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Solutions of Integral Equations by using Successive Approximation Method

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Abstract

Background/Objectives: Integral transforms and iterative techniques play a central role in the theory and solution of integral equations. In this work, generalized special functions, namely Prefunctions and Extended Prefunctions are used to obtain exact and approximate solutions of Volterra integral equations of the second kind. These functions are parameterized by α and extend the analytic structure of exponential, hyperbolic and trigonometric functions. These functions are effective tools in solving differential and integral equations. **Methods**: In this study successive approximations method is used to derive closed-form solutions for two classes of Volterra integral equations with different kernels. For the kernel $K(x,t) = x - t$ with $f\,(x,\alpha) = psin(x,\alpha)$ the iterative sequence yields a generalized solution expressed as a series of shifted Prefunctions, reducing to $y(x,0) = \frac{1}{2}(psinh(x) + sin(x))$ for α =0 confirming the consistency of the method with known classical results. For the convolution kernel $K(x,t) = e^{-(x-s)}$ with $f(x,\alpha) = psinh(x,\alpha)$ the Picard iteration process leads to the compact representation $y(x,\alpha) = psinh(x,\alpha) + sin(x,\alpha+1)$. **Findings**: Numerical illustrations and graphical results confirm uniform convergence of the successive approximations to the exact solution for various values of α. For a large value of α convergence rate is higher. **Novelty:** Prefunction -based methods provide generalized frameworks for solving Volterra integral equations and open further directions for applications in mathematical physics and engineering models.

2020 Mathematics Subject Classification: 45Dxx, 34A12.

Keywords: Integral Equations; Prefunctions; Successive Approximation Method; Convergence; Picard Iterates

1 Introduction

Integral transforms serve as powerful mathematical tools for addressing both differential and integral equations. In recent decades, numerous researchers (1–9,9–11) have successfully employed a variety of transforms for solving such problems. Among them, the Laplace, Sumudu, and Fourier transforms are extensively utilized and hold significant importance in the literature. The theoretical framework of integral equations can be traced back to the pioneering work of J. Fourier (1768–1830), whereas the terminology integral equation was first formally introduced by Du Bois Reymond in 1888. Subsequently, D.C. Sharma et al. [11] have undertaken comprehensive studies on different classes of integral equations, presenting their exact solutions as well as discussing analytical and numerical methodologies for both linear and nonlinear equations. In the framework of generalized special functions, Latpate and Dhaigude [12] introduced the concept of general Prefunctions, Laplace transform of general Prefunctions and its application, which generalize the classical exponential and trigonometric functions by incorporating a real parameter α. For the particular value of the parameter they reduce to standard ones. The presence parameter α permits Prefunctions extensive physical and mathematical phenomenon, which cannot be perfectly expressed by classical functions by oneself. These functions are constructed using power series with the Gamma function in the denominator, thereby extending the analytic structure of standard functions and offering broader applicability in solving differential and integral equations. Dhaigude et al [13] have studied the application of Prefunctions and differential equations via sumudu transform. The solutions of Volterra integral equations using prefunction are obtained by Dhaigude [14] . . Khandeparkar et al [15] have studied system of differential equations via Laplace transform incorporating prefunctions. Mane *et al* [16] have studied some integral equations and Prefunctions using Shehu Transform. Thirumalai *et al* [17] have studied Prefunctions of complex variables.

The aim of this papers is to figure out using the new special functions, namely Prefunctions in Volterra integral equations as known function $f(x) = psin(x, \alpha)$ and $f(x) = psinh(x, \alpha)$ for different kernels and also study their solution, convergence with its exact solution by method of successive approximation for different values of α. The successive approximation approach is a useful analytical methodology for obtaining exact or approximate solutions to integral equations, particularly Volterra and Fredholm integral equations of second-order when paired with Prefunctions, this approach provides an extremely effective tool for solving integral equations incorporating generalized kernels and nonlinear behaviors. The introduction of Prefunction in the integral equation provides approximate and exact solution, handle nonlinear kernels, convergence is faster than the classical methods, reduces computational complexity.

This study shows that confirmation across different values of α illustrated the convergence behaviour of the iterative scheme and highlighted the strength of the Prefunction approach.

The paper is organised as follows, in section 2, we discuss the methodology. Section 3 is devoted for the result and discussion. Finally we discuss the conclusion in Section 4.

2 Methodology

PRELIMINARIES:

2.1 Definition and Basic Concept

A Volterra integral equation is an integral equation in which the upper limit of integration is the independent variable itself:

$$u(x) = f(x) + \lambda \int_a^x K(x,t)u(t)dt \tag{1}$$

or equivalently, $f(x) = \int_a^x K(x,t)u(t)dt$

The defining feature is that the integral limit depends on x typically (a $\leq$ t $\leq$ x). This structure reflects causality and memory in physical systems: the state u(x) depends only on past values u(t), (t $\leq$ x).

Volterra equations are widely used in: Hereditary mechanics and viscoelasticity, Population dynamics, Heat conduction with memory, Control systems with delay, Neutron transport and diffusion processes

2.2 Classification

The Volterra integral equations are classified as

(a) First Kind Volterra Equation

$$\int_a^x K(x,t)u(t)dt = f(x) \tag{2}$$

where $u(t)$ is unknown function and inside the integral only.

(b) Second Kind Volterra Equation

$$u(x) = f(x) + \lambda \int_a^x K(x,t)u(t)dt \tag{3}$$

Most common and well-posed under mild conditions. The kernel acts as a memory function. Existence and uniqueness follow from Volterra–Picard successive approximations.

(c) Third Kind Volterra Equation:

$$a(x)u(x) = f(x) + \lambda \int_a^x K(x,t)u(t)dt \tag{4}$$

where $a(x)$ may vanish or change sign.

Mixes first-kind and second-kind behaviour (degenerate near zeros of $a(x)$).

2.3 Theoretical Background

Existence and Uniqueness **(Volterra–Picard Theorem)**

For the second-kind equation

$$u(x) = f(x) + \lambda \int_a^x K(x,t)u(t)dt \tag{5}$$

if $K(x,t)$ and $f(x)$ are continuous on (a ≤ t ≤ x ≤ b), then there exists a unique continuous solution $u(x)$.

The solution can be obtained via successive approximations:

$$u_{n+1}(x) = f(x) + \lambda \int_a^x K(x,t)u_n(t)dt, \quad u_0 = f(x) \tag{6}$$

The series $(u(x) = \lim_{n\to\infty} u_n(x))$ converges uniformly on $[a,b]$

2.4 Prefunctions and Extended Prefunctions

The pre-exponential function is given by[13,14]

$$pexp(t,\alpha) = 1 + \sum_{n=0}^{\infty} \frac{t^{n+\alpha}}{\Gamma(n+1+\alpha)} \tag{7}$$

where $t \in R$ and $\alpha \geq 0$.

For each fixed α, this series represents a generalized form of the classical exponential function, termed as the prefunction of t.

The pre-trigonometric functions extend the usual sine and cosine functions and are defined as follows[13,14]

$$psin(t,\alpha) = \sum_{n=0}^{\infty} \frac{(-1)^n\, t^{2n+1+\alpha}}{\Gamma(2n+2+\alpha)}\ , \ \alpha \geq 0,\ t \in \mathbb{R} \tag{8}$$

$$pcos(t,\alpha) = 1 + \sum_{n=0}^{\infty} \frac{(-1)^n\, t^{2n+\alpha}}{\Gamma(2n+1+\alpha)}\ , \ \alpha \geq 0,\ t \in \mathbb{R} \tag{9}$$

when $\alpha = 0$, these functions reduce to the classical trigonometric functions:

$$psin(t,0) = sin(t), \quad pcos(t,0) = cos(t)$$

In these definitions, the introduction of the parameter α provides an additional degree of generality, which allows these functions to bridge the gap between classical trigonometric functions and their generalized analogues.

The Extended Prefunction is defined by[13,14]

$$p\, M_{3,2}(t,\alpha) = \sum_{n=0}^{\infty} \frac{(-1)^n\, t^{3n+2+\alpha}}{\Gamma(3n+3+\alpha)},\ \alpha \geq 0,\ t \in \mathbb{R} \tag{10}$$

This extended version incorporates higher-order terms and offers new possibilities in the study of differential and integral equations. Prefunctions and their extended forms are particularly useful in obtaining generalized solutions of ordinary and partial differential equations, and they serve as powerful tools in mathematical physics and engineering models where classical functions may not provide sufficient generality.

To solve the integral equations involving Prefunctions, we use a successive approximation method. It is an iterative procedure for obtaining approximate (and often exact) solutions to integral equations and differential equations. It is particularly useful in the context of Volterra and Fredholm integral equations of the second kind.

2.5 Method

2.5.1. General Form of Integral Equation

Consider the Volterra integral equation of the second kind:

$$y(x) = f(x) + \lambda \int_a^x K(x,t)y(t)dt \tag{11}$$

where $f(x)$ is a known function, $K(x,t)$ is the kernel of the integral equation, λ is a parameter, and $y(x)$is the unknown function to be determined.

2.5.2. Basic Idea

We construct a sequence of approximations $\{y_n(x)\}$ such that

$$y(x) = \lim_{n\to\infty} y_n(x) \tag{12}$$

The first approximation is chosen as the free term:

$$y_0(x) = f(x).$$

Then, successive approximations are generated by substituting the previous approximation into the integral equation. The general term of the series reads as

$$y_{n+1}(x) = f(x) + \lambda \int_a^x K(x,t)y_n(t)dt,\ n = 0,1,2,\ldots.. \tag{13}$$

2.5.3. Convergence

If the kernel $K(x,t)$ and the parameter λ satisfy certain conditions, such as Boundedness of the kernel and smallness of λ, the sequence $\{y_n(x)\}$ converges uniformly to the exact solution $y(x)$.

3 Results and Discussion

In this section we solve the examples of Volterra integral equation and obtain the solutions in terms of Prefunctions and discuss the results with numerically as well as graphically and

also check the convergence of the solution.

Ex. 3.1: We consider the Volterra integral equation of the form:

$$y(x) = f(x,\alpha) + \int_0^x (x-t)y(t)dt \tag{14}$$

Here $K(x,t) = x-t$, λ=1, $f(x,\alpha) = psin(x,\alpha)$.

Now, we use Successive approximation method to solve Eq. (14)

Step1: The initial approximation is

$$y_0(x,\alpha) = \sum_{n=0}^{\infty} \frac{(-1)^n\, x^{2n+1+\alpha}}{\Gamma(2n+2+\alpha)}\ ,\ \alpha \geq 0 \tag{15}$$

Step2: The first iteration is

$$\begin{aligned} y_1(x,\alpha) &= f(x) + \int_0^x (x-t)y_0(t,\alpha)dt \\ &= psin(x,\alpha) + \int_0^x (x-t)\left(\sum_{n=0}^{\infty} \frac{(-1)^n\, t^{2n+1+\alpha}}{\Gamma(2n+2+\alpha)}\right)dt \end{aligned} \tag{16}$$

By interchanging sum and integral in Eq. (16), the first iteration reduces to

$$y_1(x,\alpha) = psin(x,\alpha) + \sum_{n=0}^{\infty} \frac{(-1)^n}{\Gamma(2n+2+\alpha)} \int_0^x (x-t)t^{2n+\alpha}\, dt \tag{17}$$

Evaluating the integral of Eq. (17) and substituting the required terms, we obtain

$$y_1(x,\alpha) = \sum_{n=0}^{\infty} \frac{(-1)^n\, x^{2n+1+\alpha}}{\Gamma(2n+2+\alpha)} + \sum_{n=0}^{\infty} \frac{(-1)^n\, x^{2n+3+\alpha}}{\Gamma(2n+4+\alpha)} \tag{18}$$

Eq. (18) can be expressed as

$$y_1(x,\alpha) = psin(x,\alpha) + psin(x,\alpha+2) \tag{19}$$

Similarly, we find out the next iteration as

$$y_2(x,\alpha) = psin(x,\alpha) + psin(x,\alpha+2) + psin(x,\alpha+4) \tag{20}$$

and

$$y_3(x,\alpha) = psin(x,\alpha) + psin(x,\alpha+2) + psin(x,\alpha+4) + +psin(x,\alpha+6) \tag{21}$$

By induction the general iteration reads as

$$y(x,\alpha) = \sum_{k=0}^{\infty} psin(x,\alpha+2k) \tag{22}$$

where

$$psin(x,\beta) = \sum_{n=0}^{\infty} \frac{(-1)^n\, x^{2n+1+\beta}}{\Gamma(2n+2+\beta)}$$

Eq. (22) represents the closed-form of the solution using the method of successive approximations.

Now we specialize the general result to the case $\alpha = 0$ in equation (22).

For $\alpha = 0$, we have

$$y(x,0) = \sum_{k=0}^{\infty} psin(x,2k) = psin(x,0) + psin(x,2) + psin(x,4) + \ldots\ldots \tag{23}$$

Expanding the first few terms for $\beta = 0,$ we get

$$psin(x,0) = \sum_{n=0}^{\infty} \frac{(-1)^n\, x^{2n+1}}{\Gamma(2n+2)} = \sum_{n=0}^{\infty} \frac{(-1)^n\, x^{2n+1}}{(2n+1)!} = sin(x) \tag{24}$$

$$psin(x,2) = \sum_{n=0}^{\infty} \frac{(-1)^n\, x^{2n+3}}{\Gamma(2n+4)} = \sum_{n=0}^{\infty} \frac{(-1)^n\, x^{2n+3}}{(2n+3)!} \tag{25}$$

Eq. (25) is exactly the Taylor expansion of sin(x) with the first term removed (starting from x^3).

For $\beta = 4$:

$$psin(x,4) = \sum_{n=0}^{\infty} \frac{(-1)^n\, x^{2n+5}}{\Gamma(2n+6)} = \sum_{n=0}^{\infty} \frac{(-1)^n\, x^{2n+5}}{(2n+5)!} \tag{26}$$

Continuing in this way, the series (23) becomes

$$y(x) = x + \frac{x^5}{5!} + \frac{x^9}{9!} + \ldots \tag{27}$$

Eq. (27) can be written as

$$y(x) = \frac{1}{2}\left[\sinh(x) + sin(x)\right] \tag{28}$$

Eq. (28) is the final result (for $\alpha = 0$), which matches the exact solution directly when $f(x) = sin(x)$.

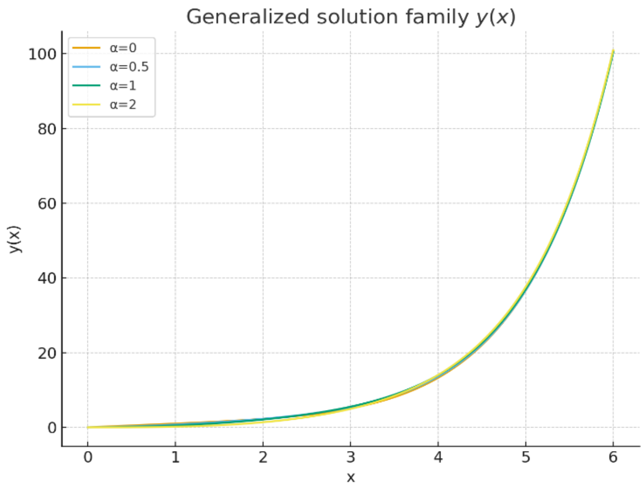

Fig 1. Family of curves for different value of α

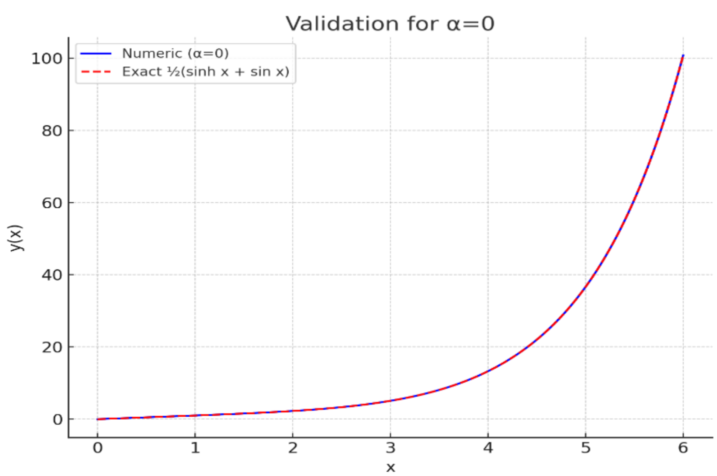

Fig 2. Validation for $\alpha = 0$ vs. closed form $\frac{1}{2}(sinhx + sinx)$

3.1 Graphical Representation for Convergence of Solution

Here, we discuss the graphical representation for the convergence of solution given by Eq. (22) and Eq. (23).

Figure 1 represent the Family of curves for different α. From Figures 2, 3 and 4, we observed that as α increase the rate of convergences of the series improves.

Ex. 3.2: Solve the second kind Volterra equation of the form

$$y(x) = f(x,\alpha) + \int_0^x K(x,s)y(s)ds \tag{29}$$

with $K(x,s) = e^{-(x-s)}$ and $f(x,\alpha) = psinh(x,\alpha) = \sum_{n=0}^{\infty} \frac{x^{2n+1+\alpha}}{\Gamma(2n+2+\alpha)}$ this Volterra equation is a standard convolution-type problem and the successive-approximation (Picard) method gives a very clean closed form. We will show the Picard iteration, prove convergence, and give the final explicit solution for power-series $psinh(x,\alpha)$.

The solution of problem by Picard (successive-approximation) iterations

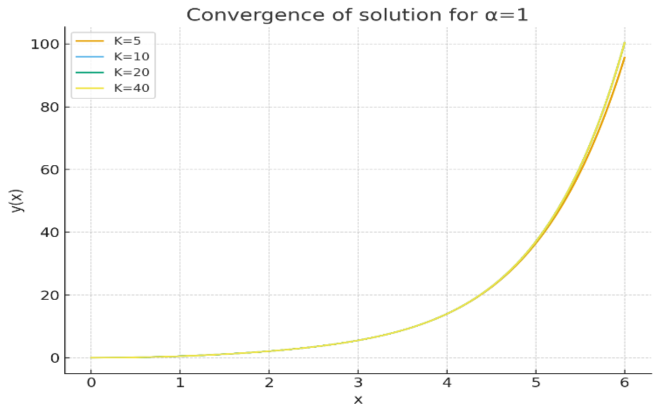

Fig 3. Convergence for $\alpha = 1$ as K increases

The Volterra integral Eq. (29) Becomes

$$y(x,\alpha) = \sum_{n=0}^{\infty} \frac{x^{2n+1+\alpha}}{\Gamma(2n+2+\alpha)} + \int_0^x K(x,s)y(s,\alpha)ds \tag{31}$$

Step 1: Picard iteration scheme

Define:

$$y_0(x) = f(x),\ \ y_{m+1}(x) = f(x) + \int_0^x e^{-(x-s)}\, y_m(s)ds\ \ ,\ m \geq 0 \tag{32}$$

Write the kernel as with $K(t) = e^{-t}$ for t $\geq$ 0. Then the iterates expand as

$$y_1 = f + k*f\ ,\ y_2 = f + k*f + k*k*f\ ,\ \ldots,\ ,\ y_m = \sum_{j=0}^{m} k^{*j} * f \tag{33}$$

where k^{*j} is the j-fold convolution of k with itself and $k^{*0} * f = f$.

Step 2: Compute the convolution powers of k

For $K(t) = e^{-t}$ (t $\geq$ 0)

$$k^{*n}(t) = \frac{t^{n-1}\, e^{-t}}{(n-1)!}\ ,\ n \geq 1 \tag{34}$$

Hence

$$\sum_{n=1}^{\infty} k^{*n}(t) = 1 \tag{35}$$

Step 3: Limit of the Picard sequence (Neumann series)

Let $y = \lim_{m\to\infty} y_m$, the equation (33) becomes

$$y(x) = \sum_{j=0}^{\infty} k^{*j} * f(x) = f(x) + \sum_{j=0}^{\infty} \int_0^x k^{*j}(x-s)f(s)ds \tag{36}$$

By using the computed sum of convolution kernels, we get

$$\sum_{j=0}^{\infty} k^{*j}\ (x-s) = 1 \tag{37}$$

Using Eq. (37) in Eq. (36) the closed form of solution is

$$y(x,\alpha) = f(x,\alpha) + \int_0^x f(s,\alpha)ds \tag{38}$$

Step 4: substitute $f(x,\alpha) = psinh(x,\alpha)$

Integrate f term-wise for each n, we get (39)

$$\int_0^x f(s,\alpha)ds = \sum_{n=0}^{\infty} \frac{x^{2n+2+\alpha}}{\Gamma(3n+3+\alpha)}$$

The Picard iterates (33) are for

$$f(s,\ \alpha) = psin(s,\alpha) = \sum_{n=0}^{\infty} a_n s^{2n+1+\alpha} \quad , \ a_n = \frac{1}{\Gamma(2n+2+\alpha)} \tag{40}$$

we get

$$y_0(x,\ \alpha) = f(x,\ \alpha) = \sum_{n=0}^{\infty} a_n x^{2n+1+\alpha} \tag{41}$$

$$y_1(x,\ \alpha) = \sum_{n=0}^{\infty} a_n x^{2n+1+\alpha} + e^{-x} \sum_{n=0}^{\infty} \sum_{p=0}^{\infty} b_{n,p} x^{2n+2+\alpha+p} \tag{42}$$

where $b_{n,p} = \frac{\Gamma(2n\,+\,2\,+\,\alpha\,+\,p)}{\Gamma(2n\,+\,2\,+\,\alpha\,)\,p!\,\Gamma(2n\,+\,3\,+\,\alpha\,+\,p)}$

$$y_2(x,\ \alpha) = \sum_{n=0}^{\infty} a_n x^{2n+1+\alpha} + e^{-x} \sum_{n,p\geq 0} A_{n,p}^{(1)} x^{2n+2+\alpha+p} + e^{-x} \sum_{n,p\geq 0} A_{n,p}^{(2)} x^{2n+3+\alpha+p} \tag{43}$$

where $A_{n,p}^{(1)} = b_{n,p}$ from the $k * f$ term and

$$A_{n,p}^{(2)} = \frac{\Gamma(2n+2+\ \alpha+p)}{\Gamma(2n+2+\alpha\ )\,p!\,\Gamma(2n+4+\alpha+p)}$$

General $y_m(x,\ \alpha)$ for j ≥ 1,

$$k^{*j} * f = \int_0^x \frac{(x-s)^{j-1} e^{-(x-s)}}{(j-1)!} f(s)ds = e^{-x} \sum_{n=0}^{\infty} \sum_{p=0}^{\infty} C_{n,j,p} x^{2n+j+1+\alpha+p} \tag{44}$$

where

$$C_{n,j,p} = \frac{\Gamma(2n+2+\alpha+p)}{p!\,\Gamma(2n+2+\alpha)\ \Gamma(2n+j+2+\alpha+p)}$$

Hence for finite m, we get

$$y_m(x,\ \ \alpha) = \sum_{n=0}^{\infty} a_n x^{2n+1+\alpha} + e^{-x} \sum_{j=0}^{m} \sum_{n=0}^{\infty} \sum_{p=0}^{\infty} C_{n,j,p} x^{2n+j+1+\alpha+p} \tag{45}$$

This is the explicit triple-sum power series representation of the m^{th} Picard iterate

From these iterations we get the solution

$$y(x,\ \alpha) = \lim_{m\to\infty} y_m(x,\ \alpha) = \sum_{n=0}^{\infty} \frac{x^{2n+1+\alpha}}{\Gamma(2n+2+\alpha)} + \sum_{n=0}^{\infty} \frac{x^{2n+2+\alpha}}{\Gamma(2n+3+\alpha)}$$

$$y(x,\ \alpha)\ = psinh(x,\alpha) + psin(x,\alpha+1)$$

The Picard iterates y_m are partial sums $y_m = \sum_{j=0}^{m} k^{*j} * f$ and converge on to the closed form above because the Neumann series for this Volterra operator converges.

Various author[18–20] studied successive approximation method for the numerical solution of linear and nonlinear integral equations.

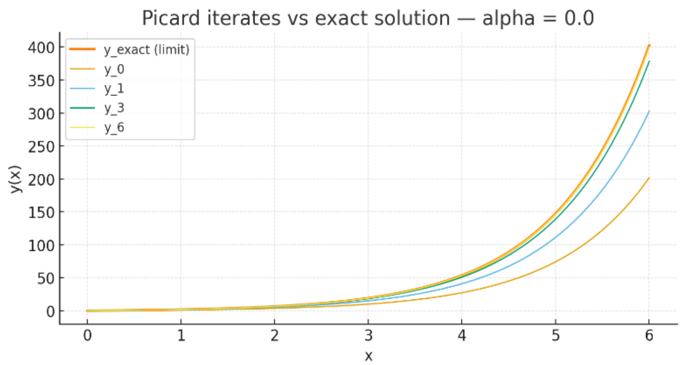

Fig 4. Picard's iterates vs. exact Solution for α = 0.0

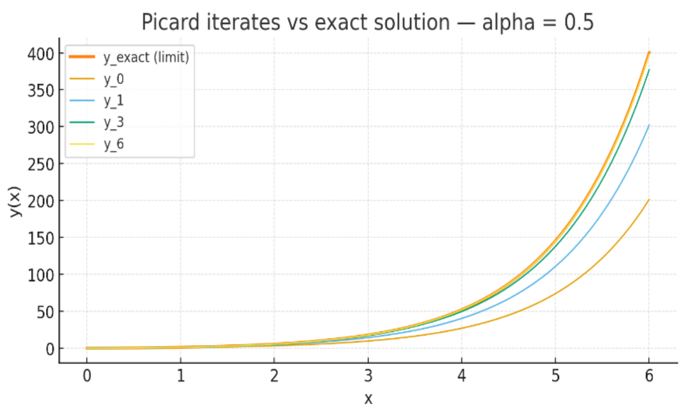

Fig 5. Picard's iterates vs. exact Solution for α = 0.5

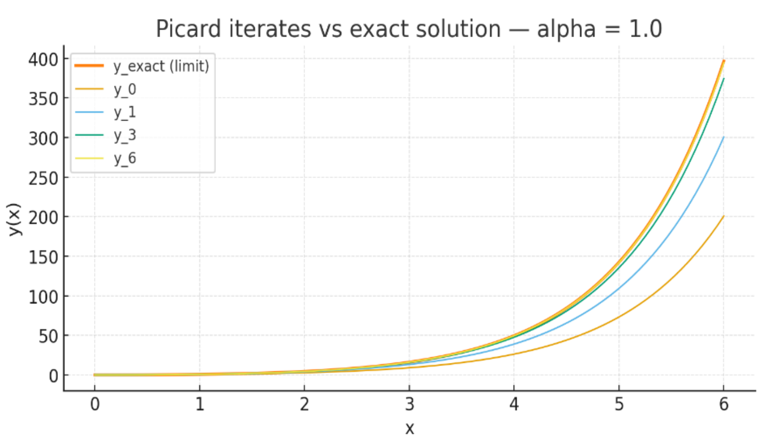

Fig 6. Picard's iterates vs. exact Solution for α = 1.0

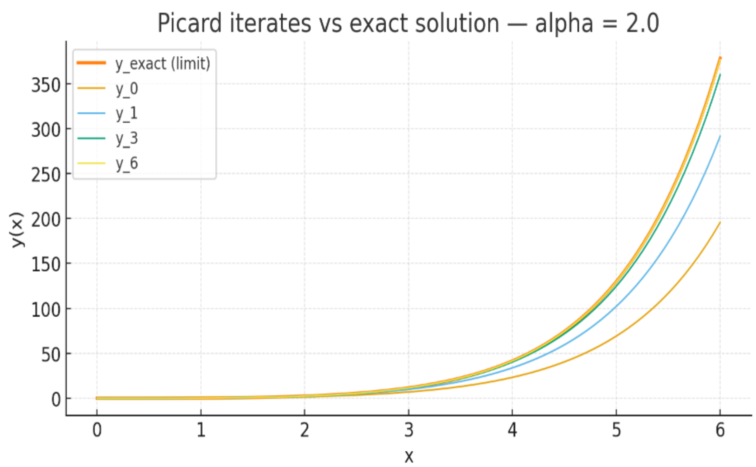

Fig 7. Picard's iterates vs. exact Solution for α = 2.0

3.2 Graphical Representation of Picard iterates vs. the exact solution and displayed convergence for:

1. For α = 0.0 *Fig.*4
2. For α = 0.5 *Fig.*5
3. For α = 1.0 *Fig.*6
4. For α = 2.0 *Fig.*7

Figures 4, 5, 6 and 7, show the Picard's iterates versus exact solution for various values of α. It is observed that for large number of iterations the solution matches with exact solution.

4 Conclusion

This study has demonstrated the applicability of Prefunctions and Extended Prefunctions in obtaining generalized solutions of Volterra integral equations of the second kind using the method of successive approximations. The incorporation of the parameter α into pre-exponential and pre-trigonometric functions provides a unifying framework that bridges classical functions with their generalized analogues. For the kernel $K(x,t) = x - t$ with $f(x,\ \alpha) = psin(x,\alpha)$, the successive approximation method generated a well-designed closed form representation as an infinite sum of shifted Prefunctions. Specialization to α = 0 led to the exact solution $y(x,0) = \frac{1}{2}(psinh(x) + sin(x))$, confirming the consistency of the method with known classical results. For the convolution kernel $K(x,t) = e^{-(x-s)}$ with $f(x,\ \alpha) = psinh(x,\alpha)$, the Picard iteration process provides a compact solution $y(x,\ \alpha) = psinh(x,\alpha) + sin(x,\alpha+1)$. Graphical validation across different values of α illustrated the convergence behavior of the iterative scheme and highlighted the robustness of the Prefunction approach. The methodology not only ensures theoretical rigor but also extends the practical toolkit for solving a wider class of Volterra equations, particularly in mathematical physics and engineering contexts where memory-dependent processes are modelled. Future work may focus on extending this framework to nonlinear integral equations, integro-differential systems, and numerical algorithms that exploit the analytic structure of Prefunctions for computational efficiency. It is observed that the approximation approach is successfully employed for obtaining exact or approximate solutions to integral equations, particularly Volterra and Fredholm integral equations of second-order when paired with Prefunctions,. The introduction of Prefunctions in the integral equation provides approximate and exact solution, handle nonlinear kernels, convergence. Further investigation will focus on improving the theoretical framework and exploring its applicability to a wider class of real problems in mathematical physics and engineering models. Extending the prefunction to generate Mittag –Leffler type functions with additional parameter which is broadly applicable in mathematical modelling.

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