS)[1],[2], in which both membership and non-membership will be in intervals for Interval Value Intuitionistic Fuzzy Vague Set. The extension for linguistic variables, IVIFVS[1], the values are simplified using the Lindo software, and after the crisp value is obtained by the score function, it also offers improved data gathering when handling uncertain and imprecise information, with a basic understanding of algebraic propositions and operations. Additionally, the relationship between the vague set and IFS is studied in greater detail using algebraic properties. This study establishes that the integrated COCOSO–IVIFVS technique can be effectively used for authenticating artefacts, particularly in complex idol-theft investigations. The COCOSO method was innovated by Yazdani to combine and compromise the attribute values for a better solution. A detailed numerical example based on a Chola sculpture theft case is constructed to illustrate in the authentication method from the expert opinions into interval-valued intuitionistic fuzzy vague evaluations and processed through the MCDM framework.

The paper is structured as follows: Section 1 introduces the necessary preliminaries on intuitionistic fuzzy sets, vague sets, and IVIFVS, while Section 2 explains the background of the Chola idol theft case and defines the criteria and alternatives used in the model. Section 3 applies the COCOSO–IVIFVS method to this case, computes the ranking of authentication strategies, and then uses the Friedman test to handle differences in rankings and to statistically validate the final order of alternatives, thereby identifying the most reliable strategy for resolving theft.

2 Methodology

2.1 Definitions

D1 Intuitionistic fuzzy value[1]:

An **Intuitionistic fuzzy set (IFS)** I in S is defined as an object of the following form I = {x, μ_I(x), γ_I(x): x ∈ S} where the functions: $\mu_I : S \rightarrow [0,1]$ and $\gamma_I : S \rightarrow [0,1]$. Define the degree of membership and the degree of non-membership of the element x ∈ S, respectively, and for every x ∈ S $0 \leq \mu_I + \gamma_I \leq 1$.

D2 Vague Fuzzy Set[3]:

Let U be a universal set. A vague set A in U is characterized by truth membership $t_{fa}x$ and false membership $f_{fa}x$, then U= $t_{fa}x$, 1 - $f_{fa}x$]where $t_{fa}x \in [1,0]$, $f_{fa}x \in [0,1]$ and $0 \leq t_{fa}x < 1 - f_{fa}x \leq 1$.

D3 Interval Value Intuitionistic Fuzzy Sets

Let X be an ordinary finite non-empty set is given by empty set. An IVIFS Å in X is given by A=$\{< x, \mu_{\mathring{A}}(x), \vartheta_{\mathring{A}}(x)\}$ where $\mu_{\mathring{A}}(x) \subseteq [0,1]$, $\vartheta_{\mathring{A}}(x) \subseteq [0,1]$ are the intervals denoting the degree of membership and non- membership of the element x in the set Å. The **Interval Value Intuitionistic Fuzzy Set** the intervals are denoted as *L* and *U* which is represent as lower and upper of the intervals then IVIFS of Å $is\mathring{A} = \left\{< x, [\mu_{\mathring{A}}(x)^L +, \mu_{\mathring{A}}(x)^U],\ [\vartheta_{\mathring{A}}(x)^L, \vartheta_{\mathring{A}}(x)^U] > / x\ \epsilon X\right\}$ where $0 \leq \mu_{\mathring{A}}(x)^U + \vartheta_{\mathring{A}}(x)^U \leq 1 \leq, \mu_{\mathring{A}}(x)^L \geq 0,\ \vartheta_{\mathring{A}}(x)^L \geq 0$. Besides L and U of membership and non-membership degrees of IVIFS, the L and U for hesitation degree of IVIFS are defined accordingly. Hesitation degree of IVIFS is obtained using

$\pi(x)^U = 1 - \mu_{\mathring{A}}(x)^L - \vartheta_{\mathring{A}}(x)^L$:

$\pi(x)^L = 1 - \mu_{\mathring{A}}(x)^U - \vartheta_{\mathring{A}}(x)^U$.

D4 Interval Value Intuitionistic Fuzzy Vague Sets[1]

An interval-valued intuitionistic fuzzy vague set (IVIFVS) in the universe of discourse X which is characterized by membership, and non-membership, functions is written as IVIFVS

$\mathring{A}_{IVIFVS} = \left\{< x, [\mu_{\mathring{A}}(x)^L +, \mu_{\mathring{A}}(x)^U],\ [\vartheta_{\mathring{A}}(x)^L, \vartheta_{\mathring{A}}(x)^U] > / x\ \epsilon X\right\}$ where $\mu_{\mathring{A}}(x)^L = [\mu_{\mathring{A}}(x)^{L-} +, \mu_{\mathring{A}}(x)^{L+}]$, $\mu_{\mathring{A}}(x)^U = [\mu_{\mathring{A}}(x)^{U-} +, \mu_{\mathring{A}}(x)^{U+}]$ similarly $\vartheta_{\mathring{A}}(x)^L = \left[\vartheta_{\mathring{A}}(x)^{L-} +, \vartheta_{\mathring{A}}(x)^{L+}\right] \vartheta_{\mathring{A}}(x)^U = \left[\vartheta_{\mathring{A}}(x)^{U-}, \vartheta_{\mathring{A}}(x)^{U+}\right]$ The symbols L and U denote the lower and upper of the intervals with the all conditions such that $\vartheta_{\mathring{A}}(x)^{L-} +, \vartheta_{\mathring{A}}(x)^{L+} \leq 1$,

$$\vartheta_{\mathring{A}}(x)^{U-}, \vartheta_{\mathring{A}}(x)^{U+} \leq 1$$

$\mu_{\mathring{A}}(x)^{L-} +, \mu_{\mathring{A}}(x)^{L+} \leq 1$ and

$$\mu_{\mathring{A}}(x)^{U-} +, \mu_{\mathring{A}}(x)^{U+} \leq 1$$

D 5 The Fuzzy entropy formulae for IVIFVS[2]:

$$E\left(c_j\right)=\ \frac{1}{m}\sum_{i=1}^{m}\frac{\min\left(\ddot{\mu}^{l}(x),\ \ddot{\vartheta}^{l}(x)\right)+\ \min\left(\ddot{\mu}^{u}(x),\ \ddot{\vartheta}^{u}(x)\right)+\ddot{\pi}^{l}(x)+\ \ddot{\pi}^{u}(x)}{\max\left(\ddot{\mu}^{l}(x),\ \ddot{\vartheta}^{l}(x)\right)+\ \max\left(\ddot{\mu}^{u}(x),\ \ddot{\vartheta}^{u}(x)\right)+\ddot{\pi}^{l}(x)+\ \ddot{\pi}^{u}(x)}$$

$\ddot{\pi}^{l}(x)=1-\ddot{\mu}^{u}(x)-\ddot{\vartheta}^{u}(x)$

$\ddot{\pi}^{u}(x)=1-\ddot{\mu}^{l}(x),\ \ddot{\vartheta}^{l}(x)$ therefor weight $w_j=\frac{1-\ E(c_j)}{\sum_{j=1}^{n}(1-\ E(c_j))}$

2.2 Methods

2.2.1. *Integrating COCOSO- IVIFVS*[1],[3],[4]

Step 1: collect the detail from expert in term of linguistics

Step 2: convert the linguistics value into IVIFVS using lingo software

Step 3: the midpoint formulae is used to turn the value as crisp

Step 4: apply Fuzzy entropy method from [D 5] to evaluate weight value from IVIFS

Step 5: Cocoso method implementation[4],[5]

The matrix's values are normalized from the previous phase i:e step3 to get crisp values for the alternatives.

$$\text{Eij}=\frac{x_{i\ -min}x_i}{\max x_{ij}-\min x_{ij}} \tag{1}$$

Step 4: Sum and Power for the weight of the matrix are calculated using

$$(C)=\sum_{j=1}^{n}(wj\ teij\ ) \tag{2}$$

$$(J)=\sum_{j=1}^{n}(\ teij\ )^{wj} \tag{3}$$

for the comparability of alternative weights.

Step 5: Three strategies for accumulating assets

$$I1=\frac{C+J}{\sum_{i=1}^{m}(C+J)} \tag{4}$$

$$I2=\frac{C}{\min C}+\frac{J}{\min J} \tag{5}$$

$$I3=\frac{C+\ (1+\lambda\ )(J)}{\max C\ +(1-\max J)}0\leq\lambda\leq 1 for \lambda=0.5 \tag{6}$$

Step 6: Organizing the choices according to the $(\text{N})_i$ Assessment

$$N_i=(I1\times I2\times I3)^{\frac{1}{3}}+\ \frac{1}{3}(I1+I2+3) \tag{7}$$

Step 5: If the Ranking of the alternatives varies use Freidman method.

2.2.2. *Numerical Illustration*

Case Study of Lord Shiva Nataraja Sculpture theft[6],[7],[8],[9]:

In 2014, India successfully raised a righteous cause and won a legal battle to preserve our state's cultural treasures and the legacy of our ancient works. This story centres on the recovery of a bronze statue of Lord Shiva Nataraja, stolen from the Peruvudaiyar Kovil temple in Sripuranthan Village, Tamil Nadu. The saga began on September 30, 2005, when Subhash Kapoor

and Asokan photographed the antique artifact at the Taj Connemara Hotel, Chennai. Kapoor, who ran the ”Art of the Past” gallery in Manhattan, used it to fence stolen artifacts; images were transmitted to him with a timestamp of October 15, 2006, after the theft. He obtained a Handicraft Certificate from the Assistant Director at the Handicraft Development Office, Ministry of Textiles in Chennai, classifying idols as artistic handicrafts for export. Asokan then illicitly exported them via Ever Star International Services Inc., which received the Nataraja idol between January 4 and February 8, 2007. In 2007, Kapoor secured a certificate from the Art Loss Register stating that the Shiva idol was not a stolen property. The theft went unnoticed until August 18, 2008, when efforts to enhance temple security revealed it to the public. Worship had ceased decades earlier, and few villagers visited the site. The case was handed over to the Tamil Nadu Criminal Investigation Department's Idol Wing. In 2011, the German police arrested Kapoor in Frankfurt, triggering a global probe into his hundreds of smuggled idols.

2.2.2.1. Repatriation. In 2013, journalists Jason Felch and Michaela Boland, along with Vijay Kumar of the India Pride Project, highlighted in-situ photos of the idol taken at the temple—first by P.Z. Pattabiraman on June 24, 1961, and later by S. Natarajan on November 23, 1991, both from the French Institute of Pondicherry. They published blogs and articles urging scrutiny of the Nataraja idol at the National Gallery of Australia. Tamil Nadu Police, aided by confessions from Kapoor's manager Aaron Freedman (arrested in the US in 2013), approached the Indian High Commission in Australia. The Commission lodged a formal complaint, backed by photographic evidence, leading to repatriation in 2014 during Prime Minister Narendra Modi's visit, overseen by Australian PM Tony Abbott.

This first international repatriation wins for India in ancient temple artifact theft cases. Lawyer opinions emphasize, the above the technique is used for sorting the theft cases related to artifact in the field of ancient temple's architecture opinion from lawyer to solve and which alternative is used to prove its originality and consider the criteria to be common need, this solved case is an International and first case which India won, criteria as Legal Strength (L) / International Recognition (IR) Accuracy(A), Cost(C), Time(T) and Alternatives for sculpture identification is Provenance Research (PR), Forensic Material Analysis (FM), Artistic & Iconographic Study (AI), AI & Digital Authentication (AD). Table 1 consists of IVIFVS is simplified using lindo software[10] where the weight for the lingo software is considered as default value 0.5 as for the criteria.

Table 1. Interval-valued intuitionistic vague set (IVIFVS) from the expert

	A	C	T	L / IR
PR	**[0.70,0.76][0.17,0.21] [0.21,0.01][0.06, 0.16]**	**[0.70,0.76] [0.17,0.21] [0.21,0.01][0.06, 0.16]**	**[0.30,0.39][0.2,0.42] [0.1, 0.42] 0.08, 0.13]**	**[0.70,0.76] [0.17,0.21] [0.21, 0.01] [0.06, 0.16]**
FM	**[0.70,0.76] [0.17,0.21] [0.21,0.01][0.06, 0.16]**	**[0.49,0.59] [0.22,0.32] [0.12,0.23][0.09, 0.11]**	**[0.70,0.76] [0.17,0.21] [0.21,0.01][0.06, 0.16]**	**[0.68,0.72] [0.16,0.23] [0.02, 0.04][0.05, 0.17]**
AD	**[0.70,0.76] [0.17,0.21] [0.21,0.01][0.06, 0.16]**	**[0.30,0.39] [0.2,0.42] [0.1, 0.42][0.08, 0.13]**	**[0.49,0.59] [0.22,0.32] [0.12,0.23][0.09, 0.11]**	**[0.49,0.59] [0.22,0.32] [0.12, 0.23] [0.09, 0.11]**
AI	**[0.68,0.72] [0.16,0.23] [0.02,0.04][0.05, 0.17]**	**[0.49,0.59] [0.22,0.32] [0.12,0.23][0.09, 0.11]**	**[0.49,0.59] [0.22,0.32] [0.12,0.23][0.09, 0.11]**	**[0.49,0.59] [0.22,0.32] [0.12, 0.23] [0.09, 0.11]**

For the crisp value and the score function applied to the IVIFVS values in Table1

$$L_k = \begin{bmatrix} 0.52 & 0.52 & 0.51 & 0.52 \\ 0.52 & 0.515 & 0.52 & 0.485 \\ 0.52 & 0.51 & 0.515 & 0.515 \\ 0.485 & 0.515 & 0.515 & 0.515 \end{bmatrix}_{4\times4}$$

The COCOSO method of normalization is done for the matrix L_k

$$L_k = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 1 & 0.857143 & 1 & 0 \\ 1 & 0 & 0.5 & 0.5 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{4\times4}$$

From [D5] fuzzy entropy method the weight values are determine

$$w_j = \begin{bmatrix} 0.6 \\ 0.2 \\ 0.16 \\ 0.14 \end{bmatrix}$$

$$(C) = \begin{bmatrix} 1.8 \\ 0.5714 \\ 0.32 \\ 0.42 \end{bmatrix} \quad (J) = \begin{bmatrix} 0.216 \\ 0.01006 \\ 0.0256 \\ 0.42274 \end{bmatrix}$$

$$I1 = \begin{bmatrix} 0.5989 \\ 0.1727 \\ 0.1026 \\ 0.1255 \end{bmatrix} \quad \text{I2} = \begin{bmatrix} 8.4533 \\ 56.7745 \\ 12.58 \\ 153.0678 \end{bmatrix} \quad \text{I3} = \begin{bmatrix} 1.8417 \\ 0.5733 \\ 0.3249 \\ 0.4205 \end{bmatrix}$$

$$\text{N} = \begin{bmatrix} 10.3541 \\ 37.331 \\ 8.3985 \\ 95.6726 \end{bmatrix}$$

3 Result and discussion

For I1, I2, I3, and K, the rank has a non-parametric order. Use the Friedman test to clarify this confusion, where the hypotheses are

Null hypothesis (H_0)	Alternative hypothesis (H_a)
There is no difference between the dependent variables I1, I2, I3 and K.	There is difference among the dependent variables of I1, I2, I3 and K.

Using the Statisty software, the Hypothesis values are analysed in Figure 1.

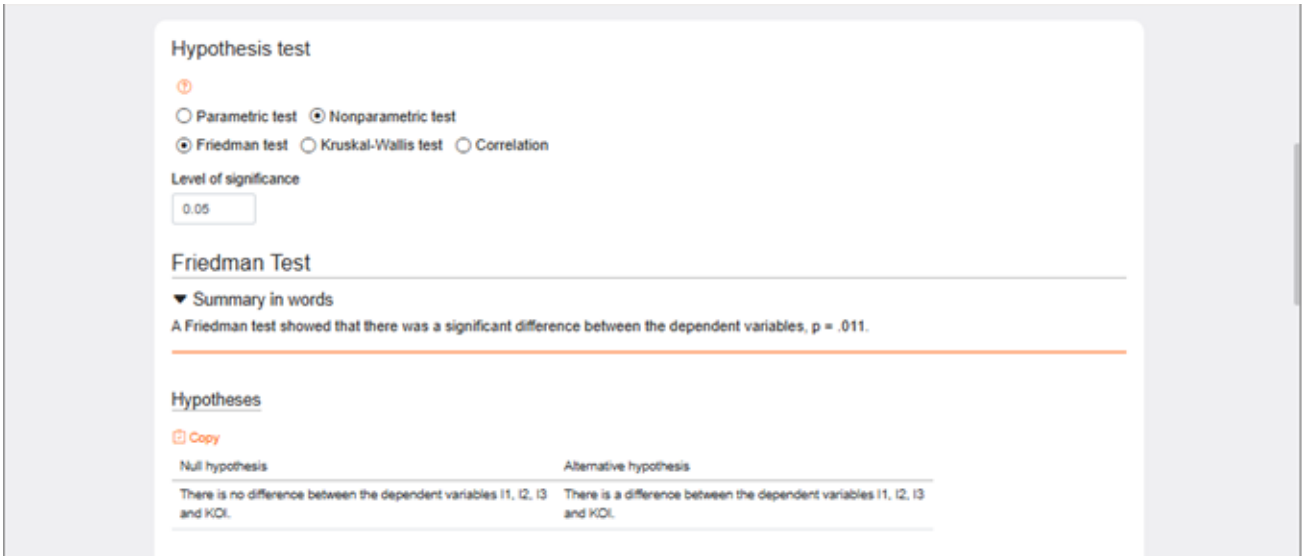

Fig 1. Hypothesis calculation using Statisty Software

Figure 2 consists of bar diagram(a) and violinen plot(b). Therefore, the significance level is (H_0) is rejected and accept(Ha) saying There is difference among the dependent variables of I1, I2, I3 and K. since the rank order of I3and I1 fall the same order and the Quality of order is high, conclude that values of 1>2>4>3 which says PR>FM>AI>AD i:e **Provenance Research (PR)** [ownership record, old photograph], Analysis Report Stand firm, that is evidence is maintained at the French Institute of Pondicherry. Thus, the first victory of our country against the international courtroom.

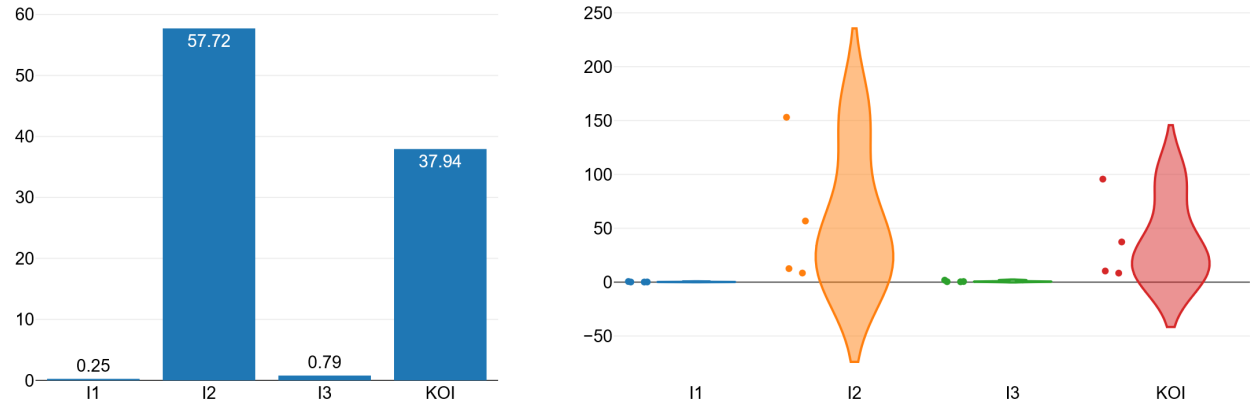

Fig 2. The H_0 and H_α are examined in bar diagram(a) and violinen plot(b)

4 Conclusion

An efficient set of authentication strategies for the Chola sculpture theft case is revealed by applying the integrated technique of COCOSO with Interval Valued Intuitionistic Fuzzy Vague Sets, analyzing the results with the Friedman test, and jointly considering accuracy for cost and time while comparing the alternatives. The Friedman test strengthens the choice of approach to prioritized by resolving the dissimilar ranks of the attributes by confirming the statistical significance for the favoured one. Future idol-theft investigations can use this integrated approach help the investigators and attorneys to choose the best authentication option for each case and make more convincing and quantitatively demonstrated defences in courtroom.

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