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Integer-Linear Preservers of Binary Matrices

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Abstract

Objectives: Let $\mathcal{B}_n$ be the set of $n \times n$ matrices with entries in $\{0, 1\}$. This study aims to characterize the $\mathbb{Z}$ -linear maps from $M_n(\mathbb{Z})$ to $M_n(\mathbb{Z})$ that map the set of binary matrices into itself. **Method:** We apply vectorization to analyze the representing matrices, evaluate disjoint-support criteria for matrix units, and establish a finite test and module rank enumeration for binary preservers. **Findings:** We determine that every valid linear map acts as a coordinate-copying transformation where each output entry is zero or a copy of one input coordinate. If the map also preserves ordinary matrix rank, it takes the classical form $T(A) = PAQ$ or $T(A) = PA^TQ$ using permutation matrices P and Q . **Novelty:** This work provides a complete classification, exact module rank enumeration, and symmetry analysis for integer-linear binary preservers, demonstrating that symmetry alone is weaker than rank preservation.

Keywords: linear preserver problem; integer matrix; disjoint support; binary matrices; combinatorial matrix theory

1 Introduction

The matrices whose entries are 0 or 1 (binary matrices) play significant role in computer science and allied areas. A complete characterization of $\mathbb{Z}$ -linear maps from $M_n(\mathbb{Z})$ into itself that preserve the set of binary matrix appears to be absent from the existing literature. The aim of this paper is to obtain a complete characterization of linear maps preserving binary matrices.

Linear preserver problems ask for the structure of linear transformations that leave a specified matrix set, function, or relation invariant. Several recent studies have contributed to the development of linear preserver problems and different notions of matrix rank. Gogic and Tomašević investigated Jordan embeddings and linear rank preservers on structural matrix algebras, providing a characterization of unital rank-one preservers and a complete description of linear rank-preserving maps on these algebras, classical examples concern determinant, singularity, rank, spectrum, positivity, and structured matrix spaces; see ^{(1), (2), (3)}. In another recent work, Ahmed et al. studied linear preserver problems on Toeplitz matrices and characterized linear maps preserving rank-one matrices, rank, and determinant within the space of Toeplitz matrices ⁽⁴⁾.

For a fixed integer $n \geq 1$, set

$$\mathcal{B}_n = \{A = (a_{ij}) \in M_n(\mathbb{Z}) : a_{ij} \in \{0, 1\}\}$$

The present paper studies the inclusion-preserver problem

$$T(\mathcal{B}_n) \subseteq \mathcal{B}_n$$

For a $\mathbb{Z}$ -linear map $T : M_n(\mathbb{Z}) \rightarrow M_n(\mathbb{Z})$ the coefficient ring is essential. Boolean addition is idempotent, whereas ordinary addition in $\mathbb{Z}$ records an overlap of two-unit contributions as $1 + 1 = 2$. From a different perspective, Haviv and Parnas⁽⁵⁾ studied the binary and Boolean ranks of regular binary matrices and established the existence of regular matrices with a significant gap between these rank functions, thereby resolving a long-standing question concerning the binary rank of regular matrices and their complements. More recently, Parnas⁽⁶⁾ provided a comprehensive survey of binary and Boolean ranks and their relationship with real rank, covering their mathematical properties, combinatorial interpretations, computational complexity, communication complexity connections, and algorithmic aspects. Guillot, Gupta and Vishwakarma in their research⁽⁷⁾ classified real and complex functions applied entry wise to square matrices that preserve positive definiteness, which they call sign preservers. Belton, Guillot, Khare and Putinar in their research work⁽⁸⁾ studied transformations that preserve the inertia and negativity of symmetric matrices, proving their rigidity and separation of variables in multi-variable settings.

Nambiar, Adiyodi and Shanker⁽⁹⁾ presented a new and efficient algorithm to transpose binary, non-binary, and complex matrices using Sum of Product operations with the Identity Matrix acting as an agent.

Overall, these studies show the broad importance of rank-related problems in matrix theory and indicate that the study of rank-preserving transformations and different rank functions remains an active area with several important theoretical questions still open.

The first result shows that every output value is simply a direct copy of a single input value (or zero) because the matrix building blocks map to completely separate areas. This strict setup means you only need to check two points to verify how it works, lets us count the exact number of ways ranks are preserved, and proves that properties like being one-to-one, onto, and reversible all mean the exact same thing here. Building on that, the second result connects this to regular matrix geometry by showing that if ranks stay the same for matrices made only of zeros and ones, the transformed parts are simply a neat rearrangement of the original coordinates. Ranks of sums of two matrix units recover adjacency in the square rook graph, whose automorphisms produce the forms $A \rightarrow PAQ$ and $A \rightarrow PA^TQ$. Finally, we classify coordinate permutations preserving symmetric binary matrices and show that symmetry alone is weaker than rank preservation.

2 Methodology

To analyze the mathematical properties of our linear operators, we utilize the framework of linear preserver problems surveyed by Li and Pierce⁽¹⁰⁾. They classify linear maps that preserve matrix functions, subsets, or relations and explain standard proof techniques like extreme point reductions and duality.

In the methodology we have defined terms like support and vectorization, we mapped $M_n(\mathbb{Z})$ to $\mathbb{Z}^{n^2}$. We fix the column-wise vectorization throughout the paper. As a result, the representing matrix of the linear transformation is obtained in $M_{n^2}(\mathbb{Z})$. With the help of this, we found the characterization of maps that preserve binary matrices. The definitions and lemmas needed for this from the preliminaries are mentioned below.

2.1 Preliminaries

Let $[n] = \{1, 2, \dots, n\}$ and let E_{ij} denote the standard matrix unit. Every $A = (a_{ij}) \in M_n(\mathbb{Z})$ has the expansion

$$A = \sum_{i=1}^n \sum_{j=1}^n a_{ij} E_{ij}$$

For $A \in M_n(\mathbb{Z})$, write

$$\text{supp}(A) = \{(i, j) \in [n]^2 : a_{ij} \neq 0\}.$$

Choose a fixed vectorization $\text{vec} : M_n(\mathbb{Z}) \rightarrow \mathbb{Z}^{n^2}$. The representing matrix

$L_T \in M_{n^2}(\mathbb{Z})$ is determined by,

$$\text{vec}(T(A)) = L_T \text{vec}(A)$$

The column of L_T indexed by (i, j) is $\text{vec}(T(E_{ij}))$.

2.2 Definition

A $\mathbb{Z}$ linear map $T : M_n(\mathbb{Z}) \rightarrow M_n(\mathbb{Z})$ is a binary preserver if $T(\mathcal{B}_n) \subseteq \mathcal{B}_n$. It is a strong binary preserver if $T(\mathcal{B}_n) = \mathcal{B}_n$.

2.3 Lemma (Disjoint-support criterion).

Let $C_1, \dots, C_r \in \mathcal{B}_n$. Then $C_1 + \dots + C_r \in \mathcal{B}_n$ if and only if the supports $\text{supp}(C_1), \dots, \text{supp}(C_r)$ are pairwise disjoint.

Proof. If the supports are pairwise disjoint, each coordinate of the sum receives at most one contribution equal to 1. Conversely, if two supports intersect, the corresponding coordinate of the sum is at least 2, so the sum is not binary.

2.4 Lemma (Two-point set).

A $\mathbb{Z}$ linear map $T : M_n(\mathbb{Z}) \rightarrow M_n(\mathbb{Z})$ is a binary preserver if and only if

1. $T(E_{ij}) \in \mathcal{B}_n$ for every $(i, j) \in [n]^2$;
2. $T(E_{ij} + E_{kl}) \in \mathcal{B}_n$ whenever $(i, j) \neq (k, l)$.

Proof. Necessity is immediate. Under the two stated conditions, 2.3 lemma shows that the supports of the distinct matrices $T(E_{ij})$ are pairwise disjoint. Every binary matrix is a sum of a subset of the matrix units, so its image is binary by another application of 2.3 lemma.

3 Result and Discussion

This section presents the main result of our paper, which is named the Binary Preserver Theorem, along with some of its corollaries and remarks using examples. Following that, Section 3.2 presents the structural consequences and enumeration, which includes Proposition and 3.2.2 Corollary, as well as the rank-preserving refinement. Then, there is 3.3.2 Theorem, titled the Binary and Rank Preserver, along with its corollaries. This will be followed by the strong preservers of symmetric binary matrices, and the discussion and relation to existing work.

3.1 Complete classification

3.1.1. Theorem (Binary-preserver theorem).

Let $T : M_n(\mathbb{Z}) \rightarrow M_n(\mathbb{Z})$ be $\mathbb{Z}$ -linear. The following statements are equivalent

1. $T(\mathcal{B}_n) \subseteq \mathcal{B}_n$
2. Every $T(E_{ij})$ is binary and the nonempty supports in the family $\{\text{supp}(T(E_{ij})) : (i, j) \in [n]^2\}$ are pairwise disjoint.
3. The matrix L_T is binary and every row of L_T contains at most one 1.
4. There exists a function

$$\pi : [n]^2 \rightarrow \{0\} \cup [n]^2$$

Such that, for every $A = (a_{ij}) \in M_n(\mathbb{Z})$ and $(p, q) \in [n]^2$,

$$(T(A))_{pq} = \begin{cases} 0, & \pi(p, q) = 0, \\ a_{ij}, & \pi(p, q) = (i, j) \end{cases}$$

Proof. Assume the condition (i) holds. Since each matrix unit is binary, every $T(E_{ij})$ is binary. If $(i, j) \neq (k, l)$, then $E_{ij} + E_{kl}$ is binary and hence

$$T(E_{ij}) + T(E_{kl}) = T(E_{ij} + E_{kl}) \in \mathcal{B}_n.$$

By 3.2 lemma, the two supports are disjoint. This (ii) holds.

Moreover, assume the condition (ii) holds and let $A \in \mathcal{B}_n$. Since

$$A = \sum_{(i,j) \in \text{supp}(A)} E_{ij}, \text{ linearity gives, } T(A) = \sum_{(i,j) \in \text{supp}(A)} T(E_{ij}).$$

The summands are binary and have pairwise disjoint supports, so 3.2 lemma yields $T(A) \in \mathcal{B}_n$.

Therefore (i) and (ii) are equivalent. The columns of L_T are the vectorized images of the matrix units. Hence all columns are binary exactly when all $T(E_{ij})$ are binary. Two column supports overlap exactly when some row of L_T contains 1 in both columns. This proves the equivalence of (ii) and (iii). Under (iii), each row of L_T is either zero or a standard basis row vector. For the row corresponding to the output position (p, q) , define $\pi(p, q) = 0$ in the first case and define $\pi(p, q) = (i, j)$ when the unique 1 lies in the column indexed by (i, j) . Matrix multiplication by L_T then gives (iv). Conversely, the formula in (iv) produces a binary representing matrix with at most one 1 in each row.

3.1.2. **Corollary** (Coordinate-copying form).

A $\mathbb{Z}$ -linear map $T : M_n(\mathbb{Z}) \rightarrow M_n(\mathbb{Z})$ is a bilinear preserver if and only if every entry of $T(A)$ is either zero or a copy of one entry of A .

3.1.3. **Remark**

The map may delete an input coordinate or copy it into several output positions. The restriction is that no output coordinate may combine two distinct input entries.

3.1.4. **Corollary** (Strong preservers).

For a $\mathbb{Z}$ -linear map $T : M_n(\mathbb{Z}) \rightarrow M_n(\mathbb{Z})$, the following statements are equivalent.

1. $T(\mathcal{B}_n) = \mathcal{B}_n$.
2. L_T is a permutation matrix of order n^2 .
3. T permutes the n^2 matrix entries.

Proof. If L_T is a permutation matrix, then T permutes the vertices of the binary cube. Conversely, assume $T(\mathcal{B}_n) = \mathcal{B}_n$. By 3.1.1 theorem, every output coordinate selects at most one input coordinate. Surjectivity on the binary cube requires every output coordinate to select an input coordinate, and it also requires the selected input coordinates to be distinct; otherwise, two output coordinates would always agree, or some input coordinate would be irrelevant and the image would have fewer than 2^{n^2} elements. Thus L_T is a permutation matrix.

3.1.5. **Corollary** (Invertible preservers).

If T is a $\mathbb{Z}$ -module automorphism and $T(\mathcal{B}_n) \subseteq \mathcal{B}_n$, then T is a coordinate permutation. In particular, $T(\mathcal{B}_n) = \mathcal{B}_n$.

Proof. By 3.1.1 theorem, every row of L_T contains at most one 1. Since L_T is invertible over $\mathbb{Z}$, no row or column is zero. Therefore, every row and every column contain exactly one 1.

3.1.6. **Example**

For $n = 2$, the map

$$T \left[\begin{pmatrix} a & b \\ c & d \end{pmatrix} \right] = \begin{pmatrix} a & a \\ d & 0 \end{pmatrix}$$

is a binary preserver. It duplicates a , copies d , and deletes b and c , so one-sided binary preservation does not force a classical two-sided matrix action.

3.1.7. **Corollary** (Rectangular extension).

Let $T : M_{m,n}(\mathbb{Z}) \rightarrow M_{p,q}(\mathbb{Z})$, be $\mathbb{Z}$ -linear. Then L_T maps every binary matrix to a binary matrix if and only if its $pq \times mn$ representing matrix is binary with at most one 1 in each row. Consequently, the number of such maps is $(mn + 1)^{pq}$.

Proof. The proof of 3.1.1 Theorem uses only the standard coordinate bases and therefore applies without the square-dimension assumption. Each of the pq output coordinates have $mn + 1$ choices: it is zero or copies one of the mn input coordinates.

3.2 Structural consequences and enumeration

Set $N = n^2$ and let π be the coordinate function in 3.1.1 Theorem. Define

$$S_T = \pi \left([n]^2 \setminus \{0\} \right).$$

3.2.1. Proposition (Module rank and bijectivity).

If T is a binary preserver, then

$$\text{rank}_Q(T) = |S_T|$$

Moreover, the following statements are equivalent:

1. T is injective;
2. T is surjective;
3. T is a $\mathbb{Z}$ -module automorphism;
4. T is a strong binary preserver;
5. L_T is a permutation matrix of order N .
6. T permutes the N matrix entries.

Proof. Every nonzero row of L_T is a standard basis row vector, and the distinct nonzero rows are indexed by the elements of S_T . Hence the row rank over Q is $|S_T|$. If T is injective or surjective, this rank equals N . All N standard basis rows must then occur, each exactly once, so L_T is a permutation matrix. The remaining equivalences follow from 3.1.4 and 3.1.5 corollaries.

3.2.2. Corollary (Exact enumeration).

The number of $\mathbb{Z}$ -linear binary preservers on $M_n(\mathbb{Z})$ is $(N+1)^N$. The number of strong binary preservers is $N!$. More generally, the number of binary preservers of Q -linear rank r is

$$\binom{N}{r} \sum_{k=0}^r (-1)^k \binom{r}{k} (r+1-k)^N.$$

Proof. Each output coordinate independently chooses zero or one of the N input coordinates, giving $(N+1)^N$ maps. Strong preservers are the $N!$ coordinate permutations. For rank r , first choose the r input coordinates that occur in the image of π . The N output coordinates are then mapped to $\{0, 1, \dots, r\}$, with every nonzero symbol required to occur at least once. Inclusion-exclusion over the missing nonzero symbols gives the stated formula.

3.2.3. Remark (Verification complexity).

If L_T is given explicitly, 3.1.1 Theorem can be checked in $O(N^2) = O(n^4)$ elementary inspections by verifying that every entry is 0 or 1 and that each row contains at most one 1.

3.3 The rank-preserving refinement

For $u = (i, j)$ and $v = (k, l)$ in $[n]^2$, write $u \sim v$

When the two positions share a row or a column. This is adjacency in the $n \times n$ rook graph, equivalently the line graph of $K_{n,n}$; see⁽¹¹⁾. The relevant rank identity is

$$\text{rank}(E_{ij} + E_{kl}) = \begin{cases} 1, & i = k \text{ or } j = l \\ 2, & i \neq k \text{ and } j \neq l \end{cases}$$

3.3.1. Lemma (Automorphisms of the square rook graph).

Let $n \geq 2$, and let σ be a

permutation of $[n]^2$ preserving rook adjacency. Then there exist permutations $\rho, \kappa \in S_n$ such that either

$$\sigma(i, j) = (\rho(i), \kappa(j))$$

for all (i, j) or

$$\sigma(i, j) = (\kappa(j), \rho(i))$$

for all (i, j) .

Proof. Assume first that $n \geq 3$. The maximal cliques of size n are precisely the row cliques and the column cliques. Distinct row cliques are disjoint, as are distinct column cliques, whereas every row clique meets every column clique in one vertex. An automorphism therefore either preserves the two clique families or interchanges them. In the first case, it induces permutations ρ of the rows and κ of the columns, and the unique-intersection property gives $\sigma(i, j) = (\rho(i), \kappa(j))$. The second case gives the transpose-switched form. For $n = 2$, the rook graph is a 4-cycle, and its eight automorphisms are exactly the maps described by the two displayed forms.

3.3.2. **Theorem** (Binary and rank preservers).

Let $n \geq 2$, and let $T : M_n(\mathbb{Z}) \rightarrow M_n(\mathbb{Z})$ be $\mathbb{Z}$ -linear. The following statements are equivalent

1. $T(\mathcal{B}_n) \subseteq \mathcal{B}_n$ and $\text{rank}T(A) = \text{rank}(A)$ for every $A \in \mathcal{B}_n$.
2. There exist permutation matrices $P, Q \in M_n(\mathbb{Z})$.

such that either $T(A) = PAQ$, or $T(A) = PA^TQ$, for every $A \in M_n(\mathbb{Z})$

Proof. Assume the condition (i) holds. By 3.1.1 Theorem, the matrices $T(E_{ij})$ are binary and have pairwise disjoint supports. Since each E_{ij} has rank one, rank preservation makes every $T(E_{ij})$ nonzero. There are N nonempty pairwise disjoint supports contained in a set of N output positions, so every support is a singleton. Hence there is a permutation σ of $[n]^2$ such that

$$T(E_{ij}) = E_{\sigma(i,j)}.$$

For distinct positions u and v , the binary matrix $E_u + E_v$

has rank one exactly when u and v are adjacent in the rook graph. Rank preservation therefore implies that σ preserves adjacency. Since σ is a vertex permutation, it is a rook-graph automorphism. By 3.3.1 lemma, σ has either the row-column form or the transpose-switched form.

The corresponding row and column permutations are implemented by permutation matrices P and Q , giving $T(A) = PAQ$ or $T(A) = PA^TQ$ by linearity. Conversely, either displayed map permutes entries, maps binary matrices to binary matrices, and preserves rank for every matrix.

3.3.3. **Corollary** (Finite rank test).

A rank-preserving binary preserver is strong and invertible. In 3.3.2 theorem, it is sufficient to test rank on the matrix units and on sums of two distinct matrix units.

Proof. These are the only matrices used in the proof of 3.3.2 theorem. Once either classical form is obtained, strong preservation and invertibility are immediate.

3.3.4. **Remark**

The proof separates two sources of rigidity. Binary preservation first reduces the map to a coordinate permutation, and rank tests on two-unit sums then recover the row-column geometry. This differs from invoking a classical rank-preserver theorem on the entire matrix algebra.

3.4 Strong preservers of symmetric binary matrices

Let $S_n^{(0,1)} = \{A \in \mathcal{B}_n : A = A^T\}$.

3.4.1. **Theorem** (Symmetry-preserving strong maps).

Let T be a strong binary preserver,

and write $T(E_{ij}) = E_{\sigma(i,j)}$ for the associated coordinate permutation σ . Then

$$T(S_n^{(0,1)}) = S_n^{(0,1)}.$$

if and only if the following conditions hold.

- (i) σ permutes the diagonal positions $\{(i, i) : i \in [n]\}$.
- (ii) For every $i < j$, the set $\{\sigma(i, j), \sigma(j, i)\}$ has the form $\{(p, q), (q, p)\}$ with $p < q$.

(iii) The induced assignment on the $\binom{n}{2}$ unordered off-diagonal pairs is a permutation.

If $m = \binom{n}{2}$, the number of such maps is $n! 2^m m!$.

Proof. Suppose T maps $S_n^{(0,1)}$ onto itself. Each diagonal unit E_{ii} is symmetric. Since $T(E_{ii})$ is a single symmetric matrix unit, it must be diagonal, so diagonal positions are permuted. For $i < j$, the matrix $E_{ij} + E_{ji}$ is symmetric. Its image consists of two distinct matrix units and must again be symmetric. The diagonal output positions are already occupied by images of diagonal units, so the two image positions form a transposed off-diagonal pair. Bijectivity of σ then gives a permutation of the unordered off-diagonal pairs. Conversely, conditions (i)–(iii) state that every orbit of the transpose operation, either a diagonal singleton or an off-diagonal pair, is sent to an orbit of the same size. Therefore, a binary matrix constant on transpose orbits is mapped to another such matrix. Applying the same argument to T^{-1} gives equality. There are $n!$ permutations of the diagonal positions, $m!$ permutations of the off-diagonal transpose pairs, and 2^m independent orientation choices within those pairs. Their product is $n! 2^m m!$.

3.4.2. Remark

Only $n!$ of the maps in 3.4.1 theorem are induced by graph relabeling $A \rightarrow PAP^T$. Thus, preservation of symmetric adjacency matrices alone does not recover vertex geometry; the rank condition in 3.3.2 theorem imposes the necessary compatibility among rows and columns.

3.5 Discussion and relation to existing work

Classical linear preserver theory typically fixes an invariant such as rank, determinant, or singularity and derives multiplicative or transpose-type forms on a vector space of matrices^{(2), (3), (4)}. The binary-preserver condition considered here is different: it requires a finite, nonconvex subset of an integer module to be mapped into itself. A data grid is made only of 0s and 1s, you can change this data using math. However, the new grid must also stay as only 0s and 1s. Because of this rule, you have only three choices. You can delete rows or columns and can make copies of them. Or, you can shuffle their order.

You cannot add two parts together. Adding 1 and 1 makes 2, which breaks the rule. Deleting and copying are not normal math moves, still, experts can connect them to classic math rules. They use a special chart to track the rows and columns and ultimately, these three choices only change the basic shape of the data. You must add extra rules if you want to keep other special math patterns safe.

Several refinements remain natural. One may require preservation of zero diagonal and row sums, examine multiplication on selected binary classes, or impose graph invariants beyond symmetry. These are distinct preserver problems because each additional condition restricts the coordinate-copying class in a different manner.

4 Conclusion

We have classified the $\mathbb{Z}$ -linear maps on $M_n(\mathbb{Z})$ that send every binary matrix to a binary matrix. Such maps are precisely the coordinate-copying transformations: each output entry is zero or reproduces one input entry. The classification gives a finite test, exact enumeration, and a complete description of the strong and invertible cases. When ordinary rank is also preserved, the coordinate permutation must respect rook-graph adjacency and therefore has one of the classical forms PAQ or PA^TQ . The symmetry result identifies a larger family and shows that symmetry preservation alone does not determine a graph relabeling.

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