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RESEARCH ARTICLE

Bipolar Intuitionistic Anti Fuzzy α-Ideal of A BP-Algebra

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Abstract

Objectives: The focus of this article is to present the idea of employing complement, union, and intersection and examine these fundamental operations in both bipolar intuitionistic anti fuzzy (BIAF) ideals. We investigate the conditions of BIAF ideal and BIAF α-ideal of a BP-algebra in this research work. Comparing the fuzzy set to a bipolar intuitionistic fuzzy set revealed a definite result. Therefore, the concept of bipolar intuitionistic fuzzy set was defined as a combination of intuitionistic fuzzy set and bipolar fuzzy set. It provides useful outcomes for problem analysis. **Methods:** We provide the conditions for both BIAF ideals. We also give characterizations of a bipolar intuitionistic fuzzy (BIF) set. **Findings:** Implementing the fuzzy theory and ideal theory of a BP-algebra is the main aim of this study that can represent uncertainty and bipolarity in data. We consider the methodology as a concept of the BIAF ideals and discuss the operations. **Novelty:** The introduction of the BIAF ideals and the thorough examination of the basic operations constitute the research's uniqueness. In BIAF ideals, the study highlighted the concepts of complement, the union of two BIAF ideals and intersection of two BIAF ideals that can be investigated to provide more flexibility in analyzing problems. This study is easy to understand, very effective, and appropriate for novices in a variety of professions. In this article, BIAF ideals play the major role with some basic operations.
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Keywords: BP-Algebra; Fuzzy Ideal; Bipolar Fuzzy Ideal; BIAF Ideal; BIAF α-Ideal

1 Introduction

To deal with the uncertainties in our everyday lives, Zadeh created the idea of fuzzy set in 1965. In applied mathematics, fuzzy sets are incredibly helpful in solving numerous problems. Many fuzzy sets of generalization are then shown, as intuitionistic fuzzy set and interval valued fuzzy set. K.T.Atanassov introduced the theory of intuitionistic fuzzy set in 1986. As a generalization of conventional fuzzy set, Zhang first proposed the idea of bipolar fuzzy set in 1994. By combining the concepts of intuitionistic fuzzy set and bipolar fuzzy set, Ezhilmaran[1] developed the term bipolar intuitionistic fuzzy set.

The notions of α-ideal, fuzzy α-ideal and bipolar fuzzy α-ideal are defined, and a lot of properties are investigated by Osama Rashad El Gendy[2] in 2020. A theoretical analysis of bipolar intuitionistic fuzzy soft ideal was produced by Balamurugan M et al.[3], who described a bipolar intuitionistic fuzzy soft set with a soft set. Natthinee Deetae et al.[4] have discussed the ideas of the neutrosophic bipolar valued fuzzy left (right, interior) ideal. On bipolar intuitionistic fuzzy matrices, Muthuraji et al.[5] defined the Hamacher sum, Hamacher product, Hamacher scalar multiplication, and Hamacher exponentiation operations. Nithyanandham D et al.[6] introduced the notions of dominance, covering, and matching in bipolar intuitionistic fuzzy graph (BIFG). Deva N et al.[7] presented the bipolar intuitionistic anti-fuzzy graph (BIAFG). It is stated that the bipolar intuitionistic fuzzy matrix and its determinant are notions by Deva N. et al.[8]. The flexible results of the problems are shown using the three-dimensional representation of the bipolar intuitionistic fuzzy matrix.

In this study, we intended to present the union of two BIF sets, the intersection of two BIF sets and the complement of the BIF set. This concept provides a new algebraic tool in many uncertainties. Therefore, the present study introduced BIAF ideal and BIAF α-ideal of BIF set. In particular, we discussed the complement of BIAF α-ideal, the union of two BIAF α-ideals, and the intersection of two BIAF α-ideals.

The fuzzy set theory encompasses a variety of fuzzy set extensions, such as interval valued fuzzy set, intuitionistic fuzzy set, ambiguous set, and others. Group theory, ring theory, graph theory, medical science, social science, computer network, expert system, decision making are actively researching the theory of BIF set.

2 Methodology

To support the theoretical realm of the BIAF ideals, certain essential definitions were covered in this section.

2.1 Materials

Definition: 2.1 [2]

If an algebra $(P, \circledast, O)$ meets the following axioms, it is referred to as BP-algebra.

(i) $p \circledast p = O$

(ii) $p \circledast (p \circledast q) = q$

(iii) $(p \circledast r) \circledast (q \circledast r) = p \circledast q$, for all $p, q, r \in P$.

A binary relation "$\leq$" in P can be defined as follows: $p \leq q$ if and only if $p \circledast q = O$.

Example: 2.1(a)

Assume that $P = \{ O, \alpha, \beta, \gamma\}$. Use the following table to define $\circledast$ on P:

Table 1.

$\circledast$	O	α	β	γ
O	O	α	β	γ
α	α	O	γ	β
β	β	γ	O	α
γ	γ	β	α	O

Then a BP-algebra is $(P, \circledast, O)$.

Definition: 2.2 [2]

Consider the BP-algebra $(P, \circledast, O)$. The following criteria determine whether a fuzzy set μ in P is an anti fuzzy ideal of P.

(i) $\mu(O) \leq \mu(p)$

(ii) $\mu(p) \leq \text{maximum}\{\mu(p \circledast q), \ \mu(q)\}$, for all $p, q \in P$.

Example: 2.2(a)

Let $P = \{O, \alpha, \beta, \gamma\}$ be a BP-algebra, where example 2.1(a) provides the operation $\circledast$. Let the $n_1 < n_2 \leq n_3$ and that n_1, n_2, and $n_3 \in [0, 1]$. The μ mapping is defined as

$P \longrightarrow [0, 1]$, according to $\mu(O) = n_1$, $\mu(\alpha) = n_2$, and $\mu(\beta) = \mu(\gamma) = n_3$. As per the standard computation, μ is an anti fuzzy ideal of P.

Definition: 2.3 [2]

Consider the BP-algebra $(P, \circledast, O)$. The following criteria determine whether a fuzzy set μ_α in P is an anti fuzzy α-ideal of P.

(i) $\mu_\alpha(O) \leq \mu_\alpha(p)$

(ii) $\mu_\alpha(q \circledast r) \leq$ maximum $\{\mu_\alpha(p \circledast r),\ \mu_\alpha(p \circledast q)\}$, for all $p, q, r \in P$.

Definition: 2.4[2]

In BP-algebra P, a bipolar fuzzy set A is defined as A = $\{(p, \mu_A^P(p), \mu_A^N(p)) / p \in P\}$, where $\mu_A^P : P \longrightarrow [0, 1]$ and $\mu_A^N : P \longrightarrow [-1, 0]$ are mappings.

Definition: 2.5 [2]

Any two bipolar fuzzy sets, A = (μ_A^P, μ_A^N) and $B = (\mu_B^P, \mu_B^N)$ in P, can be assumed, then it can be defined as:

(i) A ∩ B = $\{(p$, minimum $(\mu_A^P(p), \mu_B^P(p))$, maximum $(\mu_A^N(p), \mu_B^N(p))) / p \in P\}$

(ii) A ∪ B = $\{(p$, maximum $(\mu_A^P(p), \mu_B^P(p))$, minimum $(\mu_A^N(p), \mu_B^N(p))) / p \in P\}$

Definition: 2.6 [2]

If a bipolar fuzzy set A = (μ_A^P, μ_A^N) of BP-algebra P meets the following criteria, it is referred to as a bipolar anti fuzzy (BAF) ideal of P.

(i) $\mu_A^P(O) \leq \mu_A^P(p)\ and\ \mu_A^N(O) \geq \mu_A^N(p)$

(ii) $\mu_A^P(p) \leq$ maximum $\{\mu_A^P(p \circledast q),\ \mu_A^P(q)\}$

(iii) $\mu_A^N(p) \geq minimum\{\mu_A^N(p \circledast q),\ \mu_A^N(q)\}$, for all $p, q \in P$.

Example: 2.6(a)

Example 2.1(a) provides the operation $\circledast$ for a BP-algebra $P = \{O, \alpha, \beta, \gamma\}$. We define a bipolar fuzzy set A = (μ_A^P, μ_A^N) by

$\mu_A^P = \begin{pmatrix} O & \alpha & \beta & \gamma \\ 0.3 & 0.3 & 0.7 & 0.7 \end{pmatrix}$ and

$\mu_A^N = \begin{pmatrix} O & \alpha & \beta & \gamma \\ -0.6 & -0.6 & -0.8 & -0.8 \end{pmatrix}$.

The usual calculation gives that A = (μ_A^P, μ_A^N) is a BAF ideal of P.

Definition: 2.7 [2]

If a bipolar fuzzy set A = $(\mu_{\alpha_A}^P, \mu_{\alpha_A}^N)$ of BP-algebra P meets the following criteria, then it is referred to as a BAF α-ideal of P.

(i) $\mu_{\alpha_A}^P(O) \leq \mu_{\alpha_A}^P(p)$ and $\mu_{\alpha_A}^N(O) \geq \mu_{\alpha_A}^N(p)$

(ii) $\mu_{\alpha_A}^P(q \circledast r) \leq$ maximum $\{\mu_{\alpha_A}^P(p \circledast r),\ \mu_{\alpha_A}^P(p \circledast q)\}$

(iii) $\mu_{\alpha_A}^N(q \circledast r) \geq minimum\{\mu_{\alpha_A}^N(p \circledast r),\ \mu_{\alpha_A}^N(p \circledast q)\}$, for all $p, q, r \in P$.

Definition: 2.8

The definition of a BIF set A of BP-algebra P is
$\{(p, \mu_A^P(p), \mu_A^N(p), \nu_A^P(p), \nu_A^N(p)) / p \in P\}$, where $\mu_A^P : P \to [0, 1]$, $\mu_A^N : P \to [-1, 0]$, $\nu_A^P : P \to [0, 1]$, $\nu_A^N : P \to [-1, 0]$ are mappings, and therefore, it fulfills
$0 \leq \mu_A^P(p) + \nu_A^P(p) \leq 1$ and $-1 \leq \mu_A^N(p) + \nu_A^N(p) \leq 0$.

2.2 Methods

Definition: 2.9

Assuming A and B are any two BIF sets in P, we define

(i) A ∩ B = $\{(p$, minimum $(\mu_A^P(p),\ \mu_B^P(p))$, maximum $(\mu_A^N(p),\ \mu_B^N(p))$,
maximum $(\nu_A^P(p), \nu_B^P(p))$, minimum $\{\nu_A^N(p), \nu_B^N(p))) / p \in P\}$

(ii) A ∪ B = $\{(p$, maximum $(\mu_A^P(p),\ \mu_B^P(p))$, minimum $(\mu_A^N(p),\ \mu_B^N(p))$,
minimum $(\nu_A^P(p), \nu_B^P(p))$, maximum $\{\nu_A^N(p), \nu_B^N(p))) / p \in P\}$

(iii) $\bar{A} = \{(p, \nu_A^P(p), \nu_A^N(p), \mu_A^P(p), \mu_A^N(p)) / p \in P\}$.

Definition: 2.10

In BP-algebra, BIF set A = $\{\mu_A^P, \mu_A^N, \nu_A^P, \nu_A^N / p \in P\}$. If a BIAF ideal of P meets the following criteria, such as:

(i) $\mu_A^P(O) \leq \mu_A^P(p)$ and $\mu_A^N(O) \geq \mu_A^N(p)$

(ii) $\mu_A^P(p) \leq$ maximum$\{\mu_A^P(p \circledast q),\ \mu_A^P(q)\}$

(iii) $\mu_A^N(p) \geq minimum\ \{\mu_A^N(p \circledast q),\ \mu_A^N(q)\}$

(iv) $\nu_A^P(0) \geq \nu_A^P(p)$ and $\nu_A^N(0) \leq \nu_A^N(p)$

(v) $\nu_A^P(p) \geq$ minimum $\{\nu_A^P(p \circledast q),\ \nu_A^P(q)\}$

(vi) $\nu_A^N(p) \leq$ maximum $\{\nu_A^N(p \circledast q),\ \nu_A^N(q)\}$, for all $p, q \in P$.

Example: 2.10(a)

Let $P = \{O, \alpha, \beta, \gamma\}$ be a BP-algebra, where example 2.1(a) provides the operation $\circledast$. We define a BIF set A = $\{\mu_A^P, \mu_A^N, \nu_A^P, \nu_A^N\}$, where

$$\mu_A^P = \begin{pmatrix} O & \alpha & \beta & \gamma \\ 0.3 & 0.3 & 0.7 & 0.7 \end{pmatrix} \qquad \nu_A^P = \begin{pmatrix} O & \alpha & \beta & \gamma \\ 0.5 & 0.5 & 0.2 & 0.2 \end{pmatrix}$$

$$\mu_A^N = \begin{pmatrix} O & \alpha & \beta & \gamma \\ -0.6 & -0.6 & -0.8 & -0.8 \end{pmatrix} \qquad \nu_A^N = \begin{pmatrix} O & \alpha & \beta & \gamma \\ -0.3 & -0.3 & -0.1 & -0.1 \end{pmatrix}.$$

The usual calculation gives that A = $\{\mu_A^P, \mu_A^N, \nu_A^P, \nu_A^N\}$ is a BIAF ideal of P.

Definition: 2.11

If a BIF set A = $\{(p, \mu_{\alpha_A}^P(p), \mu_{\alpha_A}^N(p), \nu_{\alpha_A}^P(p), \nu_{\alpha_A}^N(p)) / p \in P\}$, of BP-algebra P meets the following criteria, it is referred to as a BIAF α-ideal of P.

(i) $\mu_{\alpha_A}^P(O) \le \mu_{\alpha_A}^P(p)$ and $\mu_{\alpha_A}^N(O) \ge \mu_{\alpha_A}^N(p)$

(ii) $\mu_{\alpha_A}^P(q \circledast r) \le maximum\ \{\mu_{\alpha_A}^P(p \circledast r),\ \mu_{\alpha_A}^P(p \circledast q)\}$

(iii) $\mu_{\alpha_A}^N(q \circledast r) \ge minimum\ \{\mu_{\alpha_A}^N(p \circledast r),\ \mu_{\alpha_A}^N(p \circledast q)\}$

(iv) $\nu_{\alpha_A}^P(O) \ge \nu_{\alpha_A}^P(p)$ and $\nu_{\alpha_A}^N(O) \le \nu_{\alpha_A}^N(p)$

(v) $\nu_{\alpha_A}^P(q \circledast r) \ge minimum\{\nu_{\alpha_A}^P(p \circledast r),\ \nu_{\alpha_A}^P(p \circledast q)\}$

(vi) $\nu_{\alpha_A}^N(q \circledast r) \le maximum\{\nu_{\alpha_A}^N(p \circledast r),\ \nu_{\alpha_A}^N(p \circledast q)\}$, for all $p, q, r \in P$.

3 Results and Discussion

In this section, we proved the following main theorems.

Result: 3.1

$\overline{\overline{A}}$ = A is a BIAF ideal of BP-algebra P if A is a BIAF ideal of P.

Proof: Let A be a BIAF ideal of P. Consider $O, p, q \in$ A.

a) Now $\mu_{\overline{\overline{A}}}^P(O) = \nu_{\overline{A}}^P(O) = \mu_A^P(O) \le \mu_A^P(p)$.

Therefore $\mu_{\overline{\overline{A}}}^P(O) \le \mu_A^P(p)$.

Now $\mu_{\overline{\overline{A}}}^N(O) = \nu_{\overline{A}}^N(O) = \mu_A^N(O) \ge \mu_A^N(p)$.

Therefore $\mu_{\overline{\overline{A}}}^N(O) \ge \mu_A^N(p)$.

b) Now $\mu_{\overline{\overline{A}}}^P(p) = \nu_{\overline{A}}^P(p) = \mu_A^P(p) \le$ maximum $\{\mu_A^P(p \circledast q),\ \mu_A^P(q)\}$.

Therefore $\mu_{\overline{\overline{A}}}^P(p) \le$ maximum $\{\mu_A^P(p \circledast q),\ \mu_A^P(q)\}$.

c) Now $\mu_{\overline{\overline{A}}}^N(p) = \nu_{\overline{A}}^N(p) = \mu_A^N(p) \ge$ minimum $\{\mu_A^N(p \circledast q),\ \mu_A^N(q)\}$.

Therefore $\mu_{\overline{\overline{A}}}^N(p) \ge$ minimum $\{\mu_A^N(p \circledast q),\ \mu_A^N(q)\}$

d) Now $\nu_{\overline{\overline{A}}}^P(O) = \mu_{\overline{A}}^P(O) = \nu_A^P(O) \ge \nu_A^P(p)$.

Therefore $\nu_{\overline{\overline{A}}}^P(O) \ge \nu_A^P(p)$.

Now $\nu_{\overline{\overline{A}}}^N(O) = \mu_{\overline{A}}^N(O) = \nu_A^N(O) \le \nu_A^N(p)$.

Therefore $\nu_{\overline{\overline{A}}}^N(O) \le \nu_A^N(p)$.

e) Now $\nu_{\overline{\overline{A}}}^P(p) = \mu_{\overline{A}}^P(p) = \nu_A^P(p) \ge$ minimum$\{\nu_A^P(p \circledast q),\ \nu_A^P(q)\}$

Therefore $\nu_{\overline{\overline{A}}}^P(p) \ge$ minimum$\{\nu_A^P(p \circledast q),\ \nu_A^P(q)\}$.

f) Now $\nu_{\overline{\overline{A}}}^N(p) = \mu_{\overline{A}}^N(p) = \nu_A^N(p) \le$ maximum $\{\nu_A^N(p \circledast q),\ \nu_A^N(q)\}$

Therefore $\nu_{\overset{=}{A}^N}(p) \leq maximum\{\nu_A^N(p \circledast q), \nu_A^N(q)\}$

Therefore $\overset{=}{A}$ = A is a BIAF ideal of P.

Result: 3.2

A BIAF ideal of P is once more formed by the intersection of any two BIAF ideals of BP-algebra P.

Proof: Let A and B be two BIAF ideals of P.

Consider $O, p, q \in$ A ∩ B $\Longrightarrow O, p, q \in$ A and $O, p, q \in$ B.

a) Now $\mu^P{}_{A\cap B}(O) =$ minimum $\{\mu^P{}_A(O), \mu^P{}_B(O)\}$

$\leq$ minimum $\{\mu_A^P(p), \ \mu_B^P(p)\} = \mu_{A\cap B}^P(p)$.

Therefore $\mu_{A\cap B}^P(O) \leq \mu_{A\cap B}^P(p)$.

Now $\mu_{A\cap B}^N(O) =$ maximum $\{\mu_A^N(O), \ \mu_B^N(O)\}$

$\geq$maximum $\{\mu_A^N(p), \ \mu_B^N(p)\} = \mu_{A\cap B}^N(p)$.

Therefore $\mu_{A\cap B}^N(O) \geq \mu_{A\cap B}^N(p)$.

b) Now $\mu^P{}_{A\cap B}(p) = minimum\,\{\mu^P{}_A(p), \mu^P{}_B(p)\}$

$\leq$minimum $\{\{\mu^P{}_A(p \circledast q), \{\mu^P{}_A(q)\}, \quad \max imum\{\mu^P{}_B(p \circledast q), \mu^P{}_B(q\}\}$

$= maximum\ \{\{\mu_A^P(p \circledast q), \mu_B^P(p \circledast q)\}, \ minimum\{\mu_A^P(q), \mu_B^P(q)\}\}$

= maximum $\{\mu_{A\cap B}^P(p \circledast q), \ \mu_{A\cap B}^P(q)\}$.

Therefore $\mu_{A\cap B}^P(p) \leq$ max $\{\mu_{A\cap B}^P(p \circledast q), \ \mu_{A\cap B}^P(q)\}$.

c) Now $\mu_{A\cap B}^N(p) = maximum\,\{\mu_A^N(p), \ \mu_B^N(p)\}$

$\geq$maximum $\{\{\mu_A^N(p \circledast q), \mu_A^N(q)\}, \ minimum\{\mu_B^N(p \circledast q), \mu_B^N(q)\}\}$

$= minimum\{\{\mu_A^N(p \circledast q), \mu_B^N(p \circledast q)\}, maximum\{\mu_A^N(q), \mu_B^N(q)\}\}$

= minimum $\{\mu_{A\cap B}^N(p \circledast q), \ \mu_{A\cap B}^N(q)\}$.

Therefore $\mu_{A\cap B}^N(p) \geq$ min $\{\mu_{A\cap B}^N(p \circledast q), \ \mu_{A\cap B}^N(q)\}$.

d) Now $\nu_{A\cap B}^P(O) =$ maximum $\{\nu_A^P(O), \ \nu_B^P(O)\}$

$\geq$maximum $\{\nu_A^P(p), \ \nu_B^P(p)\} = \nu_{A\cap B}^P(p)$.

Therefore $\nu_{A\cap B}^P(O) \geq \nu_{A\cap B}^P(p)$.

Now $\nu_{A\cap B}^N(O) =$ minimum $\{\nu_A^N(O), \ \nu_B^N(O)\}$

$\leq$minimum $\{\nu_A^N(p), \ \nu_B^N(p)\} = \nu_{A\cap B}^N(p)$.

Therefore $\nu_{A\cap B}^N(O) \leq \nu_{A\cap B}^N(p)$.

e) Now $\nu_{A\cap B}^P(p) = maximum\,\{\nu_A^P(p), \ \nu_B^P(p)$

$\geq$maximum $\{\{\nu_A^P(p \circledast q), \ \nu_A^P(q)\}, \{\nu_B^P(p \circledast q), \ \nu_B^P(q)\}\}$

= minimum$\{\{\nu_A^P(p \circledast q), \nu_B^P(p \circledast q)\}, \{\nu_A^P(q), \ \nu_B^P(q)\}\}$

= minimum $\{\nu_{A\cap B}^P(p \circledast q), \ \nu_{A\cap B}^P(q)\}$.

Therefore $\nu_{A\cap B}^P(p) \geq$ min $\{\nu_{A\cap B}^P(p \circledast q), \ \nu_{A\cap B}^P(q)\}$.

f) Now $\nu_{A\cap B}^N(p) = minimum\,\{\nu_A^N(p), \ \nu_B^N(p)\}$

$\leq$minimum $\{\nu_A^N\{(\mathrm{p} \circledast \mathrm{q}), \nu_A^N(\mathrm{q})\}, \{\nu_B^N(\mathrm{p} \circledast \mathrm{q}), \nu_B^N(\mathrm{q})\}\}$

= maximum $\{\min imum\,\{\nu_A^N(\mathrm{p} \circledast \mathrm{q}), \nu_B^N(\mathrm{p} \circledast \mathrm{q})\}, \quad minimum\,\{\nu_A^N(\mathrm{q}), \nu_B^N(\mathrm{q})\}\}\}$

= maximum $\{\nu_{A\cap B}^N(p \circledast q), \ \nu_{A\cap B}^N(q)\}$.

Therefore $\nu_{A\cap B}^N(p) \leq$ max $\{\nu_{A\cap B}^N(p \circledast q), \ \nu_{A\cap B}^N(q)\}$.

Hence intersection of any two BIAF ideals of P is again a BIAF ideal of P.

Result: 3.3

Union of any two BIAF ideals of BP-algebra P is a BIAF ideal of P if either is contained in the other.

Proof: Let A and B be two BIAF ideals of P.

If A ⊆ B ⟹ A ∪ B = B and if B ⊆ A ⟹ A ∪ B = A.

Consider $O, p, q \in$ A ∪ B $\Longrightarrow O, p, q \in$ A or $O, p, q \in$ B

a) Now $\mu_{A\cup B}^P(O) =$ maximum $\{\mu_A^P(O), \ \mu_B^P(O)\}$

$\leq$ maximum $\{\mu_A^P(p), \ \mu_B^P(p)\} = \mu_{A\cup B}^P(p)$.

Therefore $\mu_{A\cup B}^P(O) \leq \mu_{A\cup B}^P(p)$.

Now $\mu_{A\cup B}^N(O) =$ minimum $\{\mu_A^N(O), \ \mu_B^N(O)\}$

$\geq minimum\ \{\mu_A^N(p), \ \mu_B^N(p)\} = \mu_{A\cup B}^N(p)$

Therefore $\mu^N_{A\cup B}(O) \geq \mu^N_{A\cup B}(p)$.
b) Now $\mu^P_{A\cup B}(p) =$ maximum { $\mu^N_A(O),\ \mu^N_B(O)\}$
$\geq$maximum { $\{\mu^P_A(p\circledast q),\ \mu^P_B(p\circledast q), maximum\{\mu^P_A(q), \mu^P_B(q)\}\}$.
= =maximum { $\mu^P_{A\cup B}$(p$\circledast$q), $\mu^P_{A\cup B}$(q) }
$Therefore\ \mu^P{}_{A\cup B}(p) \leq max\{\ \mu^P{}_{A\cup B}(p\circledast q),\ \mu^P{}_{A\cup B}(q)\}$.
c) Now $\mu^N_{A\cup B}(p) =$ minimum { $\mu^N_A(p),\ \mu^N_B(p)\}$
$\geq$ minimum { $minimum\{\mu^N_A(p\circledast q),\ \mu^N_A(q)\},\ minimum\{\mu^N_B(p\circledast q),\ \mu^N_B(q)\}\}$
= minimum $\{minimum\{\mu^N_A(p\circledast q),\ \mu^N_B(p\circledast q)\},\ minimum\{\mu^N_A(q),\ \mu^N_B(q)\}\}$
= minimum { $\mu^N_{A\cup B}(p\circledast q),\ \mu^N_{A\cup B}(q)\}$.
Therefore $\mu^N_{A\cup B}(p) \geq$ minimum { $\mu^N_{A\cup B}(p\circledast q),\ \mu^N_{A\cup B}(q)\}$.
d) Now $\nu^P_{A\cup B}(O) =$ minimum { $\nu^P_A(O),\ \nu^P_B(O)\}$
$\geq$minimum { $\nu^P_A(p),\ \nu^P_B(p)\}= \nu^P_{A\cup B}(p)$.
Therefore $\nu^P_{A\cup B}(O) \geq \nu^P_{A\cup B}(p)$.
Now $\nu^N_{A\cup B}(O) =$ maximum { $\nu^N_A(O),\ \nu^N_B(O)\}$
$\leq$maximum { $\nu^N_A(p),\ \nu^N_B(p)\}= \nu^N_{A\cup B}(p)$.
Therefore $\nu^N_{A\cup B}(O) \leq \nu^N_{A\cup B}(p)$.
e) Now $\nu^P_{A\cup B}(p) = minimum\ \{\nu^P_A(p),\ \nu^P_B(p)\}$
$\geq$minimum {minimum $\{\nu^P_A(\mathrm{p}\circledast\mathrm{q}), \nu^P_A(\mathrm{q})\}$, minimum $\{\nu^P_B(\mathrm{p}\circledast\mathrm{q}), \nu^P_B(\mathrm{q})\}\}$
= minimum { minimum $\{\nu^P_A(\mathrm{p}\circledast\mathrm{q}), \nu^P_B(\mathrm{p}\circledast\mathrm{q})\}$, minimum $\{\nu^P_A(\mathrm{q}), \nu^P_B(\mathrm{q})\}\}\}$
= minimum { $\nu^P_{A\cup B}(p\circledast q),\ \nu^P_{A\cup B}(q)\}$.
Therefore $\nu^P_{A\cup B}(p)\ \geq$min { $\nu^P_{A\cup B}(p\circledast q),\ \nu^P_{A\cup B}(q)\}$
f) Now $\nu^N_{A\cup B}(p) =$ maximum { $\nu^N_A(p),\ \nu^N_B(p)\}$
$\leq$maximum { max $\{\nu^N_A(p\circledast q), \nu^N_A(q)\}$, max $\{\nu^N_B(p\circledast q), \nu^N_B(q)\}\}\}$
= maximum { max $\{\nu^N_A(\circledast), \nu^N_B(\circledast)\}$, max $\{\nu^N_A(), \nu^N_B()\}\}\}$
= maximum { $\nu^N_{A\cup B}(p\circledast q),\ \nu^N_{A\cup B}(q)\}$.
Therefore $\nu^N_{A\cup B}(p)\ \leq$maximum { $\nu^N_{A\cup B}(p\circledast q),\ \nu^N_{A\cup B}(q)\}$.
Therefore A $\cup$ B is a BIAF ideal of P.

Result: 3.4

$\overline{\overline{A}}$ = A is a BIAF α-ideal of BP-algebra P if A is a BIAF α-ideal of P.

Proof: Let A be a BIAF α-ideal of P. Consider O, p, q, $r \in$ A.

a) Now $\mu_{\alpha\frac{P}{\overline{\overline{A}}}(O)=\nu_{\alpha\frac{P}{A}(O)}} = \mu^P_{\alpha_A}(O) \leq \mu^P_{\alpha_A}(p)$.
Therefore $\mu_{\alpha\frac{P}{\overline{\overline{A}}}(O)\leq\mu^P_{\alpha_A}(p)}$.
Now $\mu^N_{\alpha\overline{\overline{A}}}(O) = \nu_{\alpha\frac{N}{A}(O)} = \mu^N_{\alpha_A}(O) \geq \mu^N_{\alpha_A}(p)$.
Therefore $\mu_{\alpha\frac{N}{\overline{\overline{A}}}(O)\geq\mu^N_{\alpha_A}(p)}$.
b) Now $\mu^P_{\alpha\overline{\overline{A}}}(q\circledast r) = \nu_{\alpha\frac{P}{A}(q\circledast r)} = \mu^P_{\alpha_A}(q\circledast r)$
$\leq max\{\ \mu^P_{\alpha_A}(p\circledast r),\ \mu^P_{\alpha_A}(p\circledast q)\}$.
Therefore $\mu_{\alpha\frac{P}{\overline{\overline{A}}}(q\circledast r)\leq max\{\mu^P_{\alpha_A}(p\circledast r),\ \mu^P_{\alpha_A}(p\circledast q)\}}$.
c) Now $\mu_{\alpha\frac{N}{\overline{\overline{A}}}(q\circledast r)=\nu_{\alpha\frac{N}{A}(q\circledast r)}} = \mu^N_{\alpha_A}(q\circledast r)$
$\geq min\{\ \mu^N_{\alpha_A}(p\circledast r),\ \mu^N_{\alpha_A}(p\circledast q)\}$.
Therefore $\mu_{\alpha\frac{N}{\overline{\overline{A}}}(q\circledast r)\geq min\{\mu^N_{\alpha_A}(p\circledast r),\ \mu^N_{\alpha_A}(p\circledast q)\}}$.
d) Now $\nu^P_{\alpha\overline{\overline{A}}}(O) = \mu_{\alpha\frac{P}{A}(O)} = \nu^P_{\alpha_A}(O) \geq \nu^P_{\alpha_A}(p)$.
Therefore $\nu_{\alpha\frac{P}{\overline{\overline{A}}}(O)\geq\nu^P_{\alpha_A}(p)}$.
Now $\nu_{\alpha\frac{N}{\overline{\overline{A}}}(O)=\mu_{\alpha\frac{N}{A}(O)}} = \nu^N_{\alpha_A}(O) \leq \nu^N_{\alpha_A}(p)$.
Therefore $\nu_{\alpha\frac{N}{\overline{\overline{A}}}(O)\leq\nu^N_{\alpha_A}(p)}$.
e) Now $\nu^P_{\alpha\overline{\overline{A}}}(q\circledast r) = \mu_{\alpha\frac{P}{A}(q\circledast r)} = \nu^P_{\alpha_A}(q\circledast r)$

$\geq \min\left\{\nu^P_{\alpha_A}(\circledast), \nu^P_{\alpha_A}(\circledast)\right\}$
Therefore $\nu_{\alpha\frac{P}{\overline{\overline{A}}}(\circledast)\geq\min\left\{\nu^P_{\alpha_A}(\circledast),\nu^P_{\alpha_A}(\circledast)\right\}}$
f) Now $\nu_{\alpha\frac{N}{\overline{\overline{A}}}(q\circledast r)=\mu_{\alpha\frac{N}{A}(q\circledast r)}}$ = $\nu^N_{\alpha_A}(q\circledast r)$
$\leq \max\left\{\nu^N_{\alpha_A}(\circledast), \nu^N_{\alpha_A}(\circledast)\right\}$
Therefore $\nu_{\alpha\frac{N}{\overline{\overline{A}}}(\circledast)\leq\max\left\{\nu^N_{\alpha_A}(\circledast),\nu^N_{\alpha_A}(\circledast)\right\}}$.
Therefore $\overline{\overline{A}}$ = A is also a BIAF α-ideal of P.

Result: 3.5

A BIAF α-ideal of P is once more formed by the intersection of any two BIAF α-ideals of BP-algebra P.

Proof: Let A and B be two BIAF α-ideals of P.
Consider $O, p, q, r\in$ A ∩ B $\Longrightarrow O, p, q, r\in$ A and $O, p, q, r\in$ B.
a) Now $\mu^P{}_{\alpha A \upsilon B}(O) = mimimum\{\mu^P{}_{\alpha_A}(O), \mu^P{}_{\alpha_B}(O)\} \leq minimum\{\mu^P{}_{\alpha_A}(p),\ \mu^P{}_{\alpha_B}(p)\}\} = \mu^P{}_{\alpha A \upsilon B}(p)$
$\leq$minimum $\{\mu^P_{\alpha_A}(p),\ \mu^P_{\alpha_B}(p)\}\}$= $\mu^P_{\alpha_{A\cap B}}(p)$.
Therefore $\mu^P_{\alpha_{A\cap B}}(O) \leq \mu^P_{\alpha_{A\cap B}}(p)$.
Now $\mu^N_{\alpha_{A\cap B}}(O) =$ maximum $\{\mu^N_{\alpha_A}(O),\ \mu^N_{\alpha_B}(O)\}$
$\geq$maximum $\{\mu^N_{\alpha_A}(p),\ \mu^N_{\alpha_B}(p)\}\}$= $\mu^N_{\alpha_{A\cap B}}(p)$.
Therefore $\mu^N_{\alpha_{A\cap B}}(O) \geq \mu^N_{\alpha_{A\cap B}}(p)$.
b) Now $\mu^P_{\alpha_{A\cap B}}(q\circledast r) = minimum\,\{\mu^P_{\alpha_A}(q\circledast r),\ \mu^P_{\alpha_B}(q\circledast r)\}$
$\leq$ minimum$\{\{\mu^P{}_{\alpha_A}(p\circledast r), \mu^P{}_{\alpha_A}(p\circledast q)\}$
$maximum\{\ \mu^P_{\alpha_B}(p\circledast r), \mu^P_{\alpha_B}(p\circledast q)\}$
= maximum $\{\{\mu^P_{\alpha_A}(p\circledast r),\ \mu^P_{\alpha_B}(p\circledast r)\}$
$minimum\{\mu^P_{\alpha_A}(p\circledast q), \mu^P_{\alpha_B}(p\circledast q)\}\}$
= maximum $\{\mu^P_{\alpha_{A\cap B}}(p\circledast r),\ \mu^P_{\alpha_{A\cap B}}(p\circledast q)\}$.
Therefore $\mu^P_{\alpha_{A\cap B}}(q\circledast r) \leq\leq maximum\,\{\mu^P_{\alpha_{A\cap B}}(p\circledast r),\ \mu^P_{\alpha_{A\cap B}}(p\circledast q)\}$.
c) Now $\mu^N_{\alpha_{A\cap B}}(q\circledast r) = maximum\,\{\mu^N_{\alpha_A}(q\circledast r),\ \mu^N_{\alpha_B}(q\circledast r)\}$
$\geq\{\ \ maximum\,\{\ \mu^N{}_{\alpha_A}(p\circledast r),\ \ \mu^N{}_{\alpha_A}(p\circledast q)\},$
= minimum$\{\{\{\mu^N_{\alpha_A}(p\circledast r),\ \mu^N_{\alpha_B}(p\circledast r)\}$
$maximum\{\mu^N_{\alpha_A}(p\circledast q), \mu^N_{\alpha_B}(p\circledast q)\}\}$
= minimum $\{\mu^N_{\alpha_{A\cap B}}(p\circledast r),\ \mu^N_{\alpha_{A\cap B}}(p\circledast q)\}$.
Therefore $\mu^N_{\alpha_{A\cap B}}(q\circledast r) \geq$ min $\{\mu^N_{\alpha_{A\cap B}}(p\circledast r),\ \mu^N_{\alpha_{A\cap B}}(p\circledast q)\}$.
d) Now $\nu^P{}_{\alpha A\cap B}(O) = maximum\,\nu^P{}_{\alpha_A}(O), \nu^P_{\alpha_B}(O)\} \geq maximum\,\{\nu^P{}_{\alpha_A}(p), \nu^P_{\alpha_B}(p)\} = \nu^P{}_{\alpha A\cap B}(p)$
Therefore $\nu^P_{\alpha_{A\cap B}}(O) \geq \nu^P_{\alpha_{A\cap B}}(p)$.
Now $\nu^N_{\alpha_{A\cap B}}(O) =$ minimum $\{\nu^N_{\alpha_A}(O),\ \nu^N_{\alpha_B}(O)\}$
$\leq$minimum $\{\nu^N_{\alpha_A}(p),\ \nu^N_{\alpha_B}(p)\}$ = $\nu^N_{\alpha_{A\cap B}}(p)$.
Therefore $\nu^N_{\alpha_{A\cap B}}(O) \leq \nu^N_{\alpha_{A\cap B}}(p)$.

e) Now
$$\begin{aligned}&\nu^P{}_{\alpha A\cap B}(q\circledast r) = maximum\,\{\nu^P{}_{\alpha A}(q\circledast r),\ \nu^P{}_{\alpha B}(q\circledast r)\}\\ &\geq maximum\{\{\nu^P{}_{\alpha A}(p\circledast r),\ \nu^P{}_{\alpha A}(p\circledast q)\},\\ &\{\nu^P{}_{\alpha B}(p\circledast r),\ \nu^P{}_{\alpha B}(p\circledast q)\}\}\\ &= minimum\,\{\{\nu^P{}_{\alpha A}(p\circledast r), \nu^P{}_{\alpha B}(p\circledast r)\},\ \{\nu^P{}_{\alpha A}(p\circledast q),\ \nu^P{}_{\alpha B}(p\circledast q)\}\}\end{aligned}$$
= minimum $\{\nu^P_{\alpha_{A\cap B}}(p\circledast r),\ \nu^P_{\alpha_{A\cap B}}(p\circledast q)\}$.
Therefore $\nu^P_{\alpha_{A\cap B}}(q\circledast r) \geq$ minimum $\{\nu^P_{\alpha_{A\cap B}}(p\circledast r),\ \nu^P_{\alpha_{A\cap B}}(p\circledast q)\}$.

f) Now
$$\begin{aligned}&\nu^N{}_{\alpha A\cap B}(q\circledast r) = minimum\,\{\nu^N{}_{\alpha A}(q\circledast r),\ \nu^N{}_{\alpha B}(q\circledast r)\}\\ &\leq\ minimum\,\{\{\nu^N{}_{\alpha A}(p\circledast r),\ \nu^N{}_{\alpha A}(p\circledast q)\},\\ &\{\nu^N{}_{\alpha B}(p\circledast r),\ \nu^N{}_{\alpha B}(p\circledast q)\}\}\\ &=\ maximum\,\{\{\nu^N{}_{\alpha A}(p\circledast r), \nu^N{}_{\alpha B}(p\circledast r)\},\\ &\{\nu^N{}_{\alpha A}(p\circledast q),\ \nu^N{}_{\alpha B}(p\circledast q)\}\}\end{aligned}$$

= maximum { $\nu^N_{\alpha_{A\cap B}}(p\circledast r),\ \nu^N_{\alpha_{A\cap B}}(p\circledast q)\}$.
Therefore $\nu^N_{\alpha_{A\cap B}}(q\circledast r)\le$ maximum { $\nu^N_{\alpha_{A\cap B}}(p\circledast r),\ \nu^N_{\alpha_{A\cap B}}(p\circledast q)\}$.
Hence intersection of any two BIAF α-ideals of P is again a BIAF α-ideal of P.

Result: 3.6

Union of any two BIAF α-ideals of BP-algebra P is also a BIAF α-ideal of P if either is contained in the other.

Proof: Let A and B be two BIAF α-ideals of P.
If A $\subseteq$ B $\Longrightarrow$ A$\cup$ B = B and if B $\subseteq$ A $\Longrightarrow$A $\cup$ B = A.
Consider $O, p, q, r\in$ A $\cup$ B $\Longrightarrow O, p, q, r\in$ A or $O, p, q, r\in$ B
a) Now $\mu^P{}_{\alpha A\upsilon B}(O) = maximum\,\{\mu^P{}_{\alpha A}(O), \mu^P{}_{\alpha B}(O)\}$
$\le$maximum { $\mu^P_{\alpha_A}(p),\ \mu^P_{\alpha_B}(p)\}\}$ = $\mu^P_{\alpha_{A\cup B}}(p)$
Therefore $\mu^P{}_{\alpha A\upsilon B}(O)\le \mu^P{}_{\alpha A\upsilon B}(p)$
Now $\mu^N_{\alpha_{A\cup B}}(O) =$ minimum { $\mu^N_{\alpha_A}(O),\ \mu^N_{\alpha_B}(O)\}$
$\ge$minimum { $\mu^N_{\alpha_A}(p),\ \mu^N_{\alpha_B}(p)\}$ = $\mu^N_{\alpha_{A\cup B}}(p)$.
Therefore $\mu^N_{\alpha_{A\cup B}}(O)\ge\mu^N_{\alpha_{A\cup B}}(p)$.
b) Now $\mu^P{}_{\alpha A\upsilon B}(q\circledast r) = \ maximum\,\{\mu^P{}_{\alpha A}(q\circledast r),\ \mu^P{}_{\alpha B}(q\circledast r)\}$
$\le$ maximum $\{\left(\ \mu^P_{\alpha_A}(p\circledast r),\ \mu^P_{\alpha_A}(p\circledast q)\right\}$,
$maximum\{\ \mu^P_{\alpha_B}(p\circledast r),\ \mu^P_{\alpha_B}(p\circledast q)\}\}$
= maximum { $\{\mu^P_{\alpha_A}(p\circledast r),\ \mu^P_{\alpha_B}(p\circledast r)\}$,
$maximum\{\mu^P_{\alpha_A}(p\circledast q), \mu^P_{\alpha_B}(p\circledast q)\}\}$
=maximum { $\mu^P_{\alpha_{A\cup B}}(p\circledast r),\ \mu^P_{\alpha_{A\cup B}}(p\circledast q)\}$.
Therefore $\mu^P_{\alpha_{A\cup B}}(q\circledast r)\le$ maximum { $\mu^P_{\alpha_{A\cup B}}(p\circledast r),\ \mu^P_{\alpha_{A\cup B}}(p\circledast q)\}$.
c) Now $\mu^N_{\alpha_{A\cup B}}(q\circledast r) =$ minimum $\{\mu^N_{\alpha_A}(q\circledast r),\ \mu^N_{\alpha_B}(q\circledast r)\}$
$\ge$ minimum $\{minimum\ \left\{\ \mu^N_{\alpha_A}(p\circledast r),\ \mu^N_{\alpha_A}(p\circledast q)\right\}$,
$minimum\{\ \mu^N_{\alpha_B}(p\circledast r),\ \mu^N_{\alpha_B}(p\circledast q)\}$
= minimum { $minimum\{\mu^N_{\alpha_A}(p\circledast r),\ \mu^N_{\alpha_B}(p\circledast r)\}$,
$minimum\{\mu^N_{\alpha_A}(p\circledast q), \mu^N_{\alpha_B}(p\circledast q)\}\}$
=minimum { $\mu^N_{\alpha_{A\cup B}}(p\circledast r),\ \mu^N_{\alpha_{A\cup B}}(p\circledast q)\}$.
Therefore $\mu^N_{\alpha_{A\cup B}}(q\circledast r)\ \ge$ minimum { $\mu^N_{\alpha_{A\cup B}}(p\circledast r),\ \mu^N_{\alpha_{A\cup B}}(p\circledast q)\}$.
d) Now $\nu^P_{\alpha_{A\cup B}}(O) =$ minimum { $\nu^P_{\alpha_A}(O),\ \nu^P_{\alpha_B}(O)\}$
$\ge$minimum { $\nu^P_{\alpha_A}(p),\ \nu^P_{\alpha_B}(p)\}$ = $\nu^P_{\alpha_{A\cup B}}(p)$.
Therefore $\nu^P_{\alpha_{A\cup B}}(O)\ge\nu^P_{\alpha_{A\cup B}}(p)$.
Now $\nu^N_{\alpha_{A\cup B}}(O) =$ maximum { $\nu^N_{\alpha_A}(O),\ \nu^N_{\alpha_B}(O)\}$
$\le$maximum { $\nu^N_{\alpha_A}(p),\ \nu^N_{\alpha_B}(p)\}$ = $\nu^N_{\alpha_{A\cup B}}(p)$.
Therefore $\nu^N_{\alpha_{A\cup B}}(O)\le\nu^N_{\alpha_{A\cup B}}(p)$.

e) Now
$$\begin{aligned}&\nu^P{}_{\alpha A\upsilon B}(q\circledast r) = \ minimum\,\{\ \nu^P{}_{\alpha A}(q\circledast r),\ \nu^P{}_{\alpha B}(q\circledast r)\}\\ &\ge\ minimum\{\{\ \nu^P{}_{\alpha A}(p\circledast r), \nu^P{}_{\alpha A}(p\circledast q),\\ &\{\ \nu^P{}_{\alpha B}(p\circledast r),\ \nu^P{}_{\alpha B}(p\circledast q)\}\\ &=\ minimum\,\{\ \{\ \nu^P{}_{\alpha A}(p\circledast r), \nu^P{}_{\alpha B}(p\circledast r)\ \},\\ &\{\nu^P{}_{\alpha A}(p\circledast q)\,, \nu^P{}_{\alpha B}(p\circledast q)\}\}\end{aligned}$$

= minimum { $\nu^P_{\alpha_{A\cup B}}(p\circledast r),\ \nu^P_{\alpha_{A\cup B}}(p\circledast q)\}$.
Therefore $\nu^P_{\alpha_{A\cup B}}(q\circledast r)\ge$ minimum { $\nu^P_{\alpha_{A\cup B}}(p\circledast r),\ \nu^P_{\alpha_{A\cup B}}(p\circledast q)\}$.
f) Now $\nu^N_{\alpha_{A\cup B}}(q\circledast r) =$ maximum { $\nu^N_{\alpha_A}(q\circledast r),\ \nu^N_{\alpha_B}(q\circledast r)\}$

$$\le \text{maximum}\ \{\ \begin{aligned}&\{\ \nu^N{}_{\alpha A}(p\circledast r),\ \nu^N{}_{\alpha A}(p\circledast q)\},\\ &\{\ \nu^N{}_{\alpha B}(p\circledast r),\ \nu^N{}_{\alpha B}(p\circledast q)\}\}\\ &= maximum\,\{\ \{\ \nu^N{}_{\alpha A}(p\circledast r), \nu^N{}_{\alpha B}(p\circledast r)\}\,,\\ &\{\ \nu^N{}_{\alpha A}(p\circledast q),\ \nu^N{}_{\alpha B}(p\circledast q)\}\end{aligned}$$

= maximum { $\nu^N_{\alpha_{A\cup B}}(p\circledast r),\ \nu^N_{\alpha_{A\cup B}}(p\circledast q)\}$.

Therefore $\nu^{N}_{\alpha_{A\cup B}}(q \circledast r) \leq$maximum $\{ \nu^{N}_{\alpha_{A\cup B}}(p \circledast r),\ \nu^{N}_{\alpha_{A\cup B}}(p \circledast q)\}$
Therefore A ∪ B is a BIAF α-ideal of P.

4 Conclusion

Bipolarity is a significant factor in applied mathematics. Many intriguing research areas have emerged from the combination of intuitionistic set theory and bipolar fuzzy set theory. This paper discussed some fundamental operations on the topic of BIAF ideal and α-ideal, as well as the transformation of bipolar fuzzy set theories to BIF set theories. It is our belief that the research can generate further new topics. In conclusion, we presented the criteria for a BIAF ideal and α-ideal. It will be possible to apply these definitions and basic findings to many algebraic structures in the future. The BIAF ideals could be used for other research work. We hope our findings in this work will provide a basis for future research.

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