INDIAN JOURNAL OF SCIENCE AND TECHNOLOGY

ORIGINAL ARTICLE

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OPEN ACCESS

Received: 24/11/2025

Accepted: 27/12/2025

Published: 22/01/2026

Citation: Arulmani K, Bharathi S (2026) Local Neutrosophic Rough Matrix, Its Energy, and Its Application. Indian Journal of Science and Technology 19(2): 44-58. https://doi.org/10.17485/IJST/v19i2.1885

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Funding: None

Competing Interests: None

Published By Indian Society for Education and Environment (iSee)

ISSN
Print: 0974-6846
Electronic: 0974-5645

Local Neutrosophic Rough Matrix, Its Energy, and Its Application

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Abstract

Objective: This study introduces the concept of the Local Neutrosophic Rough Matrix (LNRM) and energy of Local Neutrosophic Rough Set (LNRS), and explores its algebraic operations and properties. **Method:** The LNRM method is implemented to determine the truth, indeterminacy, and falsity of a matrix, and subsequently, the energy of LNRS is calculated to support the scaling of the order. **Findings:** The proposed approach is applied to validate the process of purchasing a quality machine for initiating a new project in a specific company. Finally, a figure provides a visual representation of the ranking of four machines. A comparative analysis is conducted to verify the effectiveness of the work. **Novelty:** This paper aims to address the limitations by introducing the energy concept within the framework of the local neutrosophic rough set. Furthermore, the proposed matrix-based computations are implemented using MATLAB software, which enables efficient handling of matrices of size $n \times n$. In contrast, existing studies performed computations solely through manual calculations without employing any software tools, thereby restricting matrix operations to small sizes, such as 3×3 or 4×4.

Keywords: Rough Neutrosophic set; Local Rough set; Local Neutrosophic Rough set; Rough Neutrosophic Matrix; Local Neutrosophic Rough Matrix

1 Introduction

Over the past several decades, numerous innovative mathematical tools have emerged to handle the difficulties associated with uncertainty, vagueness, and incomplete information. These advancements have greatly enhanced the efficiency and precision of the modelling process. Alcantud et al.[1] introduced the scored energy for hesitant fuzzy soft sets by integrating the score functions of hesitant fuzzy elements with singular value–based matrix energy. Başer et al.[2] introduced the idea of energy within neutrosophic soft sets, which includes both lower and upper energy, based on the singular values of the corresponding matrices. By representing neutrosophic soft sets in matrix representation and applying relevant norms and products, an efficient decision-making strategy under uncertainty was developed.

J. Boobalan and E. Mathivadhana[3] presented Neutrosophic Hyper Soft rough matrix, detailing key operators and mathematical properties. Their approach integrates a score function to assess decision-making outcomes across three operators. In[4], the properties of interval-valued fuzzy soft sets were introduced, along with the concepts of energy, pessimistic energy, and optimistic energy, to develop an effective decision-making algorithm under uncertainty. D. J. S. Martina and G. Deepa[5] introduced the concept of the rough neutrosophic matrix. This framework employed lower and upper approximation neutrosophic matrices to address uncertainty in complicated systems. They also introduced[6] its energy and demonstrated its application to a multi-criteria decision-making (MCDM) problem for selecting the best building construction site.

The local neutrosophic rough set (LNRS) was introduced by S. Bharathi and K. Arulmani[7] by combining the local rough set and the neutrosophic set. S Vijayabalaji, Srinivasan, et al.[8] have introduced Neutrosophic soft and soft-rough transition matrices, discussing their properties and applications. They demonstrated their effectiveness across various decision-making situations. Matrix energy is defined as a spectral measure that sums the eigenvalues, thereby reflecting the structural properties of matrices. This concept generalises both graph and Laplacian energy and extends to operations such as scalar multiplication, inversion and conjugation.

Existing research has examined energy concepts within the frameworks of hesitant fuzzy soft sets, neutrosophic soft sets, neutrosophic hyper-soft rough matrix, interval-valued fuzzy soft sets, and rough neutrosophic sets. This paper aims to address this shortcoming by introducing the energy concept within the framework of the local neutrosophic rough set. Furthermore, the proposed matrix-based computations are implemented using MATLAB software, which enables efficient handling of matrices of size $n \times n$. In contrast, existing studies performed computations solely through manual calculations without employing any software tools, thereby restricting matrix operations to small sizes, such as 3×3 or 4×4. The utilization of computational tools significantly reduces processing time and enhances both efficiency and accuracy. Consequently, this approach becomes more applicable for large-scale and real-world applications. A comparative analysis is conducted to verify the effectiveness of the work, and the approach is applied to validate the process of purchasing a quality machine for initiating a new project in a specific company. Finally, a figure provides a visual representation of the ranking of four machines according to their truth values.

2 Preliminaries

Definition 2.1[9]

Let Q be a non-zero set, C be an equivalence relation on Q. For some non-zero subset P of Q.

$\underline{C}P = \bigcup \{p : [p]_C \subseteq P\}$

$\overline{C}P = \bigcup \{p : [p]_C \cap P \neq \emptyset\}$

The pair $(\underline{C}P, \overline{C}P)$ called the lower and upper approximations, respectively on P. Here (Q, C) is called an approximation space and C is an indiscernibility relation. The two $(\overline{C}P,\ \underline{C}P)$ are called rough set on P.

Definition 2.1[7]

Let ρ be a non-empty set. Consider γ as an equivalence relation on ρ and ζ as a neutrosophic relation on ρ, with the membership functions for truth T_ζ, indeterminacy I_ζ and falsity F_ζ. The upper and lower values concerning ζ, along with the pair (ρ, ζ), define the approximation space denoted by $\underline{K}(\zeta)$, and $\overline{K}(\zeta)$ are detailed below.

$\underline{K}(\zeta) = \left\{ < g, T_{\underline{K}(\zeta)}(g),\ I_{\underline{K}(\zeta)}(g),\ F_{\underline{K}(\zeta)}(g) > \epsilon\ [g]_\gamma, g \epsilon \rho \right\}$

$\overline{K}(\zeta) = \left\{ < g,\ T_{\overline{K}(\zeta)}(g),\ I_{\overline{K}(\zeta)}(g),\ F_{\overline{K}(\zeta)}(g) >\ \epsilon\ [g]_\gamma,\ g\ \epsilon\ \rho \right\}$, where

$T_{\underline{K}(\zeta)}(g) = \wedge_{d \epsilon [g]_\gamma} T_\zeta(d),\ I_{\underline{K}(\zeta)}(c) = \wedge_{d \epsilon [c]_\gamma} I_\zeta(d),\ F_{\underline{K}(\zeta)}(c) = \wedge_{d \epsilon [c]_\gamma} F_\zeta(d),$

$T_{\overline{K}(\zeta)}(c) = \vee_{d \epsilon [c]_\gamma} T_{(\zeta)}(d), I_{\overline{K}(\zeta)}(c) = \vee_{d \epsilon [c]_\gamma} I_{(\zeta)}(d),\ F_{\overline{K}(\zeta)}(c) = \vee_{d \epsilon [c]_\gamma} F_{(\zeta)}(d)$

Such that $T_{\underline{K}(\zeta)}(c),\ I_{\underline{K}(\zeta)}(c),\ F_{\underline{K}(\zeta)}(c), T_{\overline{K}(\zeta)}(c), I_{\overline{K}(\zeta)}(c),\ F_{\overline{K}(\zeta)}(c) : \zeta\ \epsilon [0,1],$

so, $0 \leq\ T_{\underline{K}(\zeta)}(c) + I_{\underline{K}(\zeta)}(c) + F_{\underline{K}(\zeta)}(c) \leq 3\ and 0 \leq T_{\overline{K}(\zeta)}(c) + I_{\overline{K}(\zeta)}(c) +\ F_{\overline{K}(\zeta)}(c) \leq 3.$

Here, symbols "$\wedge$" and" $\vee$" mean maximum and minimum operators, respectively. Where $[c]_\gamma$ is an equivalence class in γ. Then $\left(\underline{K}(\zeta), \overline{K}(\zeta)\right)$ is defined as RNS in (ρ, γ).

Definition 2.3[7]

Let the pair is defined $(K, \leq)$ as a partially ordered set. For any $l, m\ \epsilon\ k$, the real number $D\left(\frac{l}{m}\right)$ complies with the conditions below

1. $0 \leq\ D\left(\frac{l}{m}\right) \leq 1$
2. $l \leq m\ \ implies\ \ D\left(\frac{l}{m}\right) = 0$
3. $l \leq m \leq r\quad implies\ D\left(\frac{l}{r}\right) \leq D\left(\frac{l}{m}\right)$. This D is an including degree of K.

Definition 2.4 [7]

Let (J, Q) be a space of approximation. Let D be an including degree in $P(J) \times P(J)$. If any $W \subseteq J$, the α – lower, β-upper approximations are defined as follows

$$\underline{Q}_{\alpha}(W) = \{w \ \ (\frac{W}{[q]_Q \ \geq \ \alpha \, , w \, \epsilon \, W \ \} \, ,}$$

$\overline{Q}_{\beta}(W) = \{w \ \ (\frac{W}{[q]_Q > \ \beta \ , w \ \epsilon \ W \ \}}$. This pair $(\underline{Q}_{\alpha}(W), \overline{Q}_{\beta}(W))$ is defined as LRS.

Definition 2.5 [7]

Let (C, R) be a space of approximation, C a non-zero set, and R a relation of equivalence in C. Let K be an LRNS in C, defined by the membership τ_K, indeterminacy δ_K and non-membership η_K . Let $0 \leq \beta < \alpha \leq 1$ at some $K \epsilon C$, the local lower and local upper approximations are denoted as $\underline{N}_{\alpha}(K)$ and $\overline{N}_{\beta}(K)$ in C respectively. Define

$$\underline{N}_{\alpha}(K) = \{(l \ , \ D(\frac{\left(\tau_{\underline{N}_{\alpha}(K)}(l) \ , \ \delta_{\underline{N}_{\alpha}(K)}(l) \ , \eta_{\underline{N}_{\alpha}(K)}(l)\right)}{[l]_R)) \ \geq \ \alpha \mid l \, \epsilon \, C} , [l]_R \neq \emptyset \ \}$$

$\overline{N}_{\beta}(K) = \{(l \ , \ D(\frac{\left(\tau_{\overline{N}_{\beta}(K)}(l) \ , \ \delta_{\overline{N}_{\beta}(K)}(l) \ , \eta_{\overline{N}_{\beta}(K)}(l)\right)}{[l]_R)) > \beta \mid l \ \epsilon \ C} , [l]_R \neq \emptyset \ \}$, where

$$\tau_{\underline{N}_{\alpha}(K)}(l) = min_{m \epsilon [l]_R} \tau_K(m), \ \delta_{\underline{N}_{\alpha}(K)}(l) = max_{m \epsilon [l]_R} \delta_K(m), \eta_{\underline{N}_{\alpha}(K)}(l) = max_{m \epsilon [l]_R} \eta_K(m),$$

$$\tau_{\overline{N}_{\beta}(K)}(l) = max_{m \epsilon [l]_R} \tau_K(m), \delta_{\overline{N}_{\beta}(K)}(l) = min_{m \epsilon [l]_R} \delta_K(m), \ \eta_{\overline{N}_{\beta}(K)}(l) = min_{m \epsilon [l]_R} \eta_K(m)$$

Here $\tau_K(l), \delta_K(l), \eta_K(l)$ denoted as membership, indeterminacy & non-membership of l in K. Therefore

$$0 \leq \ \tau_{\underline{N}_{\alpha}(K)}(l) + \delta_{\underline{N}_{\alpha}(K)}(l) + \eta_{\underline{N}_{\alpha}(K)}(l) \leq 3 \ , 0 \leq \tau_{\overline{N}_{\beta}(K)}(l) + \delta_{\overline{N}_{\beta}(K)}(l) + \eta_{\overline{N}_{\beta}(K)}(l) \leq 3.$$

The functions $\tau_{\underline{N}_{\alpha}(K)}(l), \delta_{\underline{N}_{\alpha}(K)}(l), \eta_{\underline{N}_{\alpha}(K)}(l), \tau_{\overline{N}_{\beta}(K)}(l) \ + \ \delta_{\overline{N}_{\beta}(K)}(l) \ + \ \eta_{\overline{N}_{\beta}(K)}(l) \quad : \ K \ \rightarrow]0^-, 1^+[$. The pair $(\underline{N}_{\alpha}(K) \ , \overline{N}_{\beta}(K))$ is called LNRS in C.

Definition 2.6 [4]

The pessimistic energy of the interval-valued fuzzy set $\mathbb{F}_{\mathcal{A}}$, denoted by $\mathbb{E}^{\min}_{\mathbb{F}_{\mathcal{A}}}$ is defined as

$$\mathbb{E}^{\min}_{\mathbb{F}_{\mathcal{A}}} = \sum_{\omega=1}^{p} \sigma_{\omega},$$

where $\sigma_1 \geq \sigma_2 \geq \ldots \geq \sigma_{\omega} \geq 0$ are the singular values of the matrix $\psi^{\min}_{\mathbb{F}_{\mathcal{A}}}$ of minimum values of the interval-valued fuzzy set $\mathbb{F}_{\mathcal{A}}$.

Definition 2.7 [4]

The optimistic energy of the interval-valued fuzzy set $\mathbb{F}_{\mathcal{A}}$, denoted by $\mathbb{E}^{\max}_{\mathbb{F}_{\mathcal{A}}}$ is defined as

$$\mathbb{E}^{\max}_{\mathbb{F}_{\mathcal{A}}} = \sum_{\omega=1}^{p} \sigma_{\omega},$$

where $\sigma_1 \geq \sigma_2 \geq \ldots \geq \sigma_{\omega} \geq 0$ are the singular values of the matrix $\psi^{\max}_{\mathbb{F}_{\mathcal{A}}}$ of maximum values of the interval-valued fuzzy set $\mathbb{F}_{\mathcal{A}}$.

3 Local Neutrosophic Rough Matrix (LNRM)

This section introduces the concept of LNRM, which is defined using the parameters α and β. These parameters act as lower and upper thresholds that regulate the levels of truth, indeterminacy, and falsity within the matrix, ensuring that the lower and upper approximations reflect both certainty and uncertainty. The effectiveness of this structure is successfully demonstrated through a decision-making problem.

Definition 3.1

Let (C, R) be a space of approximation, C be a universal set, and R be a relation of equivalence in C. Let G be a non-zero subset of C and a matrix $H = \left(\underline{H}_{kl}(G),\ \overline{H}_{kl}(G)\right)$ of order $m \times n$. Here $\underline{H}_{kl}$ and $\overline{H}_{kl}$ is a lower and upper approximation, respectively. Let $0 \leq \beta < \alpha \leq 1$ for some point G and D be an including degree in $P(C)$, where $P(C)$ be power set on C. Then H be a local neutrosophic rough matrix, defined as follows

$$\begin{aligned} H &= \left(D(\underline{H}_{kl}(G)/[G]_R\right) \geq \alpha, \\ &\quad D(\overline{H}_{kl}(G)/[G]_R) > \beta)_{m\times n}. \\ &= \left(D\,(\underline{T}_{kl}(G),\ \underline{I}_{kl}(G),\ \underline{F}_{kl}(G) \geq \alpha\right) \\ &\quad D(\overline{T}_{kl}(G), \overline{I}_{kl}(G), \overline{F}_{kl}(G)) > \beta)_{m\times n}. \end{aligned}$$

where,

$\underline{T}_{kl}(G) = inf\, T_{kl}(e)$, $\underline{I}_{kl}(G) = sup\, I_{kl}(e)$, $\underline{F}_{kl}(G) = sup\, F_{kl}(e)$, and

$\overline{T}_{kl}(G) = sup\, T_{kl}(e)$, $\overline{I}_{kl}(G) = inf\, I_{kl}(e)$, $\overline{F}_{kl}(G) = inf\, F_{kl}(e)$.

Here, the membership, non-membership and intermediate values of lower & upper approximations are $\underline{T}_{kl}(G)$,$\underline{I}_{kl}(G)$, $\underline{F}_{kl}(G)$ and $\overline{T}_{kl}(G)$, $\overline{I}_{kl}(G)$, $\overline{F}_{kl}(G)$ respectively, and satisfies the following conditions.

$$0 \leq \underline{T}_{kl}(G) + \underline{I}_{kl}(G) + \underline{F}_{kl}(G) \leq 3,$$

$$0 \leq \overline{T}_{kl}(G) + \overline{I}_{kl}(G) + \overline{F}_{kl}(G) \leq 3.$$

Note 3.2

It is important to observe that for $\alpha = 1$ and $\beta = 0$, the local neutrosophic rough matrix simplifies to a rough neutrosophic matrix model.

Example 3.3

Consider a set of students $\gamma = \{\mathbb{s}_1, \mathbb{s}_2, \mathbb{s}_3, \mathbb{s}_4\}$ and a set of subjects $\omega = \{\mathbb{o}_1, \mathbb{o}_2, \mathbb{o}_3, \mathbb{o}_4\}$. To determine the academic performance of students for university scholarships. The decision parameters have been set as $\alpha = 0.7$ and $\beta = 0.2$ and $X = \{\mathbb{o}_1, \mathbb{o}_2, \mathbb{o}_4\}$. Let $R = \{\{\mathbb{o}_1, \mathbb{o}_3\}, \{, \mathbb{o}_2, \mathbb{o}_4\}\}$ denote the equivalence class that is related to the truth value of $\{\mathbb{o}_1\}$ in the table. The performance of individual students is formulated as a local neutrosophic rough triplet (T, I, F). The degrees of T, I, and F are represented as strong performance, uncertainty, and weak performance, respectively, and are illustrated in Table 1.

Table 1. Students Performance

Students Subjects	$\mathbb{o}_1$	$\mathbb{o}_2$	$\mathbb{o}_3$	$\mathbb{o}_4$
$\mathbb{s}_1$	$(0.9, 0.7, 0.3)$, $(0.7, 0.3, 0.6)$	$(0.3, 0.6, 0.7)$, $(0.8, 0.5, 0.2)$	$(0.4, 0.2, 0.5)$, $(0.6, 0.8, 0.1)$	$(0.7, 0.2, 0.4)$, $(0.8, 0.2, 0.5)$
$\mathbb{s}_2$	$(0.5, 0.9, 0.4)$, $(0.6, 0.7, 0.3)$	$(0.7, 0.7, 0.6)$, $(0.8, 0.5, 0.3)$	$(0.4, 0.8, 0.4)$, $(0.7, 0.9, 0)$	$(0.6, 0.6, 0.3)$, $(0.6, 0.5, 1)$
$\mathbb{s}_3$	$(0.9, 0.5, 0.7)$, $(0.5, 0.1, 0.9)$	$(0.2, 0.7, 0.8)$, $(0.6, 0.5, 0.7)$	$(0.6, 0.3, 0.5)$, $(0.8, 0.9, 0.3)$	$(0.8, 0.7, 0.5)$, $(0.9, 0.2, 0.4)$
$\mathbb{s}_4$	$(0.5, 0.7, 0.6)$, $(0.7, 0.7, 0.1)$	$(0.3, 0.4, 0.8)$, $(0.6, 0.7, 0.3)$	$(0.8, 0.7, 0.6)$, $(1, 0.5, 0.3)$	$(0.2, 0.6, 0.7)$, $(0.4, 0.7, 0.1)$

$$D(X/[\mathbb{o}_1]_R) = \{\mathbb{o}_1, \mathbb{o}_3\} = \tfrac{1}{2} = 0.5,\ D(X/[\mathbb{o}_2]_R) = \{\mathbb{o}_2, \mathbb{o}_4\} = 1,\ D(X/[\mathbb{o}_4]_R) = \{\mathbb{o}_2, \mathbb{o}_4\} = 1$$

$$\underline{P}_{0.7}(X) = [\mathbb{o}_2]_R \cup [\mathbb{o}_4]_R$$

$$\underline{P}_{0.7}(X) = \{\mathbb{o}_2, \mathbb{o}_4\}$$

$$\overline{P}_{0.2}(X) = [\mathbb{o}_1]_R \cup [\mathbb{o}_2]_R \cup [\mathbb{o}_4]_R$$

$$\overline{P}_{0.2}(X) = \{\text{a}_1, \text{a}_2, \text{a}_3, \text{a}_4\}$$

For this problem, consider only the upper approximation values. The LNRM is presented below.

$$H = \begin{pmatrix} [(0.7,0.7,0.6), & [(0.3,0.6,0.7), & [(0.4,0.8,0.5), & [(0.7,0.2,0.5), \\ (0.9,0.3,0.3)] & (0.3,0.6,0.7)] & (0.6,0.2,0.1)] & (0.8,0.2,0.4)] \\ [(0.5,0.9,0.4), & [(0.7,0.7,0.6), & [(0.4,0.9,0.4), & [(0.6,0.6,1), \\ (0.6,0.7,0.3)] & (0.8,0.5,0.3)] & (0.7,0.8,0)] & (0.6,0.5,0.3)] \\ [(0.5,0.5,0.9), & [(0.2,0.7,0.8), & [(0.6,0.9,0.5), & [(0.8,0.7,0.5), \\ (0.9,0.1,0.7)] & (0.6,0.5,0.7)] & (0.8,0.3,0.3)] & (0.9,0.2,0.4)] \\ [(0.9,0.3,0.3), & [(0.3,0.7,0.8), & [(0.8,0.7,0.6), & [(0.2,0.7,0.7), \\ (0.9,0.3,0.3)] & (0.6,0.4,0.3)] & (1,0.5,0.3)] & (0.4,0.6,0.1)] \end{pmatrix}$$

4 Algebraic operations on LNRM

Let Q and S be two LNRM and R- Truth, N- Indeterminacy, and L- False are denoted as follows.

$$\begin{aligned} Q &= [\left(D\left(\underline{Q}_{kl}^{R}, \underline{Q}_{kl}^{N}, \underline{Q}_{kl}^{L}\right) \geq \alpha\right), \\ &\quad (D\left(\overline{Q}_{kl}^{R}, \overline{Q}_{kl}^{N}, \overline{Q}_{kl}^{L}\right) > \beta)]_{m\times n}, \quad \text{and} \\ S &= [\left(D\left(\underline{S}_{kl}^{R}, \underline{S}_{kl}^{N}, \underline{S}_{kl}^{L}\right) \geq \alpha\right), \\ &\quad (D\left(\overline{S}_{kl}^{R}, \overline{S}_{kl}^{N}, \overline{S}_{kl}^{L}\right) > \beta)]_{m\times n}. \end{aligned}$$

1. **Addition**

$$\begin{aligned} Q+S &= [sup\,\{\underline{Q}_{kl}^{R}, \underline{S}_{kl}^{R}\},\, inf\{\underline{Q}_{kl}^{N}, \underline{S}_{kl}^{N}\},\, inf\{\underline{Q}_{kl}^{L}, \underline{S}_{kl}^{L}\}], \\ &\quad [\sup\{\overline{Q}_{kl}^{R}, \overline{S}_{kl}^{R}\},\, inf\{\overline{Q}_{kl}^{N}, \overline{S}_{kl}^{N}\}, \inf\{\overline{Q}_{kl}^{L}, \overline{S}_{kl}^{L}\}]. \end{aligned}$$

2. **Multiplication**

$$\begin{aligned} Q.S &= [inf\,\{\underline{Q}_{kl}^{R}, \underline{S}_{kl}^{R}\},\, \sup\{\underline{Q}_{kl}^{N}, \underline{S}_{kl}^{N}\},\, \sup\{\underline{Q}_{kl}^{L}, \underline{S}_{kl}^{L}\}], \\ &\quad [inf\{\overline{Q}_{kl}^{R}, \overline{S}_{kl}^{R}\}, \sup\{\overline{Q}_{kl}^{N}, \overline{S}_{kl}^{N}\}, sup\{\overline{Q}_{kl}^{L}, \overline{S}_{kl}^{L}\}]. \end{aligned}$$

3. **Component-wise Addition**

$$\begin{aligned} Q \oplus S &= (sup\{Q,S\}) \\ &= [sup\,\{\underline{Q}_{kl}^{R}, \underline{S}_{kl}^{R}\},\, inf\{\underline{Q}_{kl}^{N}, \underline{S}_{kl}^{N}\},\, inf\{\underline{Q}_{kl}^{L}, \underline{S}_{kl}^{L}\}], \\ &\quad [\sup\{\overline{Q}_{kl}^{R}, \overline{S}_{kl}^{R}\},\, inf\{\overline{Q}_{kl}^{N}, \overline{S}_{kl}^{N}\}, \inf\{\overline{Q}_{kl}^{L}, \overline{S}_{kl}^{L}\}]. \end{aligned}$$

4. **Component-wise Multiplication**

$$\begin{aligned} Q \odot S &= (\inf\{Q,S\}) \\ &= [inf\,\{\underline{Q}_{kl}^{R}, \underline{S}_{kl}^{R}\},\, \sup\{\underline{Q}_{kl}^{N}, \underline{S}_{kl}^{N}\},\, \sup\{\underline{Q}_{kl}^{L}, \underline{S}_{kl}^{L}\}], \\ &\quad [inf\{\overline{Q}_{kl}^{R}, \overline{S}_{kl}^{R}\}, \sup\{\overline{Q}_{kl}^{N}, \overline{S}_{kl}^{N}\}, sup\{\overline{Q}_{kl}^{L}, \overline{S}_{kl}^{L}\}]. \end{aligned}$$

5. **Average**

$$\begin{aligned} Q\,@S &= [\left(\frac{\underline{Q}_{kl}^{R}, \underline{S}_{kl}^{R}}{2}, \frac{\underline{Q}_{kl}^{N}, \underline{S}_{kl}^{N}}{2}, \frac{\underline{Q}_{kl}^{L}, \underline{S}_{kl}^{L}}{2}\right) \\ &\quad \left(\frac{\overline{Q}_{kl}^{R}, \overline{S}_{kl}^{R}}{2}, \frac{\overline{Q}_{kl}^{N}, S_{kl}^{N}}{2}, \frac{\overline{Q}_{kl}^{L}, \overline{S}_{kl}^{L}}{2}\right)]. \end{aligned}$$

6. **Transpose of H**

$$tr\ (Q) = < \left(\underline{Q}_{lk}^{R},\ \underline{Q}_{lk}^{N},\ \underline{Q}_{lk}^{L} \right), \left(\overline{Q}_{lk}^{R}, \overline{Q}_{lk}^{N}, \overline{Q}_{lk}^{L} \right) >$$

7. **Complement of H**

$$Q^{C} = [\left(\underline{Q}_{kl}^{R}, 1 - \underline{Q}_{kl}^{N}, \underline{Q}_{kl}^{L} \right), \left(\overline{Q}_{kl}^{R}, 1 - \overline{Q}_{kl}^{N}, \overline{Q}_{kl}^{L} \right)]$$

5 The algebraic properties in an LNRM

This section focuses on the algebraic properties of an LNRM, like associativity, commutativity, distributivity, identity and transpose of LNRM.

Proposition 5.1

Consider Q, S and W are three LNRMs of equal order, which satisfy the following properties.

Commutative

1. $Q + S = S + Q$
2. $Q.S = S.Q$
3. $Q \oplus S = S \oplus Q$
4. $Q \bigodot\ S = S \bigodot\ Q$

Associative

1. $(Q + S) + W\ = Q + (S + W)$
2. $(Q.S).W = Q.(S.W)$
3. $(Q \oplus S) = Q \oplus (S \oplus W)$
4. $(Q \bigodot S) \bigodot W\ = W \bigodot (S \bigodot W)$

Proposition 5.2

Let Q be an LNRM, then it satisfies the identity property mentioned below.

1. $Q + I_a = I_a + Q = Q$
2. $Q.I_m = I_m.Q = Q$

Proposition 5.3

The three LNR matrices Q, S and W satisfies the distributive property mentioned below.

$$Q.(S + W) = Q.S + Q.W$$

$$(Q + S).W = Q.W + S.W$$

Proposition 5.4

If Q and S are two LNRM and if, $tr(Q)$ and $tr(S)$ are the transpose of LNRM, Q and S respectively. Then the following conditions are satisfied.

1. $Q + Q = Q$
2. $tr(Q) = tr\,(tr(Q))$
3. $tr(Q + S) = tr(Q) + tr(S)$

   $tr(Q.S) = tr(Q).tr(S)$

6 Energy of Local Neutrosophic Rough Set

Definition 6.1

Consider $Z=\left\{\mathbb{h}_1,\mathbb{h}_2,\ldots,\mathbb{h}_p\right\}, K=\{\mathbb{f}_1,\mathbb{f}_2,\ldots,\mathbb{f}_q\}\subseteq\mathbb{E}$. The matrix of local lower approximation values of the LNRS $\underline{N}(K)$ form an $p\times q$ matrix $\varphi(\underline{N}(K))$, defined by

$$\varphi(\underline{N}(K))=\begin{pmatrix}\underline{\mathbb{M}}_{\mathbb{f}_1}(\mathbb{h}_1) & \underline{\mathbb{M}}_{\mathbb{f}_2}(\mathbb{h}_1) & \cdots & \underline{\mathbb{M}}_{\mathbb{f}_p}(\mathbb{h}_1)\\ \underline{\mathbb{M}}_{\mathbb{f}_1}(\mathbb{h}_2) & \underline{\mathbb{M}}_{\mathbb{f}_2}(\mathbb{h}_2) & \cdots & \underline{\mathbb{M}}_{\mathbb{f}_p}(\mathbb{h}_2)\\ \vdots & \vdots & \ddots & \vdots\\ \underline{\mathbb{M}}_{\mathbb{f}_1}(\mathbb{h}_q) & \underline{\mathbb{M}}_{\mathbb{f}_2}(\mathbb{h}_q) & \cdots & \underline{\mathbb{M}}_{\mathbb{f}_p}(\mathbb{h}_q)\end{pmatrix}$$

Definition 6.2

Consider $Z=\left\{\mathbb{h}_1,\mathbb{h}_2,\ldots,\mathbb{h}_p\right\}, K=\{\mathbb{f}_1,\mathbb{f}_2,\ldots,\mathbb{f}_q\}\subseteq\mathbb{E}$. The matrix of local upper approximation values of the LNRS $\overline{N}(K)$ form an $p\times q$ matrix $\varphi\left(\overline{N}(K)\right)$, defined by

$$\varphi\left(\overline{N}(K)\right)=\begin{pmatrix}\overline{\mathbb{M}}_{\mathbb{f}_1}(\mathbb{h}_1) & \overline{\mathbb{M}}_{\mathbb{f}_2}(\mathbb{h}_1) & \cdots & \overline{\mathbb{M}}_{\mathbb{f}_p}(\mathbb{h}_1)\\ \overline{\mathbb{M}}_{\mathbb{f}_1}(\mathbb{h}_2) & \overline{\mathbb{M}}_{\mathbb{f}_2}(\mathbb{h}_2) & \cdots & \overline{\mathbb{M}}_{\mathbb{f}_p}(\mathbb{h}_2)\\ \vdots & \vdots & \ddots & \vdots\\ \overline{\mathbb{M}}_{\mathbb{f}_1}(\mathbb{h}_q) & \overline{\mathbb{M}}_{\mathbb{f}_2}(\mathbb{h}_q) & \cdots & \overline{\mathbb{M}}_{\mathbb{f}_p}(\mathbb{h}_q)\end{pmatrix}$$

Example 6.3

Considering the set of mobiles $\mathbb{U}=\{\mathbb{h}_1,\mathbb{h}_2,\mathbb{h}_3,\mathbb{h}_4\}$. Let $\mathbb{F}=\{\mathbb{f}_1,\mathbb{f}_2,\mathbb{f}_3,\mathbb{f}_4,\mathbb{f}_5\}$ be the set of features, denoted respectively as Processor & Performance, Battery & Charging, Camera System, Software Updates, and Storage. The LNRS $N(K)$ is formally defined as follows.

Table 2. Specification Values of Mobiles

N(K)	$\mathbb{f}_1$	$\mathbb{f}_2$	$\mathbb{f}_3$	$\mathbb{f}_4$	$\mathbb{f}_5$
$\mathbb{h}_1$	(.4,.8,.7)	(.9,.2,.5)	(.9,.5,.1)	(.8,.1,.9)	(.2,.5,.1)
	(.3,.9,.2)	(.7,.1,.5)	(.6,.1,.3)	(.1,.8,.6)	(.6,.9,.7)
$\mathbb{h}_2$	(.2,.3,.8)	(.1,.4,.6)	(.9,.8,.0)	(.2,.3,.7)	(.4,.9,.5)
	(.1,.8,.3)	(.6,.4,.7)	(.7,1,.2)	(1,.8,.8)	(.3,.7,.5)
$\mathbb{h}_3$	(.7,.4,.1)	(.2,.8,.9)	(.8,.1,.7)	(.9,.3,.0)	(.6,.8,.1)
	(.5,.6,.7)	(.7,.8,.1)	(.9,.5,.4)	(.4,.2,.7)	(.8,.2,.7)
$\mathbb{h}_4$	(.6,.4,.7)	(.9,.6,.2)	(.8,.5,.2)	(.5,.8,.2)	(.7,.7,.1)
	(.7,.4,.5)	(.4,.4,.4)	(.3,.6,.4)	(.8,.9,.4)	(.6,.1,.9)

The following matrix represents the values of the local lower and local upper approximations. The local lower approximation values are determined by choosing the minimum truth value and the maximum levels of indeterminacy and falsity. Hence, this work only takes the truth values.

$$\varphi(\underline{N}(K))=\begin{pmatrix}(.3,.9,.7) & (.7,.2,.5) & (.6,.5,.3) & (.1,.8,.9) & (.2,.9,.7)\\ (.1,.8,.8) & (.1,.4,.7) & (.7,1,.2) & (.2,.8,.8) & (.3\ ,.9,.5)\\ (.5,.6,.7) & (.2,.8,.9) & (.8,.5,.7) & (.4,.3,.7) & (.6,.8,.7)\\ (.6,.4,.7) & (.4,.6,.4) & (.3,.6,.4) & (.5,.9,.4) & (.6,.7,.9)\end{pmatrix}$$

$$\varphi(N(\underline{T}))=\begin{pmatrix}.3 & .7 & .6 & .1 & .2\\ .1 & .1 & .7 & .2 & .3\\ .5 & .2 & .8 & .4 & .6\\ .6 & .4 & .3 & .5 & .6\end{pmatrix},\varphi(N(\underline{I}))=\begin{pmatrix}.9 & .2 & .5 & .8 & .9\\ .8 & .4 & 1 & .8 & .9\\ .6 & .8 & .5 & .3 & .8\\ .4 & .6 & .6 & .9 & .7\end{pmatrix},$$

$$\varphi(N(\underline{F}))=\begin{pmatrix}.7 & .5 & .3 & .9 & .7\\ .8 & .7 & .2 & .8 & .5\\ .7 & .9 & .7 & .7 & .7\\ .7 & .4 & .4 & .4 & .9\end{pmatrix}$$

The local upper approximation values are determined by choosing the maximum truth value and the minimum levels of indeterminacy and falsity. Hence, this study only takes the truth values.

$$\varphi\left(\overline{N}(K)\right)=\begin{pmatrix}(.4,.8,.2) & (.9,.1,.5) & (.9,.1,.1) & (.8,.1,.6) & (.6,.5,.1)\\ (.2,.8,.8) & (.6,.4,.7) & (.9,.8,0) & (.2,.8,.8) & (.4\ ,.9,.5)\\ (.7,.6,.7) & (.7,.8,.9) & (.9,.5,.7) & (.9,.3,.7) & (.8,.8,.7)\\ (.7,.4,.7) & (.9,.6,.4) & (.8,.6,.4) & (.8,.9,.4) & (.7,.7,.9)\end{pmatrix}$$

$$\varphi\left(N(\overline{T})\right)=\begin{pmatrix}.4 & .9 & .9 & .8 & .6\\ .2 & .6 & .9 & .2 & .4\\ .7 & .7 & .9 & .9 & .8\\ .7 & .9 & .8 & .8 & .7\end{pmatrix},\varphi\left(N(\overline{I})\right)=\begin{pmatrix}.8 & .1 & .1 & .1 & .5\\ .8 & .4 & .8 & .8 & .9\\ .6 & .8 & .5 & .3 & .8\\ .4 & .6 & .6 & .9 & .7\end{pmatrix}$$

$$\varphi\left(N(\overline{F})\right)=\begin{pmatrix}.2 & .5 & .1 & .6 & .1\\ .8 & .7 & 0 & .8 & .5\\ .7 & .9 & .7 & .7 & .7\\ .7 & .4 & .4 & .4 & .9\end{pmatrix}$$

Definition 6.4

The pessimistic energy associated with the LNRS $\underline{N}(K)$, represented as $\mathbb{E}^p\left(\underline{N}(K)\right)$, is defined as

$\mathbb{E}^p\left(\underline{N}(K)\right)=\sum_{\omega=1}^{p}\gamma_\omega$,

where $\gamma_1\geq\gamma_2\geq\ldots\geq\gamma_\omega\geq0$ represent the singular values of the matrix $\varphi\left(\underline{N}(K)\right)$ of local lower approximation values of the LNRS $\underline{N}(K)$.

Example 6.5

In Example 5.3, the singular values of the matrix $\varphi\left(\underline{N}(K)\right.$ are computed by constructing $\varphi\left(\underline{N}(K)\right.\!.(\varphi(\underline{N}(K))^T$ and determining its eigenvalues. Thus, the singular values of $\varphi\left(\underline{N}(K)\right.$ correspond to the square roots of the eigenvalues of $\varphi\left(\underline{N}(K)\right.\!.(\varphi(\underline{N}(K))^T$.

$$\gamma_1=1.9213,\ \gamma_2=0.5690,\ \gamma_3=0.5273,\ \gamma_4=0.0831,\gamma_5=0$$

The pessimistic energy values are

$$\mathbb{E}^p\left(N\left(\underline{T}\right)\right)=\sum_{\omega=1}^{5}\gamma_\omega=1.9213+0.5690+0.5273+0.0831+0=3.1007.$$

Definition 6.6

The optimistic energy associated with the LNRS $\overline{N}(K)$, represented as $\mathbb{E}^o(\overline{N}(K))$, is defined as

$$\mathbb{E}^o\left(\overline{N}(K)\right)=\sum_{\omega=1}^{p}\gamma_\omega.$$

where $\underline{\gamma_1}\geq\gamma_2\geq\ldots\geq\gamma_\omega\geq0$ represent the singular values of the matrix $\varphi\left(\underline{N}(K)\right)$ of local upper approximation values of the LNRS $\overline{N}(K)$.

Example 6.7

In Example 5.3, the singular values of the matrix $\varphi\left(\overline{N}(K)\right)$ are,

$$\gamma_1=3.1859,\ \gamma_2=0.5208,\ \gamma_3=0.2309,\ \gamma_4=0.1599,\gamma_5=0$$

The optimistic energy values are

$\mathbb{E}^o\left(N(\overline{T})\right)=\sum_{\omega=1}^{5}\gamma_\omega=3.1859+0.5208+0.2309+0.1599+0=4.0975.$

Definition 6.8

The energy of LNRS $N(K)$, represented as $\mathbb{E}(N(K))$, is defined by

$$\mathbb{E}\left(N(K)\right)=\frac{\mathbb{E}^p\left(\underline{N}(K)\right)+\mathbb{E}^o\left(\overline{N}(K)\right)}{2}$$

Example 6.9

In Example 5.3, the energy value of the matrix $\varphi(N(K))$ is

$$\mathbb{E}(N(K)) = \frac{\mathbb{E}^p\left(N\left(\underline{T}\right)\right) + \mathbb{E}^o\left(N(\overline{T})\right)}{2} = 3.5991.$$

7 Results and discussion

To validate the proposed method, we consider the problem of purchasing a quality machine to initiate a new project in a specific company. The authority to decide with the chief executive officer (d_1), chief financial officer (d_2), chief marketing officer (d_3), chief operating officer (d_4). The following criteria help decide whether to purchase a machine. Production capacity (h_1), cost of equipment (h_2), availability of replacement parts (h_3), safety measures (h_4), and quality (h_5). According to the criteria, the decision authority will choose the most suitable machine among the following alternatives. The alternatives are Machine P, Machine Q, Machine R, Machine S.

First, determine the linguistic values based on the production level for LNRM. They are categorized as Very Low (VL), Low (L), Medium (M), Better (B), and Excellent (E), and are determined using the truth values. These values are displayed in Table 3.

Table 3. Linguistic values

S. No	Production Level	LNRMs
1	very low (VL)	<.1,.8,1>
2	low (L)	<.2,.6,.7>
3	medium (M)	<.5,.4,.5>
4	better (B)	<.8,.3,.2>
5	Excellent (E)	<.9,.1,.0>

According to the linguistic values, the decision authority can estimate the criteria and each alternative. It is displayed in Tables 4 and 5.

Table 4. Weights of criteria

Criteria	d_1	d_2	d_3	d_4
h_1	<.8,.3,.2>	<.2,.6,.7>	<.9,.1,.0>	<.5,.4,.5>
h_2	<.5,.4,.5>	<.1,.8,1>	<.8,.3,.2>	<.2,.6,.7>
h_3	<.8,.3,.2>	<.9,.1,.0>	<.1,.8,1>	<.2,.6,.7>
h_4	<.2,.6,.7>	<.8,.3,.2>	<.5,.4,.5>	<.1,.8,1>
h_5	<.9,.1,.0>	<.2,.6,.7>	<.1,.8,1>	<.5,.4,.5>

The decision authority has assigned the following weight values.

$d_1 = < 0.8, 0.3, 0.2 >$, $d_2 = < 0.5, 0.4, 0.5 >$

$d_3 = < 0.9, 0.1, 0.0 >$, $d_4 = < 0.8, 0.3, 0.2 >$

Utilizing the above-mentioned weight values to determine the LNRM for the criteria. Table 6 illustrates the relationship between the weights allocated to decision makers and the criteria, represented as the LNRM values for each criterion.

$$\begin{aligned} W(d_1 h_1) &= [(\min(.8,.8), max(.3,.3), max(.2,.2)), (max(.8,.8), min(.3,.3), min(.2,.2)] \\ &= [< .8,.3,.2 >< .8,.3,.2 >] \end{aligned}$$

Table 7 illustrates the relationship between the weights allocated to decision makers and the alternatives, represented by the LNRM values for each alternative.

$$\begin{aligned} MP(d_2 h_1) &= [(\min(.2,.5), max(.6,.4), max(.7,.5)), (max(.2,.5), min(.6,.4), min(.7,.5)] \\ &= [< .2,.6,.7 >< .5,.4,.5 >] \end{aligned}$$

Local lower and local upper approximations were computed below.

Let $U = \{h_1, h_2, h_3, h_4, h_5\}$, suppose that the parameter $\alpha = 0.7$ and $\beta = 0.4$, $X = \{h_2, h_3, h_4\}$

Let the equivalence relation $R = \{\{h_1, h_3\}, \{h_2\}, \{h_4\}\{h_5\}\}$

Table 5. Evaluation based on linguistic values for each alternative

Machines	Decision Authorities	h_1	h_2	h_3	h_4	h_5
Machine P (MP)	d_1	<.8,.3,.2>	<.5,.4,.5>	<.9,.1,.0>	<.2,.6,.7>	<.5,.4,.5>
	d_2	<.5,.4,.5>	<.1,.8,1>	<.8,.3,.2>	<.9,.1,.0>	<.2,.6,.7>
	d_3	<.8,.3,.2>	<.5,.4,.5>	<.9,.1,.0>	<.2,.6,.7>	<.8,.3,.2>
	d_4	<.5,.4,.5>	<.2,.6,.7>	<.1,.8,1>	<.2,.6,.7>	<.8,.3,.2>
Machine Q (MQ)	d_1	<.2,.6,.7>	<.1,.8,1>	<.8,.3,.2>	<.5,.4,.5>	<.5,.4,.5>
	d_2	<.9,.1,.0>	<.8,.3,.2>	<.2,.6,.7>	<.5,.4,.5>	<.8,.3,.2>
	d_3	<.8,.3,.2>	<.5,.4,.5>	<.2,.6,.7>	<.1,.8,1>	<.9,.1,.0>
	d_4	<.5,.4,.5>	<.8,.3,.2>	<.5,.4,.5>	<.9,.1,.0>	<.2,.6,.7>
Machine R (MR)	d_1	<.8,.3,.2>	<.9,.1,.0>	<.5,.4,.5>	<.2,.6,.7>	<.8,.3,.2>
	d_2	<.5,.4,.5>	<.9,.1,.0>	<.8,.3,.2>	<.9,.1,.0>	<.5,.4,.5>
	d_3	<.2,.6,.7>	<.2,.6,.7>	<.8,.3,.2>	<.5,.4,.5>	<.1,.8,1>
	d_4	<.9,.1,.0>	<.2,.6,.7>	<.8,.3,.2>	<.1,.8,1>	<.2,.6,.7>
Machine S (MS)	d_1	<.5,.4,.5>	<.2,.6,.7>	<.8,.3,.2>	<.1,.8,1>	<.5,.4,.5>
	d_2	<.9,.1,.0>	<.8,.3,.2>	<.1,.8,1>	<.2,.6,.7>	<.9,.1,.0>
	d_3	<.8,.3,.2>	<.9,.1,.0>	<.5,.4,.5>	<.5,.4,.5>	<.1,.8,1>
	d_4	<.2,.6,.7>	<.5,.4,.5>	<.9,.1,.0>	<.8,.3,.2>	<.2,.6,.7>

Table 6. Local Neutrosophic Rough Matrix with criteria

Criteria	d_1	d_2	d_3	d_4
h_1	<.8,.3,.2> <.8,.3,.2>	<.2,.6,.7> <.5,.4,.5>	<.9,.1,.0> <.9,.1,.0>	<.5,.4,.5> <.8,.3,.2>
h_2	<.5,.4,.5> <.8,.3,.2>	<.1,.8,1> <.5,.4,.5>	<.8,.3,.2> <.9,.1,.0>	<.2,.6,.7> <.8,.3,.2>
h_3	<.8,.3,.2> <.8,.3,.2>	<.5,.4,.5> <.9,.1,.0>	<.1,.8,1> <.9,.1,.0>	<.2,.6,.7> <.8,.3,.2>
h_4	<.2,.6,.7> <.8,.3,.2>	<.5,.4,.5> <.8,.3,.2>	<.5,.4,.5> <.9,.1,.0>	<.1,.8,1> <.8,.3,.2>
h_5	<.8,.3,.2> <.9,.1,.0>	<.2,.6,.7> <.5,.4,.5>	<.1,.8,1> <.9,.1,.0>	<.5,.4,.5> <.8,.3,.2>

Table 7. Local Neutrosophic Rough Matrix with alternative

Decision Authorities	h_1	h_2	h_3	h_4	h_5
d_1	<.8,.3,.2> <.8,.3,.2>	<.5,.4,.5> <.5,.4,.5>	<.8,.3,.2> <.9,.1,.0>	<.2,.6,.7> <.2,.6,.7>	<.5,.4,.5> <.9,.1,.0>
d_2	<.2,.6,.7> <.5,.4,.5>	<.1,.8,1> <.1,.8,1>	<.8,.3,.2> <.9,.1,.0>	<.8,.3,.2> <.9,.1,.0>	<.2,.6,.7> <.2,.6,.7>
d_3	<.8,.3,.2> <.9,.1,.0>	<.5,.4,.5> <.8,.3,.2>	<.1,.8,1> <.9,.1,.0>	<.2,.6,.7> <.5,.4,.5>	<.1,.8,1> <.8,.3,.2>
d_4	<.5,.4,.5> <.5,.4,.5>	<.2,.6,.7> <.2,.6,.7>	<.1,.8,1> <.2,.6,.7>	<.1,.8,1> <.2,.6,.7>	<.5,.4,.5> <.8,.3,.2>

$D(X/[h_2]_R) = \{h_2\} = 1$, $D(X/[h_3]_R) = \{h_1, h_3\} = \frac{1}{2} = 0.5$, $D(X/[h_4]_R) = \{h_4\} = 1$

$$\begin{aligned} \underline{P}_{0.7}(X) &= [h_2]_R \cup [h_4]_R \\ \underline{P}_{0.7}(X) &= \{h_2, h_4\} \\ \overline{P}_{0.4}(X) &= [h_2]_R \cup [h_3]_R \cup [h_4]_R \\ \overline{P}_{0.4}(X) &= \{h_1, h_2, h_3, h_4\} \end{aligned}$$

For feature reduction, only the local upper approximation was adopted. The feature h_5 was removed from further calculations to ensure that potentially relevant features are preserved. Subsequently, matrices were constructed to remove the features from the tables, and the following matrix represents the truth lower approximation matrix.

$$MP(\underline{T}) = \begin{pmatrix} 0.8 & 0.5 & 0.8 & 0.2 \\ 0.2 & 0.1 & 0.8 & 0.8 \\ 0.8 & 0.5 & 0.1 & 0.2 \\ 0.5 & 0.2 & 0.1 & 0.1 \end{pmatrix} \begin{pmatrix} 0.8 & 0.2 & 0.9 & 0.5 \\ 0.5 & 0.1 & 0.8 & 0.2 \\ 0.8 & 0.5 & 0.1 & 0.2 \\ 0.2 & 0.5 & 0.5 & 0.1 \end{pmatrix}$$

$$MP(\underline{T}) = \begin{pmatrix} 1.57 & 0.71 & 1.3 & 0.68 \\ 1.01 & 0.85 & 0.74 & 0.36 \\ 0.01 & 0.36 & 1.23 & 0.54 \\ 0.6 & 0.22 & 0.67 & 0.32 \end{pmatrix}$$

Singular values of this matrix are 3.3417,0.5127,0.1580, and 0.0116. The pessimistic energy of the matrix $MP(\underline{T})$ is the sum of the obtained singular values,

$$\mathbb{E}^p(MP(\underline{T})) = 4.024080$$

The following matrix represents the truth upper approximation matrix.

$$MP(\overline{T}) = \begin{pmatrix} 0.8 & 0.5 & 0.9 & 0.2 \\ 0.5 & 0.2 & 0.9 & 0.9 \\ 0.9 & 0.8 & 0.8 & 0.5 \\ 0.5 & 0.2 & 0.2 & 0.2 \end{pmatrix} \begin{pmatrix} 0.8 & 0.5 & 0.9 & 0.8 \\ 0.8 & 0.5 & 0.9 & 0.8 \\ 0.8 & 0.9 & 0.9 & 0.8 \\ 0.8 & 0.8 & 0.9 & 0.8 \end{pmatrix}$$

$$MP(\overline{T}) = \begin{pmatrix} 1.92 & 1.62 & 2.16 & 1.92 \\ 1.92 & 1.83 & 2.16 & 1.92 \\ 2.48 & 2.06 & 2.79 & 2.48 \\ 0.88 & 0.69 & 0.99 & 0.88 \end{pmatrix}$$

Singular values of this matrix are 7.5729,0.1862, 0, and 0. The optimistic energy of the matrix $MP(\overline{T})$ is the sum of the obtained singular values,

$$\mathbb{E}^o(MP(\overline{T})) = 7.759074$$

Hence, the energy of the matrix is

$$\mathbb{E}(MP) = \frac{\mathbb{E}^p(MP(\underline{T})) + \mathbb{E}^o(MP(\overline{T}))}{2}$$

$$\mathbb{E}(MP) = 5.891577$$

Similarly, the matrices of pessimistic and optimistic energy values can be determined using the upper and lower approximations of truth, falsity, and the intermediate values of the four machines given below.

Energy Values Table:

Machines	Energy Values of LNRM
{'MP'}	5.8916
{'MQ'}	6.0452
{'MR'}	6.1786
{'MS'}	6.2345

Fig 1. Energy values of four machines

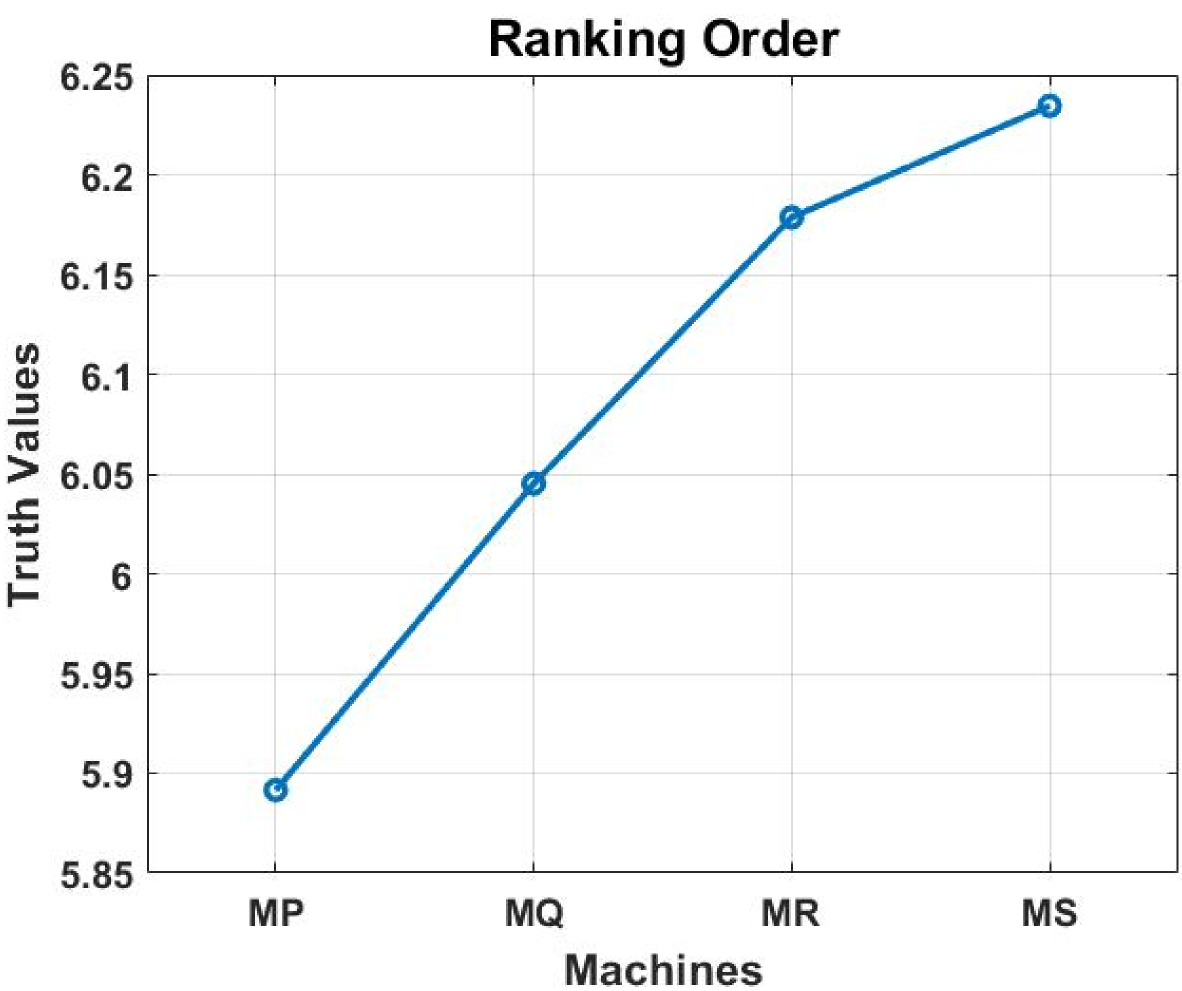

Fig 2. Ranking Order Based on Energy Values

Table 8. Pessimistic and Optimistic Energy Values

Machines	Truth		Indeterminacy		Falsity	
	Pessimistic Energy	Optimistic Energy	Pessimistic Energy	Optimistic Energy	Pessimistic Energy	Optimistic Energy
MP	4.024080	7.759074	4.661002	1.752074	6.586915	1.598292
MQ	3.023986	9.066343	5.144180	1.337670	7.409816	1.255817
MR	3.443631	8.913604	4.645878	1.395605	6.584849	1.131643
MS	3.279699	9.189341	5.063598	1.338237	7.268073	1.144835

Table 9. Ranking Order

Alternative	Truth values	Scale of order
MP	5.8916	IV
MQ	6.0452	III
MR	6.1786	II
MS	6.2345	I

The calculations for this problem were performed using MATLAB, and the results based on the truth values are illustrated in the diagram below. The figure shows the energy values of the four machines.

To determine the ranking order based on truth values. The values vary between approximately 6.23 and 5.89.

In Figure 2, the line links four circular points, each representing one of the machines. The height of each point on the vertical axis reflects the truth value for that particular machine. The graph provides a visual representation of the ranking of the four machines, MP, MQ, MR, and MS according to their truth values as assessed by the LNRS.

The scale of order is $MS > MR > MQ > MP$. Figure 1 shows that the machine S (MS) is the best machine to compare with other machines.

8 Comparative analysis of existing methods

This section introduces the two existing methods that are used to address decision-making problems on Rough Neutrosophic Sets (RNS). The method proposed by Jeni et al.'s method [6] is based on the energy of RNS, which evaluates the eigenvalues of the matrix along with their mean. The method proposed by Jeni et al.'s method [5] applies the same approach to multicriteria decision-making, using the ranking of RNS.

The proposed method, based on the energy of LNRM, was applied to the existing methodology, and a comparison of decision outcomes indicates that they are equivalent. This section presents the results, which serve to validate the findings.

Jeni et al.'s method [6] and its limitations 8.1

Here, we demonstrate the method by applying the energy of the LNRM to the Jeni method [6]. The Local lower and local upper approximations were computed below.

Let $U = \{C_1, C_2, C_3, C_4, C_5, C_6\}$, suppose that the parameter $\alpha = 0.7$ and $\beta = 0.4$, $X = \{C_2, C_3, C_4\}$. Let the equivalence relation $R = \{C_1, C_3\}, \{C_2\}, \{C_4\}\{C_5, C_6\}$

$\underline{P}_{0.7}(X) = \{C_2, C_4\}$, $\overline{P}_{0.4}(X) = \{C_1, C_2, C_3, C_4\}$

For feature reduction, only the local upper approximation was adopted. The feature C_5, C_6 was removed from further calculations to ensure that potentially relevant features are preserved. Subsequently, matrices were constructed to remove the features from the tables, and the following matrix represents the truth lower approximation matrix.

$$A_1(\underline{T}_{ij})_{n\times m} * W(\underline{T}_{ij})_{m\times n} = \begin{pmatrix} 0.8 & 0.5 & 0.8 & 0.5 \\ 0.5 & 0.8 & 0.5 & 0.8 \\ 0.35 & 0.5 & 0.8 & 0.5 \\ 0.5 & 0.1 & 0.5 & 0.8 \end{pmatrix} \begin{pmatrix} 0.8 & 0.5 & 0.8 & 0.8 \\ 0.8 & 0.8 & 0.9 & 0.8 \\ 0.5 & 0.5 & 0.5 & 0.5 \\ 0.5 & 0.8 & 0.8 & 0.8 \end{pmatrix}$$

Singular values of this matrix are 6.3942, 0.1082, 0.0495, and 0.0161. The pessimistic energy of this matrix is the sum of the obtained singular values. $\mathbb{E}^p\left(A_1\left(\underline{T}\right)\right) = 6.568147$. Similarly, finding the optimistic energy of this matrix is equivalent to summing the obtained singular values. $\mathbb{E}^o\left(A_1\left(\overline{T}\right)\right) = 10.385029$. Hence, the energy of this matrix is $\mathbb{E}\left(A_1\right) = 8.476588$. Similarly, calculate the energy of the remaining alternatives. The truth values of the energy are considered as the scaling order.

Table 10. Energy values and ranking order

Procedure	Decision Values	Obtained ranking
Jeni et al.'s method [6]	$Place\ A = 19.3015,\ Place\ B = 17.3679,$ $Place\ C = 20.2027, Place\ D = 16.8292,$ $Place\ E = 17.8091.$	$A_3 > A_1 > A_5 > A_2 > A_4$
Method based on LNRM	$\mathbb{E}(A_1) = 8.476588, \mathbb{E}(A_2) = 7.618253, \mathbb{E}(A_3) = 9.1125, \mathbb{E}(A_4) = 7.70179,$ $\mathbb{E}(A_5) = 7.8455.$	$A_3 > A_1 > A_5 > A_4 > A_2$

In both methods, the decision criterion is based on selecting the alternative with the highest value. Accordingly, both approaches identify **Place C** as the optimal location, indicating that the proposed method is consistent with Jeni's method in terms of decision outcomes.

However, Jeni et al.'s method [6] involves computing eigenvalues and substituting them into the energy formula, which is a lengthy and computationally intensive process. Moreover, eigenvalue-based techniques require the decision matrix to be square; hence, Jeni et al.'s method [6] cannot be applied to non-square matrices and requires transforming them into square matrices. In contrast, the proposed method overcomes this limitation, in singular value calculation, even when the original matrix is non-square, forming the products $A^T A$ or AA^T results in square, and positive semidefinite matrices. The eigenvalues of these matrices determine the singular values of the original matrix. Consequently, singular value computation can be applied to both square and non-square matrices. Additionally, the proposed approach involves fewer computational steps, reduces computational time, and simplifies the overall calculation process. Hence, the proposed approach demonstrates higher efficiency and flexibility while producing the same optimal decision as Jeni's method.

Jeni et al.'s method [5] and its limitations 8.2

Next, we demonstrate the method by applying the energy of LNRM to Jeni et al.'s method [5] based on the ranking of RNS. The Local lower and local upper approximations were computed below.

Let $U = \{C_1, C_2, C_3\}$, suppose that the parameter $\alpha = 0.7$ and $\beta = 0.4$, $X = \{C_1, C_2\}$

Let the equivalence relation $R = \{C_1\}, \{C_2,\ C_3\}$

$\underline{P}_{0.7}(X) = \{C_2, C_4\}, \overline{P}_{0.4}(X) = \{C_1, C_2, C_3\}$

For feature reduction, only the local upper approximation was adopted. In this case, no features have been removed. The pessimistic energy of this matrix is the sum of the obtained singular values. $\mathbb{E}^p\left(F_1\left(\underline{T}\right)\right) = 4.958161$. Similarly, finding the optimistic energy of this matrix is equivalent to summing the obtained singular values. $\mathbb{E}^o\left(F_1\left(\overline{T}\right)\right) = 8.476976$. Hence, the energy of this matrix is $\mathbb{E}(F_1) = 6.717569$. Similarly, calculate the energy of the remaining alternatives. The truth values of the energy are considered as the scaling order.

Table 11. Energy values and ranking order

Procedure	Decision Values	Obtained ranking
Jeni et al.'s method [5]	$F_1 = 3.0074, F_2 = 2.1009,\ F_3 = 2.7143,$ $F_4 = 2.9480, F_5 = 3.1869, F_6 = 2.5328, F_7 = 1.7069, F_8 = 3.4087.$	$F_8 > F_5 > F_1 > F_4 > F_3 > F_6 > F_2 > F_7$
Method based on LNRM	$\mathbb{E}(F_1) = 6.717569, \mathbb{E}(F_2) = 6.5532,$ $\mathbb{E}(F_3) = 6.7798, \mathbb{E}(F_4) = 6.6568,$ $\mathbb{E}(F_5) = 6.5557, \mathbb{E}(F_6) = 6.672,$ $\mathbb{E}(F_7) = 6.3613, \mathbb{E}(F_8) = 6.8912$.	$F_8 > F_3 > F_1 > F_6 > F_4 > F_5 > F_2 > F_7$

Both methods determine the decision by selecting the alternative with the highest value. Consequently, each approach identifies $\mathbf{F_8}$**(Factory 8)** as the optimal choice, demonstrating that the proposed method is consistent with Jeni's method in terms of decision outcomes.

Hence, Jeni et al.'s method [5] requires eigenvalue computation along with the evaluation of score and accuracy functions, which involve multiple computational steps to obtain the final decision. The proposed method involves fewer calculations, reduces computational time, and simplifies the overall decision-making process. Therefore, the proposed approach provides greater efficiency and flexibility while yielding the same optimal decision as Jeni's method.

Additionally, both the methods proposed by Jeni et al. [5] and [4] require matrix-based calculations. In manual computations, decision outcomes can usually be obtained only for matrices of small order, such as 4×4 or 5×5. As the matrix order increases, deriving decision outcomes becomes more challenging. In this context, the proposed method can efficiently assess matrices of order $n \times n$ using MATLAB. The implementation in MATLAB coding considerably reduces computational time and simplifies the calculation of decision values for large matrices, which is a notable advantage of the proposed approach.

9 Limitations and Future Works

The proposed method has a limitation in that the calculation of local lower and local upper approximation values depends on the target concept and the selected parameters. Therefore, any changes in the target set or the values of the parameters may cause variations in the approximation results. This behaviour is inherent to approximation-based rough decision models and indicates the need for further research on parameter selection and stability analysis.

Future research may focus on optimal parameter selection and stability analysis to enhance the robustness of the proposed decision-making framework.

10 Conclusion

This study introduced the idea of the LNRM, along with its algebraic operations and properties, as well as the energy of the LNRS. To address the gap by introducing the energy concept within the framework of the LNRS. By employing MATLAB software for matrix-based computations, the approach overcomes the limitations of manual calculations that are confined to small matrices, enabling efficient processing of larger $n \times n$ matrices. The use of computational tools significantly enhances both accuracy and efficiency, making the method suitable for large-scale and real-world applications. A comparative analysis is conducted to verify the effectiveness of the proposed work, and the approach is further applied to validate the process of selecting a quality machine for initiating a new project in a specific company.

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