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ORIGINAL ARTICLE

Effective Reproduction number of Conjunctivitis Viral Disease

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Abstract

Objectives: In this study, an SEIR mathematical model of conjunctivitis viral disease is formulated to analyze the effects of the control measures including social distancing and treatment. **Method:** The basic reproduction number is computed using the next generation matrix method while the effective reproduction number is calculated for the controlled model. Numerical simulations were used to evaluate the effects of control measures on the model. **Findings:** The results show that the disease can be eradicated by combining simultaneously social distancing and treatment at specific levels. **Novelty:** An expression for the effective reproduction number related to conjunctivitis viral disease is determined, and control measures level are used to estimate intervention levels that minimize conjunctivitis disease transmission.

Keywords: Conjunctivitis; Effective reproduction number; Social distancing; Treatment; Heat map

1 Introduction

Viral conjunctivitis is one of the diseases that affect the eyes of humans and animals without exception[(1)]. It is therefore very important to take precautions against this disease, which affects both animals and their owners. The results in reference[(2)] prove that personal protection and disinfection of the environment have an advantage over the other measures while reference[(3)] has proved that quarantine and treatment measures reduce disease prevalence. Brad et al.[(4)] prove in their work that proper hygiene and hand washing associated with antibiotics lead to quicker recovered in the first 2-5 days of treatment while authors in[(5)] show that a combination of different therapies including antiseptics, corticosteroids, and immunomodulators have the best results especially for severe and complicated cases. Reference[(6)] uses $SEVIR_uR_i$ (S: Susceptible; E:

Exposed;V: Vaccinated;I : Infected;R_u: Recovered without treatment; R_u: Recovered with treatment) model to examined the early detection and treatment of conjunctivitis using vaccination and recovery measures with and without medication. The results show that individuals with a strong immune system can recover with or without medication. A highest transmission where noticed in Central India[7] which affects negatively the productivity in the concerned community and the use of prophylaxis has revealed to be the control measure that impacts on transmission dynamics of conjunctivitis disease while in China, the results in[8] show that isolating infected individuals, administering treatment, disinfecting locations and providing health education are more than 90% effective. Focusing on the fuzzy mathematical modeling of the dynamics of viral conjunctivitis[9] with particular emphasis on asymptomatic carriers and environmental influences and by incorporating time-delay terms as well as other latent effects, the findings of this study demonstrates the advantage of incorporating fuzzy logic and time delays into epidemic modeling in uncertain and dynamic environments. Using public health awareness as a control measure and incorporating real-world data for validation, reference[10] analyzes the dynamics of viral conjunctivitis transmission. The results show that this measure effectively reduces infection rates and highlight the importance of educational campaigns in combating the transmission of conjunctivitis.

The use of disinfectants, along with other strict measures[11], helps reduce the spread of the disease, which enables hospital staff to continue their activities while vaccination and early detection were used, and have proven effective[12].

The manuscript develops a mathematical model for the conjunctivitis virus, incorporating control measures such as social distancing and treatment of those already infected. Similar results involving control measures have been reported previously to provide a comprehensive overview and understanding of the new technique introduced, which differs from existing ones. Much research on conjunctivitis disease has focused on control measures but has not estimated the effective reproduction number, which differs from the basic reproduction number. In our study, we propose an alternative approach to controlling viral conjunctivitis by combining social distancing measures with treatment while calculating and analyzing the effective reproduction number of the model under control measures. This threshold is a very important value in disease surveillance once control measures have been implemented throughout the population.

2 Methodology

2.1 Conjunctivitis model without control measures

This section describes compartmental model of conjunctivitis viral without control measures and, identify the parameters (see Table 1) used in numerical simulations. The population is subdivided into four compartments. The population is subdivided into four compartments: susceptible individuals S, exposed human E, Infected individuals$I$, and recovered individuals R with $N(t) = S(t) + E(t) + I(t) + R(t)$. The following scheme form[13] pictures the dynamics of the conjunctivitis without any control measures (Figure 1).

From Figure 1, we deduce the following system of equations below:

$$\begin{aligned}
\frac{dS}{dt} &= bN - \left(\frac{1-\rho}{bN}\right) SI - \mu S + \varepsilon R,\\
\frac{dE}{dt} &= \left(\frac{1-\rho}{bN}\right) SI - (\mu+\delta)E,\\
\frac{dI}{dt} &= \delta E - (\mu+\gamma)I,\\
\frac{dR}{dt} &= \gamma I \ - (\mu+\varepsilon)R.
\end{aligned} \tag{1}$$

At the disease free equilibrium, i.e.$I = 0$, we have

$$E_0\left(S^*, E^*, I^*, R^*\right) = E_0\left(\frac{b}{\mu}, 0, 0, 0\right). \tag{2}$$

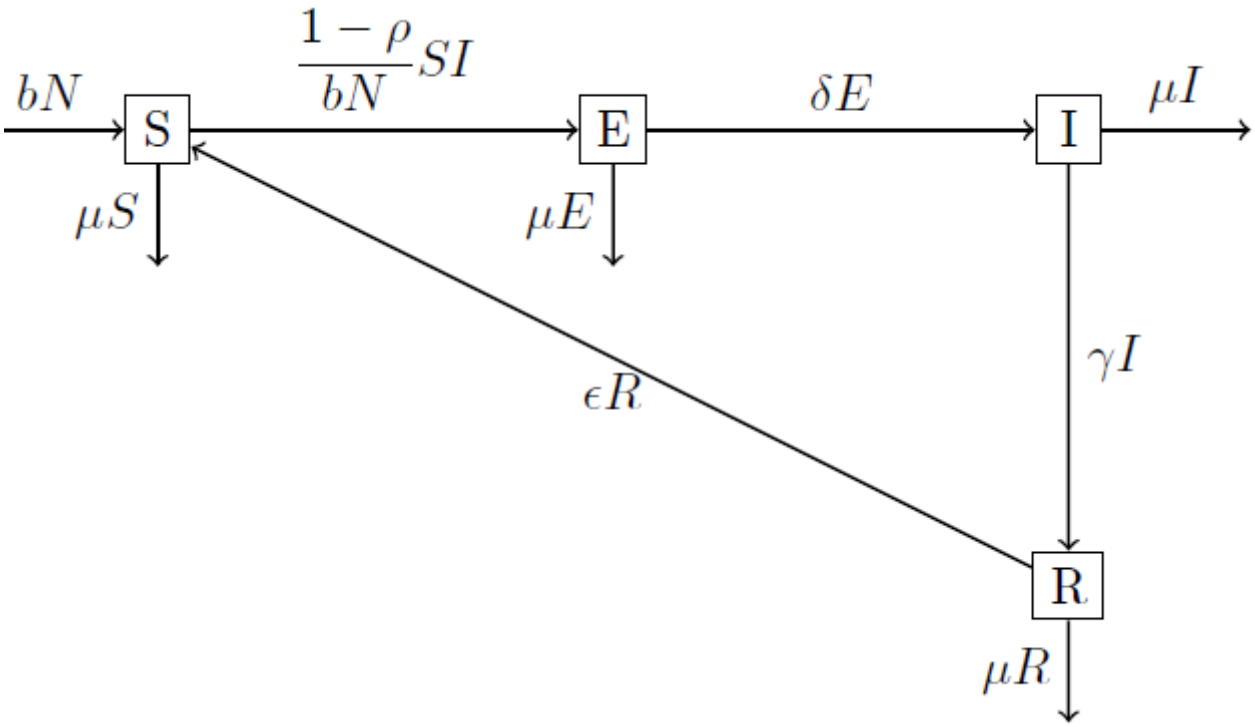


Fig 1. Conjunctivitis viral scheme without control measures

Conversely, if $I \neq 0$, there is a steady state of the endemic disease $E_1\,(S^*, E^*, I^*, R^*)$ where (S^*, E^*, I^*, R^*) are obtained by solving System (1) equated at 0 as follows

$$\begin{aligned} S^* &= \frac{b+\varepsilon R^*}{\left(\frac{1-\rho}{b}\right) I^* + \mu} \\ E^* &= \frac{1}{\mu+\delta}\left[\frac{(1-\rho)b}{b\mu+(1-\rho)I^*}\right] \\ I^* &= \frac{\delta E^*}{\gamma+\mu} = \left[\frac{\delta b(1-\rho)-b\mu(\gamma+\mu)(\delta+\mu)}{(\gamma+\mu)(\delta+\rho)(1-\rho)}\right] \\ R^* &= \frac{\gamma I^*}{\mu+\epsilon} = \frac{\gamma}{\mu+\epsilon}\left[\frac{\delta b(1-\rho)-b\mu(\gamma+\mu)(\delta+\mu)}{(\gamma+\mu)(\delta+\rho)(1-\rho)}\right] \end{aligned} \tag{3}$$

Table 1. Baseline parameters used in the model

Parameters	Interpretation	Values	Units	References
b	birth rate of human population	0.000456	day^{-1}	(12)
ρ	transmission rate of infection	0.02	day^{-1}	(14)
μ	natural death rate	0.04	day^{-1}	assumed
δ	Outflow rate of exposed subjects to infectious compartment	0.3	day^{-1}	assumed
γ	recovery rate	0.08	day^{-1}	assumed
ε	rate of immunity after recovery which is lost and individuals become susceptible again	0.01	day^{-1}	assumed

2.2 Basic reproduction number

The basic reproduction number $(R_0)$for the System (1) is calculated using the next generation matrix[15]. Setting X and X' vectors that represent respectively the infected and uninfected compartments, we obtain the following system

$$\frac{dX}{dt} = F(X) - V(X), \tag{4}$$

$$\frac{dX'}{dt} = W(X), \tag{5}$$

where $X = (X, X')$, $F(X)$ is the vector of in-flows into infected compartments (including new infections) and $V(X)$ the vector of out-flows. The functions F and V are chosen so that $F(X) \geq 0$ and $V(X) \geq$. The disease free equilibrium, calculated at Equation (2), which is defined as the steady state solution where all infectious variables are null, is denoted by $\left(0, \overline{X}'\right)$. Replacing in Equation (4), we have $F\left(0, \overline{X'}\right) = 0$ and $V\left(0, \overline{X'}\right) = 0$. The next generation matrix is given by FV^{-1}, where $F = \left(\frac{\partial F}{\partial X}\right)\Big|_{\left(0, \overline{X}'\right)}$ and $F = \left(\frac{\partial V}{\partial X}\right)\Big|_{\left(0, \overline{X}'\right)}$.

From System of equations (1), we compute matrices F and V:

$$F = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \frac{1-\rho}{\mu} \\ 0 & 0 & 0 \end{pmatrix} \text{ and } V = \begin{pmatrix} \mu & 0 & \frac{1-\rho}{\mu} \\ 0 & \mu+\delta & 0 \\ 0 & -\delta & \gamma+\mu \end{pmatrix}. \tag{6}$$

The basic reproduction number is obtained by calculating the spectral radius of the matrix obtained by multiplying F and V^{-1}

$$R_0 = \rho\left(FV^{-1}\right) = \frac{\delta(\rho-1)}{\mu(\mu+\delta)(\gamma+\mu)}. \tag{7}$$

2.3 Conjunctivitis model with control measures

In this subsection, we modify System (1) by including the controls measures defined by u_1 as the social distanciation with $0 < u_1 \leq 1$, and u_2 the treatment with $0 < u_2 \leq 1$. Social distancing avoids the contact between the infected (infectious) individuals and susceptible. It combines hygiene measures like washing hand, stay away from infected individuals if personal protective equipment is not available, and the use of disinfection products (Figure 2).

The resulting explicit equations are as follows

$$\begin{aligned} \frac{dS}{dt} &= bN - \left(\frac{1-\rho}{bN}\right)(1-u_1)SI - \mu S + \varepsilon R, \\ \frac{dE}{dt} &= \left(\frac{1-\rho}{bN}\right)(1-u_1)SI - (\mu+\delta)E, \\ \frac{dI}{dt} &= \delta E - (\mu+\gamma+u_2)I, \\ \frac{dR}{dt} &= (\gamma+u_2)I - (\mu+\varepsilon)R. \end{aligned} \tag{8}$$

2.4 Effective reproduction number

The basic reproduction number (R_0) is used in the early days of an outbreak, when the population has not yet developed herd immunity. However, as time passing, the population acquires a certain level of herd immunity, either through vaccination or because it has recovered from infection. In this case, we no longer refer to the basic reproduction number but rather to the effective reproduction number.

To calculate the effective reproduction number, we follow the same steps as for the basic reproduction number defined in Section 3. However, we use System (8), which includes control parameters, instead of System (1). From System (8), we compute

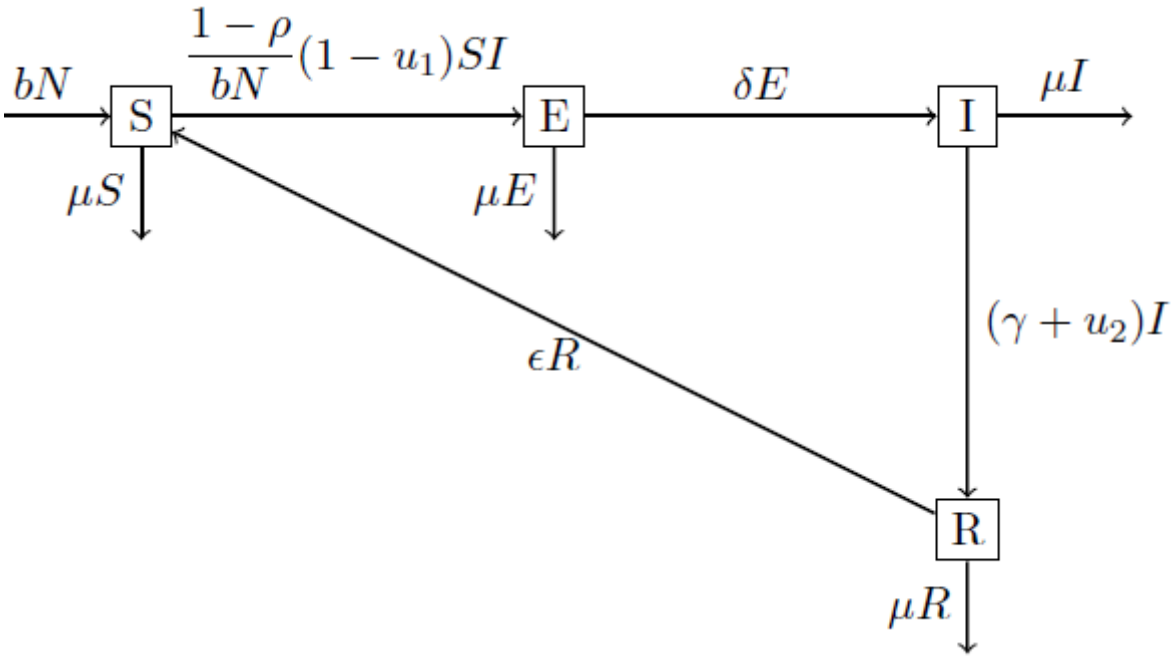


Fig 2. Conjunctivitis viral scheme with control measures

the matrices F and V respectively:

$$F=\begin{pmatrix}0 & 0 & 0\\ 0 & 0 & \frac{1-\rho}{\mu}(1-u_1)\\ 0 & 0 & 0\end{pmatrix} \text{ and } V=\begin{pmatrix}\mu & 0 & \frac{1-\rho}{\mu}(1-u_1)\\ 0 & \mu+\delta & 0\\ 0 & -\delta & \gamma+\mu+u_2\end{pmatrix}. \tag{9}$$

Computing V^{-1}, we obtain

$$V^{-1}=\begin{pmatrix}\frac{1}{\mu} & \frac{\delta(-\rho u_1+\delta\rho+\delta u_1-1)}{\gamma^2+\mu^2+2\gamma\mu^3+\gamma\mu^2u_2+\mu^4+\mu^3u_2} & \frac{-\rho u_1+\rho+u_1-1}{\gamma\mu^2+\mu^3+\mu^2u_2}\\ 0 & \frac{1}{\gamma+\mu} & 0\\ 0 & \frac{\delta}{\gamma^2+2\gamma\mu+\gamma u_2+\mu^2+\mu u_2} & \frac{1}{\gamma+\mu+u_2}\end{pmatrix}. \tag{10}$$

The matrix product FV^{-1} gives

$$FV^{-1}=\begin{pmatrix}0 & 0 & 0\\ 0 & \frac{\delta}{\gamma^2+2\gamma\mu+\gamma u_2+\mu^2+\mu u_2} & 0\\ 0 & \frac{\delta(1-\rho)(1-u_2)}{\mu(\gamma^2+2\gamma\mu+\gamma u_2+\mu^2+\mu u_2)} & \frac{(1-\rho)(1-u_2)}{\mu(\gamma+\mu+u_2)}\end{pmatrix} \tag{11}$$

From Equation (11), we calculate the effective reproduction number given by

$$R_e=\rho(FV^{-1})=\frac{\delta(\rho-1)(u_1-1)}{\mu(\mu+\delta)(\gamma+\mu+u_2)}. \tag{12}$$

Simulation model is used [mention it] to establish the relationship between the measure control levels and the values of the effective reproduction number. The parameters used in the simulations are taken from Table 1, along with the values of the control parameters $0<u_1\leq 1$and $0<u_2\leq 1$, social distancing and treatment respectively.

3 Results and Discussion

In this section, using python software, simulations are performed in order to visualize the relationship between the control measure levels and the values of the effective reproduction number. The parameters used in the simulations are taken from Table 1, along with the values of the control parameters $0 < u_1 \leq 1$and $0 < u_2 \leq 1$, social distancing and treatment respectively.

Figure 3 demonstrates the relationship between the applied social distancing and treatment threshold and the effective reproduction number, R_e.

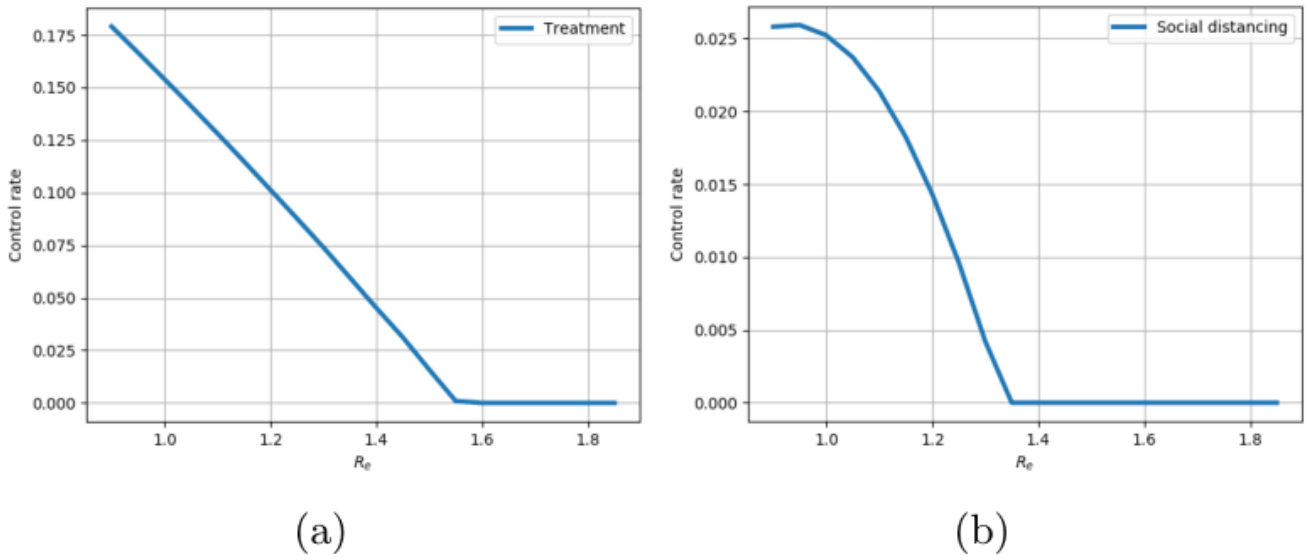


Fig 3. Figure (a) shows the effective reproduction number when social distancing $u_1 = 0$ while Figure (b) shows the effective reproduction number when treatment $u_2 = 0$.

Figure 3(a) analyzes the relationship between the effective reproduction number $R_e$and the treatment level. We observe that as the value of $R_e$increases from 0.9 to 1.55 , the treatment level decreases from $0.18$to 0, which corresponds to $R_e = 1.55$. When $R_e < 1.55$, the treatment threshold is set at a higher level in order to reduce R_e to a lower degree. However, for $R_e > 1.55$, the treatment threshold remains at zero, indicating that in this case, treatment is no longer considered necessary to control the disease, as it is ineffective against an epidemic. Under these conditions, the use of another disease control intervention is essential to limit its spread. Thus, the higher the treatment threshold, the more controllable the epidemic is. On the other hand, as transmissibility increases, treatment becomes less effective and is no longer an optimal intervention strategy. Thus, beyond $R_e = 1.55$, the disease spreads more rapidly within the population. Figure 3(b) shows that to achieve an effective reproduction number R_e between 0.5 and 1, a social distancing threshold of approximately $0.016$to $0.018$is required. When the effective reproduction number (R_e)increases from 1 to 1.25, the level of control progressively decreases to zero. For $R_e = 1.25$ and beyond, the level of control is zero, which implies that no social distancing measures are applied, and thus there is no effect on the disease dynamics. In this case, other more effective control measures can be considered to eradicate the disease. This graph indicates that social distancing measures are effective when $R_e \leq 1.25$, and that their impact gradually decreases as the value of R_e increases.

Figure 4 shows the control level for social distancing (the blue curve) and treatment (the orange curve) used to mitigate the transmission of the conjunctivitis viral. At the beginning, when the treatment is estimated at 0.8 and the level of social distancing is 0.45, the value of R_e is less than 1 i.e $R_e \approx 0.5$. At this point, the treatment declines slowly while the social distancing declines precipitously until to 0.16. The two control measures are applied intensively by emphasizing more on social distancing which is effective when the contacts are reduced. Adjusting the value of R_e at 1 require the social distancing, $u_1 \approx 0.78$and treatment, $u_2 \approx 0.18$.

When $1 < R_e < 1.6$, it requires the use of treatment at high level comparing to the social distancing level even. Notice that when $1 < R_e < 1.4$, social distancing level is increasing without exceed the level of treatment which still decreasing gradually, and declines after $R_e = 1.5$. In this situation, social distancing is no more playing a significant role rather than the treatment. That is, when the number of infected individuals is increasing, treatment is consistent and more preferred than

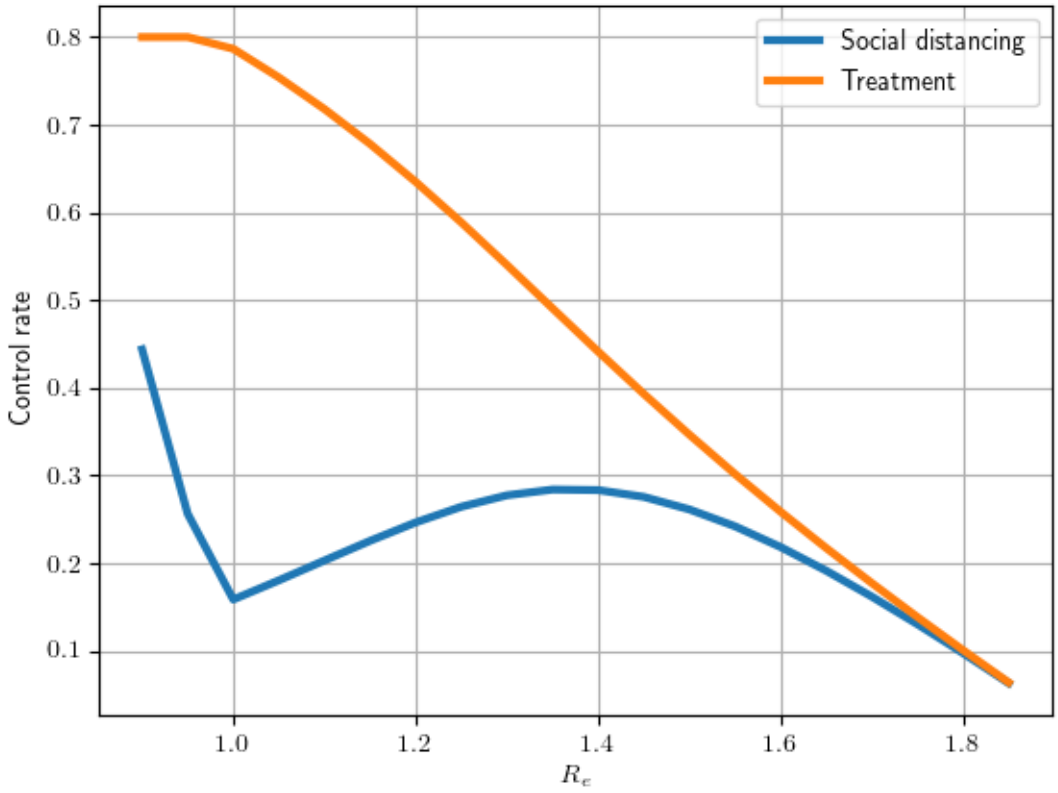


Fig 4. Effective reproduction number (R_e) value based on the control level measures.

hygiene measures. After $R_e > 1.6$, the more the value of R_e is increasing (i.e. when the transmission is at the high level), the more the level of the control measures used is decreased until they converge on the same point $0.5\,(u_1 = u_2 \approx 0.5)$ and $R_e \approx 1.85$. In this case, the applied control measures become ineffective. Therefore, this figure reveals that treatment is more applicable and preferred in the study of disease dynamics that is still propagating while social distancing is used as a complement to support the treatment.

Figure 5 describes the dependence of the effective reproduction (R_e) on the control measures using the heat-map. The heat-map shows that the value of $R_e = 1.28$ is obtained when $u_1 = 0.1$ and $u_2 = 0.15$. To achieve a value of $R_e = 1.2$, the combination $u_1 = 0.25$ and $u_2 = 0.38$ is required while $R_e = 1.12$ requires a combination of $u_1 \approx 0.38$ and $u_2 \approx 0.59$. To get the value $R_e = 1.04$, one must simultaneously use $u_1 \approx 0.49$ and $u_1 \approx 0.78$. The value of R_e that is less than $1\,(R_e < 1)$ is achieved in the case $u_1 \approx 0.6$ and $u_2 > 0.8$. For $R_e \geq 1.28$ there is a high transmission with the dark blue color (bottom-left) while from $R_e \leq 1.28$ the disease is under control transitioning from sky blue to yellow color (right at the top).

Figures 4-5 show that when attempting to adjust the value of R_e, much greater emphasis is placed on treatments than on social distancing measures. It is noticeable that if one aims to achieve a certain level of R_e, the required level of social distancing consistently remains lower than the treatment level as time progresses.

Most of the published studies cited in the introduction compute the basic reproduction number which is used to study the stability of the model. However, our approach emphasizes the case where the control measures are already incorporated in the model, compute the effective reproduction number in order to analyze the dynamics of the disease under controls measures. The literature results show that if the disease is not mitigated early, the basic reproduction number R0>1 and the disease propagates through the population whereas if $R_0 < 1$, disease dies out. Our findings indicate that misused, inappropriate or insufficient control measures lead to the disease propagation. Furthermore, combining treatment and social distancing reveal that the positive effect occurs only at specific levels of control measures, otherwise the disease persists.

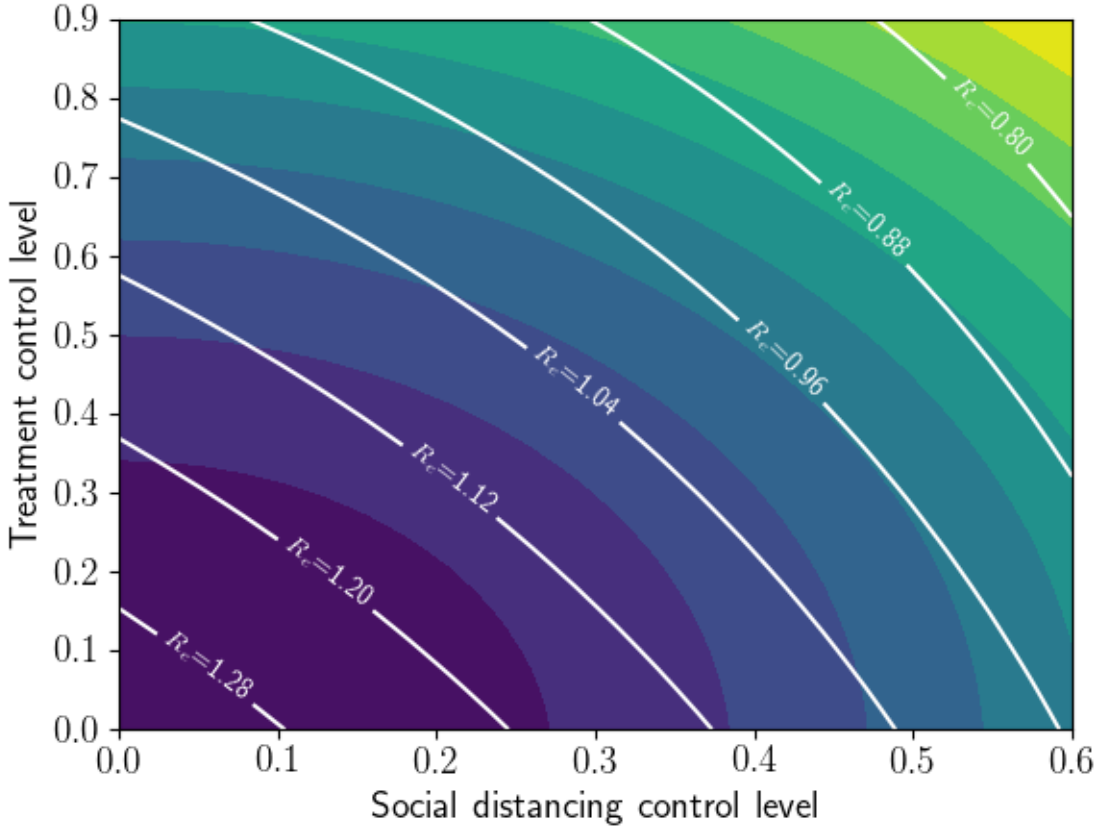


Fig 5. Heat-map showing the relationship between social distancing control level-treatment control level vs effective reproduction number described by the white contour plot.

4 Conclusion

This study presents an improved $SEIR$ mathematical model of conjunctivitis viral disease by incorporating the control measures such as social distancing and treatment. The effective reproduction number for the improved model has been calculated in order to analyze the dynamics of disease under controls. The findings show that the higher the threshold for treatment or social distancing, the more manageable the epidemic becomes. Conversely, as transmissibility increases, treatment and social distancing become less effective and are no longer an optimal intervention strategy. To achieve the value of $R_e < 1$ by combining these two pandemic control measures, it requires simultaneously maintaining social distancing at a level estimated to be at least 0.55 and treatment at a level of at least 0.85. Below these thresholds, the control measures have no significant effect on the dynamics of the disease. This highlights the fact that successfully combating diseases involves not only mitigating their spread but also requires careful organization and coordination of the capacity to provide appropriate care and of public awareness regarding the preventive measures to be implemented. In the future, the result will be improved by constructing stochastic model incorporated with some sources of randomness instead of compartmental model.

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