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REVIEW ARTICLE

Non-Local Reality with No-Signalling

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Abstract

Objectives: To derive assumptions of Bell Inequalities from one single postulate – Absolute Postulate of Universe – as mentioned by the author S.Bagaria. **Method**: It focuses on the assumptions – locality, reality, measurement independence and counter-factual definiteness - made in proving Bell's Inequality and derive them from Absolute Postulate of the Universe. Then, its consequences are studied like PR Boxes being real, DI-QKD, quantum information theory, randomness and foundations of quantum mechanics. **Findings:** From Absolute Postulate of the Universe, it has been showed that there is absolute determinism in world, resulting in zero theoretical uncertainty. Thus, the algebraically maximal correlations (S = 4) are possible. Thus, the PR-boxes can be real. But, Tsirelson Bound can't be violated in quantum computing as HUP can't be violated in Quantum Physics. **Novelty**: The four assumptions of Bell Inequality are derived from Absolute Postulate of the Universe. The correlations in the universe can be maximal yet unverifiable experimentally. EPR paradox is resolved without assuming faster than light transfer of information as the world is shown to be "non-local with no-signalling".

Keywords: Bell inequality; Popescu–Rohrlich box; Tsirelson bound; Quantum nonlocality; No-signalling; CHSH inequality; Quantum violation; absolute determinism

1 Introduction

Einstein believed that Physics should satisfy: a) Realism – Physical properties exist before measurement. b) Locality – No influence can travel faster than light. Einstein *et al.*[(1)] argued that if QM is complete, then measuring one particle can instantly determine the state of another distant particle. Schrodinger[(2)] observed that entanglement is a "characteristic trait of QM". Bohr continued to defend QM till his last breath. Bell[(3)] proved mathematically that "Any local hidden variable theory must satisfy certain inequalities". Clauser et al.[(4)] standardized the Bell Inequalities. A simplified argument is given in the Appendix A of this paper for curious readers. Brunner[(5)] provides a comprehensive overview of Bell nonlocality, showing that quantum predictions cannot always be explained by local hidden-variable theories. Clauser and Shimony[(6)] experimentally tested these inequalities and observed the violations consistent with the prediction of quantum mechanics. QM predicts violation of at least one of these assumptions. The experiments showed that (1) Bell inequality violation is true, (2)

Quantum Entanglement also exists, and (3) the experiments are incompatible with hidden variable theories.

In [7], experimentally showed loophole-free violation of Bell's inequality using superconducting qubits. The main aim is to get non-local correlations predicted by quantum mechanics experimentally without the loopholes that can weaken the Bell test. The Bell violation obtained tells that the correlation cannot be explained by local causality. In [8] detected indefinite causal order directly from the observed correlation instead of causal witnesses. However, the author mentioned that the experiment is not device-independent or loophole-free. The study [9], violates the Bell inequality from quantum indistinguishability and path identity without entanglement. The authors mention that source information never exists when the paths are completely indistinguishable, which allows different probability amplitudes to interfere. In [10], experimentally demonstrated a high-dimensional Bell violation with loophole being genuinely closed. It showed that increasing the dimensionality of entanglement can make loophole-free Bell tests experimentally more feasible.

If S = E (a, b) + E (a, b') + E (a', b)- E (a', b').

1. Classical Bell Limit – If outcomes are pre-determined local values: |S| <= 2. This is the Bell Inequality
2. Quantum Limit (Tsirelson Bound – see Appendix B for proof): Quantum Entangled States can reach |S| <= $2\sqrt{2}$ Eg, Singlet State with measurement angles differing by 45 degrees
3. Logical Algebraic Maximum: Since each of the 4 terms can be max 1, |S| <= 4, but ordinary QM never reaches that

This study evaluates all four core assumptions used in deriving Bell Inequalities. And uses the Absolute Postulate of the Universe [11] to argue that the universe is non-local with no signalling. It advocates that all the four assumptions – reality, locality, measurement independence and counter-factual definiteness can be derived from absolute determinism – in accordance with Absolute Postulate of the Universe [11]. Also, this study argues that the universe violates the classical and quantum world-limit of correlations algebraically maximally. This study then discusses consequences like PR boxes being real, DI-QKD, quantum information theory, quantum randomness and implications for foundations of quantum mechanics.

2 Review Methodology

Einstein-Bohr Debates – Since the early days of quantum mechanics, the physical meaning of quantum mechanics has been debated. Einstein, whose explanation of photoelectric effect was one of the key contributions to quantum mechanics, believed that QM is incomplete. Reality must be deterministic. Bohr, on the other hand, advocated that as QM explains all the quantum experiments well, it is the "real" thing even if outcomes of an experiment are probabilistic only.

EPR Argument and Bell Inequalities:

EPR Paradox challenges Copenhagen Interpretation. It has been observed experimentally that classical limit of correlations (S=2) is violated. Present day QM allows such violations up to (S = $2\sqrt{2}$). But that violation implies instantaneous (faster than light) transfer of information about quantum states from Alice to Bob. But faster than light travel - violates Einstein's relativity. That is the essence of EPR Paradox.

Absolute Postulate of Universe [11]:

If position and momentum of all the particles at all the instants is fixed, the universe is completely determined.

Absolute determinism:

Laplace's Demon [12]: $d^2r/dt^2 = F(r)$ with $r(t_0)$ as position r_0 and $(dr/dt)(t_0)$ as velocity v_0 are known for all particles. Then the past, present, and future can be precisely and accurately calculated.

Heisenberg Uncertainty Principle:

Heisenberg (German [13] and in English [14]) showed that the position and momentum of a particle cannot be measured accurately simultaneously. $\Delta x \bullet \Delta p \geq \hbar/2$.

Re-interpretation of HUP:

Bagaria [11] argues that even if precise measurement/ calculation is not possible, the particle has a definite position and momentum at all time.

Laplace's Demon in light of Re-interpreted HUP:

If position and momentum of all the particles at any one instant and the laws of physics (in particular Newton's Second Law) is known, the position and momentum of any particle at any other instant in the Universe is precisely determined – even if incalculable/ immeasurable – due to HUP and large number of particles involved.

Redefining Assumptions of Bell Inequalities:

1. Reality

In present day QM, the physical properties that can't be measured/ calculated are not part of reality. Bagaria's[11] study makes a distinction between reality and measurement. e.g., even if the position and momentum of a particle can't be measured/ calculated simultaneously, at any instant, they have definite and fixed values.

2. Locality

   Einstein's relativity implies that nothing can travel faster than light. Classical correlations can maximally be S = 2. Absolute Postulate of the Universe[11] shows absolute determinism (meaning universe is fixed) signals need not travel from Alice's location to Bob's location instantaneously as the physical properties of the particles, detector settings and the outcomes of the experiment(s) are fixed since inception of the universe which makes universe non-local and relativity holds.

   Jonathan Oppenheim and Stephanie Wehner[15] questioned why quantum mechanics is not maximally non-local stop at $2\sqrt{2}$(Tsirelson bound) not algebraic maximum (S = 4) and it finds out that uncertainty[16] restricts quantum correlation bound from going beyond $2\sqrt{2}$. By absolute determinism[11] (meaning uncertainty = 0) the universe is maximally non-local and algebraic maximal (S=4) correlation is possible.

3. Measurement Independence

   Measurement Independence means freedom of choice of detector settings by experimenter (or random number generator) Bagaria's[11] paper suggests that the universe is absolutely deterministic. Thus, all choices of experimenters, detector settings, outcomes of experiments are determined and fixed. So, measurement independence is not true but can be used for measurements/ calculations. Also, as the universe is determined, outcomes of random number generators are theoretically deterministic.

4. Counter-factual Definiteness

   Counter-factual means all outcomes for all possible detector settings have definite values, even for measurements that were not actually performed. Bagaria's[11] paper assumes that the world is completely deterministic and there is no alternative outcome to an experiment. But we can continue to assume counter-factualness for our experiments/ calculations.

Consequences of Revised Bell Inequalities:

1. PR Boxes[17] – For PR-Boxes to be "real", uncertainty must be zero so that correlations are algebraically maximal (s = 4), and it must satisfy PR correlation $a \oplus b = x \cdot y$ (see Appendix C for details). PR-Boxes don't assume instantaneous (or even faster -than light) travel. Relativity holds.
2. DI-QKD[18] – Device Independent Quantum Key Distribution – To establish a secure channel between Alice and Bob, without trusting the devices and relying solely on outcomes, quantum violations may be observed. If Eve eavesdrops, correlation falls to classical limit (S=2) and channel is not secure. It may seem from this study that DI-QKD needs to be modified, but that is not so because this study claiming S = 4 is possible but it cannot be experimentally verified directly because of Tsirelson Bound S = $2\sqrt{2}$.
3. Quantum Information Theory

   When uncertainty = 0, the information is complete. There is no error and the observer can calculate information precisely – in principle. It remains incalculable and immeasurable precisely in reality in accordance with well-established classical/ quantum limits as per experiment in question.

4. Randomness Certification

   Randomness relying on physical devices (classical) or quantum measurement (present day QM) are not truly random because all outcomes are pre-determined since inception of universe. But for most experiments, TRNG is not required and present-day systems suffice.

5. Foundations of QM

   This study highlights the distinction between reality (PR-Box) and what can be measured/ calculated (Tsirelson Bound of $2\sqrt{2}$). So, both Einstein's relativity and QM continue to hold without affecting the present-day experiments and most theory.

3 Overview

Table 1 lists the various interpretations of QM that have been used previously to address the various assumptions made in derivations of Bell Inequalities. This study derives all four assumptions of Bell Inequalities from absolute determinism – a direct

Table 1. Assumptions of Bell Inequalities Vs Various QM Interpretations

Interpretation	Assumption Addressed
Copenhagen interpretation	Realism
Bohmian mechanics	Locality
Many-worlds interpretation	Single definite outcomes / counterfactual definiteness
Superdeterminism	Measurement independence

consequence of Absolute Postulate of Universe [11]. The postulate and some of its consequences like absolute determinism are discussed in this paper [11]. Though a mathematical framework still needs to be developed and is in process. Table 2 sums up the literature contributions – starting from EPR Argument, Bell Inequalities, CHSH, Important Experiments and up to Bagaria's paper; Table 3 highlights the current research gaps, in particular, experiments required to test various aspects of the theory that have been proposed here and those in Bagaria's paper [11].

Table 2. Literature-summary

Period	Reference	Major contribution
1935	Einstein, Podolsky & Rosen [1]	Introduced the EPR paradox and questioned the completeness of quantum mechanics.
1935	Schrödinger [2]	Detailed concept of quantum entanglement.
1964	Bell [3]	Derived Bell's theorem which showed incompatibility between local hidden-variable theories and quantum.
1969	Clauser *et al.* [4]	Introduced the CHSH inequality for analysing the Bell theorem.
1978	Clauser & Shimony [6]	Reviewed experimental tests and effects of Bell's theorem.
1991	Ekert [19]	Introduced Bell correlations for quantum cryptography.
1994	Popescu & Rohrlich [17]	Introduced algebraic Bell limit for PR-boxes correlations such that no-signalling is satisfied.
2014	Brunner et al. [5]	Provided a comprehensive review of Bell nonlocality and Bell inequalities.
2022	Hu *et al.* [10]	High-dimensional Bell test was demonstrated without a detection loophole.
2023	Storz *et al.* [7]	Using superconducting circuits loophole-free Bell inequality violation was demonstrated.
2025	Wang *et al.* [9]	Using unentangled photons Bell-inequality violation was reported.
2026	Lu *et al.* [18]	Using single atoms device-independent QKD over 100 km was demonstrated.
2026	Guo *et al.* [8]	Investigated Bell-like inequality violation associated with causal order experimentally.
2026	Bagaria [11]	Proposed an absolute-deterministic interpretation of quantum mechanics.

4 Conclusion

Classical limit for Bell Inequalities with S = 2 and present-day quantum mechanics limit (Tsirelson bound) with S = $2\sqrt{2}$ are well-established. Bell Inequalities assumes – locality, reality, measurement independence and counter-factual definiteness. This paper assumes Absolute Postulate of the Universe and its consequence - absolute determinism. Its consequence is that the four assumptions of Bell Inequalities can be derived from the single Absolute Postulate. Uncertainty is zero in the universe. So, universe is maximally non-local with S = 4. Resulting in PR-boxes being real in our universe. PR-boxes are useful as mathematical models of correlations that are stronger than present-day quantum correlations but still don't assume instantaneous (or faster than light) messaging, compatible with relativity's no-signalling principle.

This study cannot be proved using experimental results because that is prevented by the requirement of (1) a sheer large number of particles to achieve even a simple experimental result (2) Tsirelson Bound (parallel to Heisenberg's Uncertainty Principle in continuous studies). But indirect result like (1) Inability to rule out that PR-Boxes can't be real (2) Existence of maximal algebraic bound – although we can't go beyond Tsirelson Bound (3) Bagaria's new interpretation of QM, (4) Theory of Everything (ToE) joining QM and determinism (5) Re-interpretation of QM (6) Illusion of free will (7) Chaos Theory (8) Universe being one 4N-dimenstional entity – Consciousness.

It is acknowledged that his paper lacks a rigorous mathematical framework. We hope readers will find this study interesting and work with us so that mathematics can also be developed.

Table 3. Research gaps and future direction

Identified gap	Current status	Future research direction
Quantum correlations stop at the Tsirelson bound	$2\sqrt{2}$ is the quantum limit under standard quantum theory. This study proposes 4 as the maximal limit	Investigate whether stronger no-signalling correlations are only theoretical
Physical interpretation of Bell non-locality	Bell Inequality Violation has been verified many times. This study proposes stronger non-locality (S = 4)	With the Tsirelson Bound being equivalent to HUP, maximal correlation (S = 4) exists but can't be proved experimentally. To be verified.
Determinism and unpredictability	Bagaria's [11] new interpretation suggests that particles are real with real position and momentum, but that can't be verified experimentally or calculated.	Study alternate ways to prove or disprove the distinction of determinism and unpredictability. Eg, like the illusion of free will.
Reality	As per Copenhagen interpretation of QM - the outcomes of an experiments are probabilistic until measured/ observed	The world is completely deterministic as per the absolute postulate of universe. The implication for reality needs to be further studied and mathematically proved.
Counter-factual definiteness	Counter-factual means all outcomes for all possible detector settings have definite values, even for measurements that were not actually performed	Bagaria's [11] paper assumes that the world is completely deterministic and there is no alternative outcome to an experiment. This needs to be further studied and mathematically proved.
Measurement independence	Measurements of the entangled particles by Alice and Bob at their ends are independent of each other – as per this study	See if this can be verified experimentally
No-signalling vs. non-locality	Quantum Theory permits Bell non-locality without destroying no-signalling. This study shows that.	Can it be verified experimentally?
PR-box physical realizability	PR boxes were mathematical/ hypothetical models. This study proposes they exist and are real.	Can they be verified experimentally? Does Tsirelson Bound prevent direct detection of PR-Boxes? Can they be verified using indirect methods?
Practical implementation	Device-independent protocols are already supported by Bell correlations.	Investigate how quantum-information protocols can be further improved by stronger correlations.

5 Appendix A

The CHSH [6] inequality (Clauser-Horne-Shimony-Holt inequality) is a mathematical limit on the strength of correlations between two separated systems, assuming the universe obeys local realism (objects have definite properties before they are measured, and signals cannot travel faster than light).

The derivation is elegantly simple and requires nothing more than basic algebra.

The Setup

Imagine two observers, Alice and Bob, who are far apart.

- Alice can choose to measure property A or property A'.
- Bob can choose to measure property B or property B'.
- Each measurement can only yield a result of +1 or -1.

The relationship between their average measurement results.

The Algebraic Derivation

Let A, A', B, and B' be numbers that are strictly either +1 or -1.

1. Consider the algebraic combination: S = A B + A' B + A B' - A' B'
2. Factor out the terms B and B': S = (A + A') B + (A - A') B'
3. Look at the two terms individually.
    - If A and A' are the same value (both +1 or both -1), then (A + A') is either +2 or -2, while (A - A') becomes 0.
    - If A and A' are different (one is +1, one is -1), then (A + A') becomes 0, while (A - A') is either +2 or -2.

4. Because B and B' are also strictly +1 or -1, the term (A + A') B can only ever be $\pm$ 2, and the term (A - A') B' will be 0. Conversely, if the first term is 0, the second term will be $\pm$ 2.
5. Therefore, adding these scenarios together means the maximum possible value of S is +2, and the minimum possible value is -2. In mathematical terms: -2 <= A B + A' B + A B' - A' B' <= 2 or |S| <= 2

The Quantum Violation

According to, if particles are entangled, quantum mechanics predicts this value can reach up to $2\sqrt{2}$ (approximately 2.828) given in Appendix B. Because 2.828 > 2, entangled particles break the CHSH[6] inequality, proving that our universe fundamentally defies the rules of local classical reality.

6 Appendix B

The maximum value of the CHSH[4] quantity S achievable by present day quantum mechanics is known as the Tsirelson bound[20].

Setup:

Consider the maximally entangled Bell state,

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right).$$

From the CHSH scenario, we consider measurements at different angles performed by two qubits. Let qubit q_1's measurement settings be A_1, A_2, and qubit q_2's measurement settings be B_1, B_2, with outcomes $A_i, B_i \in \{+1, -1\}$ (this is what CHSH requires).

To achieve this, we use the *rotated measurement operator*

$$M(\theta) = cos\theta\sigma_z + sin\theta\sigma_x,$$

Where $\sigma_z \; and \; \sigma_x$ are,

$$\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \qquad \sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

are the Pauli matrices, with σ_z corresponding to measurement along the vertical axis and σ_x to measurement along the horizontal axis. Any point on the 2D plane can then be parametrized using

$$\cos\theta\,\hat{x} + sin\theta\,\hat{y}.$$

Explicitly,

$$M(\theta) = \begin{pmatrix} \cos\theta & \sin\theta \\ \sin\theta & -cos\theta \end{pmatrix}.$$

Correlation function:

Let θ be q_1's measurement angle and ϕ be q_2's measurement angle. Define the correlation function

$$E(\theta, \phi) = \langle\psi| M(\theta) \otimes M(\phi) |\psi\rangle.$$

We know that,

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right).$$

Start by finding,

$$\left(M(\theta) \otimes M(\phi)\right)|\psi\rangle.$$

Using,

$$\begin{aligned} \left(M(\theta) \otimes M(\phi)\right)|00\rangle &= \left(M(\theta)|0\rangle\right) \otimes \left(M(\phi)|0\rangle\right), \\ \left(M(\theta) \otimes M(\phi)\right)|11\rangle &= \left(M(\theta)|1\rangle\right) \otimes \left(M(\phi)|1\rangle\right), \end{aligned}$$

together with,

$$\begin{aligned} M(\theta)|0\rangle &= \begin{pmatrix}\cos\theta\\ \sin\theta\end{pmatrix} = cos\theta\,|0\rangle + sin\theta|\,1\rangle, \\ M(\theta)|1\rangle &= \begin{pmatrix}\sin\theta\\ -\cos\theta\end{pmatrix} = sin\theta\,|0\rangle - \cos\theta|\,1\rangle, \end{aligned}$$

Similarly, for ϕ.

and substituting the above terms,

$$\begin{aligned} (M(\theta)\otimes M(\phi))\,|00\rangle &= (\cos\theta\,|0\rangle + sin\theta|\,1\rangle)\otimes(\cos\phi\,|0\rangle + sin\phi|\,1\rangle)\,, \\ (M(\theta)\otimes M(\phi))\,|11\rangle &= (\sin\theta\,|0\rangle - \cos\theta|\,1\rangle)\otimes(\sin\phi\,|0\rangle - \cos\phi|\,1\rangle)\,. \end{aligned}$$

Then,

$$\begin{aligned} (M(\theta)\otimes M(\phi))\,|\psi\rangle = \tfrac{1}{\sqrt{2}}[\quad & \underbrace{(\cos\theta\,|0\rangle + sin\theta|\,1\rangle)\otimes(\cos\phi\,|0\rangle + sin\phi|\,1\rangle)}_{\text{Bracket (A)}} \\ + \quad & \underbrace{(\sin\theta\,|0\rangle - \cos\theta|\,1\rangle)\otimes(\sin\phi\,|0\rangle - \cos\phi|\,1\rangle)}_{\text{Bracket (B)}}]. \end{aligned}$$

Expanding each bracket:

$$\begin{aligned} \text{Bracket (A)} &= cos\theta\cos\phi\,|00\rangle + cos\theta\sin\phi\,|01\rangle + sin\theta\cos\phi\,|10\rangle + sin\theta\sin\phi\,|11\rangle, \\ \text{Bracket (B)} &= sin\theta\sin\phi\,|00\rangle - \sin\theta\cos\phi\,|01\rangle - \cos\theta\sin\phi\,|10\rangle + cos\theta\cos\phi\,|11\rangle. \end{aligned}$$

Adding Bracket (A) + Bracket (B):

$$\begin{aligned} (M(\theta)\otimes M(\phi))\,|\psi\rangle &= \tfrac{1}{\sqrt{2}}[(\cos\theta\cos\phi + sin\theta\sin\phi)\,|00\rangle + (cos\theta\sin\phi - \sin\theta\cos\phi)|\,01\rangle \\ &\quad + (\sin\theta\cos\phi - \cos\theta\sin\phi)\,|10\rangle + (sin\theta\sin\phi + cos\theta\cos\phi)|\,11\rangle]. \end{aligned}$$

The next step to derive,

$$\langle\psi|M(\theta)\otimes M(\phi)|\,\psi\rangle.$$

Define,

$$\begin{aligned} C_{00} &= cos\theta\cos\phi + sin\theta\sin\phi, \\ C_{11} &= sin\theta\sin\phi + cos\theta\cos\phi, \end{aligned}$$

so that,

$$(M(\theta)\otimes M(\phi))\,|\psi\rangle = \frac{1}{\sqrt{2}}(C_{00}|00\rangle + C_{11}|11\rangle)$$

(the $|01\rangle$ and $|10\rangle$ terms do not contribute once the inner product is taken with $\langle\psi|$, since $\langle\psi|$ has support only on $|00\rangle$ and $|11\rangle$).

Applying,

$$\begin{aligned} \langle\psi|M(\theta)\otimes M(\phi)|\,\psi\rangle &= \tfrac{1}{\sqrt{2}}\,(\langle 00|+\langle 11|)\cdot\tfrac{1}{\sqrt{2}}\,(C_{00}\,|00\rangle + C_{11}\,|11\rangle) \\ &= \tfrac{1}{\sqrt{2}}\cdot\tfrac{1}{\sqrt{2}}\,(\langle 00\,|00\rangle C_{00} + \langle 00|\,11\rangle C_{11} \\ &\qquad + \langle 11\,|00\rangle C_{00} + \langle 11|\,11\rangle C_{11}). \end{aligned}$$

By orthonormality of the computational basis,

$$\langle 00\,|00\rangle = 1, \quad \langle 11|\,11\rangle = 1, \quad \langle 00\,|11\rangle = 0, \quad \langle 11|\,00\rangle = 0.$$

Hence,

$$\langle\psi|M(\theta)\otimes M(\phi)|\,\psi\rangle = \frac{1}{\sqrt{2}}\cdot\frac{1}{\sqrt{2}}\,(C_{00} + C_{11})\,.$$

Substituting C_{00} and C_{11}:

$$\begin{aligned} &= \tfrac{1}{2}\left[(\cos\theta\cos\phi + sin\theta\sin\phi) + (\sin\theta\sin\phi + cos\theta\cos\phi)\right] \\ &= \tfrac{1}{2}\left[2\cos\theta\cos\phi + 2\sin\theta\sin\phi\right] \\ &= cos\theta\cos\phi + sin\theta\sin\phi \\ &= cos(\theta-\phi). \end{aligned}$$

Therefore,

$$\boxed{E(\theta,\phi) = cos(\theta-\phi).}$$

Choice of measurement settings:

Choose the settings

$$A_0 \Rightarrow \theta = 0^\circ, \qquad A_1 \Rightarrow \theta = 90^\circ, \qquad B_0 \Rightarrow \phi = 45^\circ, \qquad B_1 \Rightarrow \phi = -45^\circ.$$

Evaluating the CHSH[4] quantity

From the CHSH formula,

$$S = E(A_0,B_0) + E(A_0,B_1) + E(A_1,B_0) - E(A_1,B_1).$$

Using $E(\theta,\phi) = cos(\theta-\phi)$:

$$\begin{aligned} E(A_0,B_0) &= cos(0^\circ - 45^\circ) = cos(-45^\circ) = \tfrac{\sqrt{2}}{2}, \\ E(A_0,B_1) &= cos(0^\circ + 45^\circ) = cos(45^\circ) = \tfrac{\sqrt{2}}{2}, \\ E(A_1,B_0) &= cos(90^\circ - 45^\circ) = cos(45^\circ) = \tfrac{\sqrt{2}}{2}, \\ E(A_1,B_1) &= cos(90^\circ + 45^\circ) = cos(135^\circ) = -\tfrac{\sqrt{2}}{2}. \end{aligned}$$

Substituting into S:

$$S = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} - \left(-\frac{\sqrt{2}}{2}\right) = \frac{4\sqrt{2}}{2} = 2\sqrt{2}.$$

$$\boxed{S = 2\sqrt{2}}$$

This is the **Tsirelson bound** — the maximum value of the CHSH[4] quantity S achievable by present day quantum mechanics, exceeding the classical bound of $S \leq 2$. The algebraic maximum $S \leq 4$ permitted by no-signalling is described in Appendix C.

To find the maximum value of the expression, which represents the expectation value combination of the Tsirelson bound derivation:

$$S = E(A_0,B_0) + E(A_0,B_1) + E(A_1,B_0) - E(A_1,B_1)$$

$$S = \cos(A_0 - B_0) + \cos(A_0 - B_1) + \cos(A_1 - B_0) - \cos(A_1 - B_1)$$

Let's consider,

$$x = A_0 - B_0;\ y = A_0 - B_1;\ z = A_1 - B_0;\ A_1 - B_1 = z - x + y$$

Let $u = z - x + y$. To find the critical points, we compute the partial derivatives of S with respect to x, y, and z, and set each to zero:

$$\frac{\partial S}{\partial x} = -\sin(x) - \sin(u)\cdot(-1) = -\sin(x) + \sin(u) = 0 \Longrightarrow \sin(x) = \sin(u)$$

$$\frac{\partial S}{\partial y} = -\sin(y) - \sin(u) \cdot (1) = -\sin(y) - \sin(u) = 0 \Longrightarrow \sin(y) = -\sin(u)$$

$$\frac{\partial S}{\partial z} = -\sin(z) - \sin(u) \cdot (1) = -\sin(z) - \sin(u) = 0 \Longrightarrow \sin(z) = -\sin(u)$$

At the symmetric peak critical point, we set $x = \theta$, $y = -\theta$, $z = -\theta$, which gives:

$$u = (-\theta) - \theta + (-\theta) = -3\theta$$

Substituting these angle relationships into S:

$$S = \cos(\theta) + \cos(-\theta) + \cos(-\theta) + \cos(-3\theta) = 3\cos(\theta) + \cos(3\theta)$$

Using the triple-angle identity $\cos(3\theta) = 4\cos^3(\theta) - 3\cos(\theta)$:

$$S = 3\cos(\theta) - \ (4cos^3(\theta) - 3\cos(\theta)) \ = \ 6\cos(\theta) \ - \ 4 \ \cos^3(\theta)$$

Alternatively, if we express the function in terms of angles ϕ where $\cos(\phi) = c$, the maximum value of $S(x, y, z) = \cos x + \cos y + \cos z \ - \cos(x + y + z)$ under standard CHSH/Tsirelson parameterization gives:

$$S(c) = 6c - 4c^3 \quad \text{where } c = \cos(\theta)$$

Find the Peak Critical Point of$\mathbf{S}(\mathbf{c}) = \mathbf{6c} - \mathbf{4c^3}$

To find the maximum of $S(c) = 6c - 4c^3$:

1. **First Derivative:**

$$S'(c) = \frac{d}{dc}(6c - 4c^3) = 6 - 12c^2$$

Setting $S'(c) = 0$:

$$6 - 12c^2 = 0 \Longrightarrow c^2 = \frac{6}{12} = \frac{1}{2} \Longrightarrow c = \frac{1}{\sqrt{2}}$$

2. **Second Derivative Test:**

$$S''(c) = -24c$$

Evaluating at $c = \frac{1}{\sqrt{2}}$:

$$S''\left(\frac{1}{\sqrt{2}}\right) = -24\left(\frac{1}{\sqrt{2}}\right) = -12\sqrt{2} < 0$$

Since $S'' < 0$, $c = \frac{1}{\sqrt{2}}$ is indeed a local maximum (peak).

Step 4: Calculate the Maximum Value$\mathbf{S}_{\max}$

Substitute $c = \frac{1}{\sqrt{2}}$ into $S(c)$:

$$S_{\max} = 6\left(\frac{1}{\sqrt{2}}\right) - 4\left(\frac{1}{\sqrt{2}}\right)^3$$

$$S_{\max} = \frac{6}{\sqrt{2}} - 4\left(\frac{1}{2\sqrt{2}}\right) = \frac{6}{\sqrt{2}} - \frac{2}{\sqrt{2}} = \frac{4}{\sqrt{2}} = 2\sqrt{2}$$

Hence, the maximum value for S should be $\leq \ 2\sqrt{2}$.

7 Appendix C

The maximum value of the CHSH[4] quantity S achievable using a PR box[17], showing that it saturates the logical algebraic maximum, i.e., achieves maximal correlation, the PR[17] box

The term "PR box" refers to a hypothetical device satisfying

$$a \oplus b = x \cdot y,$$

where $a, b \in \{0,1\}$ are the outputs of the PR box, and $x, y \in \{0,1\}$ are the inputs to the PR box (Table 4 and 5).

Table 4. Truth table

x	y	$x \cdot y$	**Relation for output**
0	0	0	$a = b$
0	1	0	$a = b$
1	0	0	$a = b$
1	1	1	$a \neq b$

Table 5. Possible outputs of the PR box

x	y	**Possible output** (a, b)
0	0	$(0,0),\ (1,1)$
0	1	$(0,0),\ (1,1)$
1	0	$(0,0),\ (1,1)$
1	1	$(0,1),\ (1,0)$

The probability of getting each possible output for a given x, y is $\frac{1}{2}$

Calculating CHSH values

The outputs $a, b \in \{0,1\}$, but the CHSH value needs to be expressed in terms of ± 1. Hence,

$$A = (-1)^a, \qquad B = (-1)^b.$$

To find $E(x,y)$ for input x, y, CHSH uses the term

$$E(x,y) = \sum_{a,b \in \{0,1\}} (-1)^{a+b} P(a,b \mid x,y).$$

The above term is obtained as follows:

Generally, E (x, y) represented as,

$$\begin{aligned} E(x,y) \quad &= (\text{Probability of getting the same output for } A \text{ and } B) \\ &\quad - (\text{Probability of getting different values for } A \text{ and } B), \end{aligned}$$

$$E(x,y) = P(A = B) - P(A \neq B).$$

Expand $\sum_{a,b \in \{0,1\}} (-1)^{a+b} P(a,b \mid x,y)$,

$$E(x,y) = \underbrace{P(0,0 \mid x,y) + P(1,1 \mid x,y)}_{\text{same output}} - \underbrace{P(0,1 \mid x,y) - P(1,0 \mid x,y)}_{\text{different output}}.$$

Simplifying using the PR box[17] constraint

Hence, this can be written as (Table 6):

$$(-1)^{a+b} = (-1)^{a \oplus b},$$

Table 6. verification

a	b	$(-1)^{a+b}$	$(-1)^{a\oplus b}$
0	0	1	1
0	1	−1	−1
1	0	−1	−1
1	1	1	−1

So that

$$E(x,y) = \sum_{a,b\in\{0,1\}} (-1)^{a\oplus b} P(a,b\,|\,x,y)\,.$$

Using the PR box[17] property $a\oplus b = x\cdot y$:

$$E(x,y) = \sum_{a,b\in\{0,1\}} (-1)^{x\cdot y} P(a,b\,|\,x,y)$$

Find S from the CHSH expression

$$S = E(0,0) + E(0,1) + E(1,0) - E(1,1).$$

Evaluating each correlation term

For each pair (x,y), only the two outputs (a,b) satisfying $a\oplus b = x\cdot y$ occur, each with probability $\frac{1}{2}$ and all other outputs have probability 0.

$E(0,0)$:

Here $x\cdot y = 0$, so the valid outputs are $(0,0)$ and $(1,1)$, each with probability $\frac{1}{2}$:

$$\begin{aligned} E(0,0) &= (-1)^0 P(0,0\,|\,0,0) + (-1)^0 P(1,1\,|\,0,0) \\ &= (-1)^0\cdot\tfrac{1}{2} + (-1)^0\cdot\tfrac{1}{2} = 1. \end{aligned}$$

$E(0,1)$:

Here $x\cdot y = 0$, so the valid outputs are $(0,0)$ and $(1,1)$, each with probability $\frac{1}{2}$:

$$\begin{aligned} E(0,1) &= (-1)^0 P(0,0\,|\,0,1) + (-1)^0 P(1,1\,|\,0,1) \\ &= (-1)^0\cdot\tfrac{1}{2} + (-1)^0\cdot\tfrac{1}{2} = 1. \end{aligned}$$

$E(1,0)$:

Here $x\cdot y = 0$, so the valid outputs are $(0,0)$ and $(1,1)$, each with probability $\frac{1}{2}$:

$$\begin{aligned} E(1,0) &= (-1)^0 P(0,0\,|\,1,0) + (-1)^0 P(1,1\,|\,1,0) \\ &= (-1)^0\cdot\tfrac{1}{2} + (-1)^0\cdot\tfrac{1}{2} = 1. \end{aligned}$$

$E(1,1)$:

Here $x\cdot y = 1$, so the valid outputs are $(0,1)$ and $(1,0)$, each with probability $\frac{1}{2}$:

$$\begin{aligned} E(1,1) &= (-1)^1 P(0,1\,|\,1,1) + (-1)^1 P(1,0\,|\,1,1) \\ &= (-1)^1\cdot\tfrac{1}{2} + (-1)^1\cdot\tfrac{1}{2} = -1. \end{aligned}$$

Finally,

$$\begin{aligned} S &= E(0,0) + E(0,1) + E(1,0) - E(1,1) \\ &= 1+1+1-(-1) \\ &= 4. \end{aligned}$$

$$\boxed{S = 4}$$

Thus, it has been proved that the PR box[17] violates the Bell (CHSH) inequality maximally, getting the logical algebraic maximum $|S| \le 4$, and represents a correlation stronger than any correlation permitted by present-day quantum mechanics (which is bounded by the Tsirelson bound $2\sqrt{2}$, as derived in Appendix B) – still allowing the no-signalling condition.

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