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Bulk Viscous Kantowski-Sachs Universe with Modified Chaplygin Gas in Lyra Geometry

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Abstract

Objective: To get a solution for Kantowski-Sachs model with modified Chaplygin gas in the framework of in Lyra's geometry. **Methods**: To acquire an exact solution of field equations, power law expansion was considered. **Findings**: It is observed that the jerk parameter $j(t)$ is positive which indicates that the universe smoothly transits from deceleration phase to an acceleration phase. Also observed some physical and kinematic aspects of the resulting model. It is found that the resultant model shows anisotropic in nature. **Novelty**: The dark energy model under consideration provides deep insight to understand biggest cosmological mysteries.

Keywords: Kantowski-Sachs metric; Chaplygin gas; Bulk viscous fluid; Lyra geometry; Power law expansion

1 Introduction

The origin of the universe remains one of the greatest cosmological mysteries to this day. Cosmology, as a field of study, seeks to understand the large-scale structure of the physical universe. In order to explore and explain the complexities of the universe, various cosmological models have been introduced. These models serve as frameworks that help researchers and scientists make sense of the universe's origin, evolution, and behaviour. By studying the large-scale structure of the universe, cosmologists aim to uncover the fundamental laws and principles that govern its existence. The present day observations indicate that, the universe is isotropic and homogeneous in the large scale but not true for the small scale. The exact solutions of the general theory of relativity for homogeneous space-time's can be categorized into either Bianchi types or Kantowski-Sachs models. These solutions provide valuable insights into the behaviour of gravity in different spatially homogeneous scenarios and are essential for understanding the dynamics of the universe on a large scale.

It was first noticed from high red shift supernova la that our universe is currently an accelerating. This discovery, made by observing the distant explosions of stars, indicated a surprising phenomenon: rather than slowing down, the expansion of the universe is actually speeding up. The high red shift of these supernova provided evidence for this accelerated expansion, challenging the prevailing understanding of the universe's behaviour. Subsequent studies further confirmed this astonishing observation. The large-scale structure of the universe, which refers to the distribution of galaxies and other cosmic structures on vast scales, also provided compelling evidence for the acceleration. Moreover, the cosmic microwave background radiation, a remnant of the early universe, played a crucial role in confirming the acceleration. By studying the properties of this radiation, scientists were able to glean valuable insights into the nature of the universe. In Einstein's general relativity, one must introduce an element with a large negative pressure to have this type of acceleration and this element is usually referred to as dark energy (DE). According to astronomical observations our universe is flat, and our universe has approximately $\frac{2}{3}$ dark energy and $\frac{1}{3}$ dark matter. The behaviour of dark energy as well as dark matter is not known, and many different models have been proposed positively.

It can be seen from recent observations that the Chaplygin gas (CG) may be useful to relate dark energy due to its positive energy density and negative pressure. Chaplygin gas is taken with an equation of state of the form,

$$p = -\frac{A}{\rho},$$

where $A > 0$ is a constant.

Several generalizations of Chaplygin gas has been suggested in the literature. Several modifications have been emerged with an extra degree of freedom to be adequate for the observations. One of these types of models is generalized Chaplygin gas (GCG) with an equation of state in the form of

$$p = -\frac{A}{\rho^{\gamma}},$$

where γ is a constant. Note that for $\gamma = -1$ & $A = 1$ we end up with the cosmological constant.

As generalized Chaplygin gas model has two free constants, so the main idea behind this is that they can make model behave as a pressure less fluid at early stage and as a cosmological constant later on. In addition to the generalized Chaplygin gas model, another type of Chaplygin gas model is termed as the modified Chaplygin gas (MCG) with an equation of state in the form of

$$p = B\rho - \frac{A}{\rho^{\gamma}},$$

where *B, A*, and γ are constants. For $B = 0$ we obtain the GCG model while setting $A = 0$ & $B = -1$, we get the cosmological constant. In spite of the MCG model being simple fluid model, it unifies dark matter and dark energy and conforms to many cosmological observations. Several authors[1–6] have investigated cosmological models with modified Chaplygin gas.

In the field of cosmology, bulk viscosity is an important for studying the evolution of the universe because it produces excellent results in the framework of spatially homogeneous cosmological models. This bulk viscosity contains negative pressure, which produces repulsive gravity, which overcomes the attractive gravity of matter and causes the universe to expand rapidly. As a result, cosmologists are now paying close attention to the cosmological models with bulk viscosity. There has been wide range of an investigations into the role of viscosity as a dissipative mechanism helping to smooth out the initial large anisotropies of the universe. Recently Bhoyar & Ingole[7] studied Cosmic evolution of the Kantowski-Sachs universe in the context of a bulk viscous string in teleparallel gravity.

Lyra has introduced a modification of Riemannian geometry by imposing a gauge function into the structure less manifold which is similar to Weyl's geometry. Lyra's geometry embodies the spirit of Einstein's principle of geometrization by endowing both the scalar and tensor fields with intrinsic geometrical significance. This alignment highlights the fundamental geometric nature of Lyra's framework and underscores its potential as a valuable tool in understanding the gravitational interactions within the universe. In consecutive investigations, a new scalar tensor theory of gravitation and constructed an analogue of the Einstein field equation based on Lyra's geometry. In the normal gauge, this equation can be written as follows,

$$R_{ij} - \frac{1}{2}Rg_{ij} + \frac{3}{2}\psi_i\psi_j - \frac{3}{4}g_{ij}\psi^k\psi_k = -T_{ij}$$

where, ψ_i is a displacement field and we take 8πG = 1. There are number of authors[8–15] which have analysed cosmological models depending on Lyra's manifold with constant as well as time dependent displacement vectors.

Motivated by these recent investigations, we have explored modified Chaplygin gas in the framework of Kantowski-Sachs in Lyra geometry. Some physical and kinematical aspects are also discussed.

2 Methodology

2.1 Metric and Field Equations

Consider Kantowski-Sachs universe represented by the metric,

$$ds^2 = -dt^2 + C_1^2 dr^2 + C_2^2 (d\theta^2 + \sin^2\theta d\varphi^2) \tag{1}$$

where, C_1 and C_2 are the functions of cosmic time *t*.

The field equation in the normal gauge for Lyra's geometry are given by,

$$R_{ij} - \frac{1}{2} R g_{ij} + \frac{3}{2}\psi_i \psi_j - \frac{3}{4} g_{ij}\psi^k \psi_k = -T_{ij} \tag{2}$$

where, ψ_i is a displacement field and we take $8\pi G = 1$. Here we assume ψ_i to be time like so that,

$$\psi_i = (0,0,0,\beta)$$

where, β is a function of comic time *t*.

The energy momentum tensor is in the form,

$$T_i^j = (\rho + \overline{p}) u_i u^j + \overline{p} g_i^j \tag{3}$$

where, ρ is the energy density and the total pressure $\overline{p}$ involves the proper pressure *p*, bulk viscosity ξ and Hubble parameter which is expressed as follows,

$$\overline{p} = p - 3\xi H \tag{4}$$

Here u_i is the velocity four vector with $u_i u^i = -1$.

The modified Chaplygin gas obeying an equation of state

$$p = \omega_0 \rho - \frac{D_0}{\rho^\alpha} \tag{5}$$

with $D_0 > 0$

For $\alpha = 1$, **Equation (5)** reduces to,

$$p = \omega_0 \rho - \frac{D_0}{\rho} \tag{6}$$

Here ω_0 and D_0 represents the feature of Dark Energy and Chaplygin gas respectively.

3 Results and discussion

3.1 Solution of the Field Equations

For the Kantowski-Sachs metric **Equation (1)**, the field **Equations (2)** leads to

$$2\frac{\dot{C}_1}{C_1}\frac{\dot{C}_2}{C_2} + \frac{\dot{C}_2^2}{C_2^2} + \frac{1}{C_2^2} - \frac{3}{4}\beta^2(t) = \rho, \tag{7}$$

$$2\frac{\ddot{C}_2}{C_2} + \frac{\dot{C}_2^2}{C_2^2} + \frac{1}{C_2^2} + \frac{3}{4}\beta^2(t) = -\left(\omega_0 \rho - \frac{D_0}{\rho} - 3\xi H\right) \tag{8}$$

$$\frac{\ddot{C}_1}{C_1} + \frac{\ddot{C}_2}{C_2} + \frac{\dot{C}_1}{C_1}\frac{\dot{C}_2}{C_2} + \frac{3}{4}\beta^2(t) = -\left(\omega_0 \rho - \frac{D_0}{\rho} - 3\xi H\right) \tag{9}$$

where, $(\cdot)$ denotes the derivative with respect to cosmic time t.

For the Kantowski-Sachs metric, we define the dynamical parameters average scale factor, spatial volume, Hubble parameter, scalar expansion, shear scalar, average anisotropy parameter, jerk parameter, deceleration parameter given by

$$a = (C_1 C_2^2)^{\frac{1}{3}} V = a^3 = C_1 C_2^2 \; H = \frac{\dot{a}}{a} = \frac{1}{3}\left(\frac{\dot{C}_1}{C_1} + 2\frac{\dot{C}_2}{C_2}\right) \tag{10}$$

Scalar expansion,

$$\theta = 3H = 3\frac{\dot{a}}{a} = \frac{\dot{C}_1}{C_1} + 2\frac{\dot{C}_2}{C_2} \tag{11}$$

Shear scalar,

$$\sigma^2 = \frac{1}{2}\left(\frac{\dot{C}_1^2}{C_1^2} + 2\frac{\dot{C}_2^2}{C_2^2}\right) - \frac{\theta^2}{6} \tag{12}$$

Anisotropy parameter,

$$\Delta = \frac{1}{3}\Sigma\left(\frac{H_i - H}{H}\right)^2 \tag{13}$$

Jerk parameter,

$$j(t) = \frac{a^2 a'''}{a'^3} \tag{14}$$

Deceleration parameter,

$$q = \frac{d}{dt}\left(\frac{1}{H}\right) - 1 \tag{15}$$

3.2 Physical Aspects of the Model

Now we examine the important above mentioned physical and kinematical parameters for the Kantowski-Sachs metric by assuming power law expansion as follows,

$$C_1 = m_1 t^p \;\&\; C_2 = m_2 t^s, p, s > 0 \tag{16}$$

where m_1 and m_2 are the non-zero positive constants.

By using **Equation (16)** in **Equation (10)**, we obtain the expressions for average scale factor, Spatial volume and Hubble parameter are as follows

$$a = (m_1 {m_2}^2 t^{p+2s})^{\frac{1}{3}} \quad V = m_1 {m_2}^2 t^{p+2s} \quad H = \frac{p+2s}{3t} \tag{17}$$

The expression for scalar expansion, shear scalar and average anisotropy parameter are given by,

$$\theta = \frac{p+2s}{t} \quad \sigma^2 = \frac{(p-s)^2}{3t^2} \quad \Delta = \frac{2(p-s)^2}{(p+2s)^2} \tag{18}$$

The expression for deceleration parameter is given by,

$$q = \frac{3}{p+2s} - 1 \tag{19}$$

Jerk parameter is obtained as follows,

$$j(t)=\frac{(p+2s-3)(p+2s-6)}{(p+2s)^2} \tag{20}$$

It is clear that, the jerk parameter $j(t)$ is constant and for the suitable values of constant, it signifies a smooth transition from a deceleration phase to an acceleration phase in the universe.

Also, we find,

$$\frac{\sigma}{\theta}=\frac{p-s}{\sqrt{3}(p+2s)} \tag{21}$$

Now, by solving **Equation (8) and Equation (9)**, we get,

$$\frac{s(s-1)}{t^2}+\frac{s^2}{t^2}+\frac{1}{\beta^2 t^{2s}}=\frac{p(p-1)}{t^2}+\frac{ps}{t^2} \tag{22}$$

By solving **Equation (7)** and **Equation (8)** and using **Equation (22)**, one can obtain the density as,

$$\rho=\frac{\emptyset(t)\pm\sqrt{\emptyset^2(t)-4(1-\omega_0)D_0}}{2(1-\omega_0)} \tag{23}$$

$$\text{where } \emptyset(t)=\frac{6p(2s+p-1)-3\xi_0(p+2s)-\xi_1(p+2s)^2}{3t^2} \tag{24}$$

Here to get density to be real and positive, we consider

$$\rho=\frac{\emptyset(t)+\sqrt{\emptyset^2(t)-4(1-\omega_0)D_0}}{2(1-\omega_0)} \tag{25}$$

where

$$\emptyset^2(t)>4(1-\omega_0)D_0 \;\;\&\;\; 1-\omega_0>0 \tag{26}$$

In the absence of Chaplygin gas i.e. $D_0=0$, then **Equation (6)** leads to,

$$p=\frac{\omega_0\emptyset(t)}{1-\omega_0} \tag{27}$$

With the help of **Equation (16)** and **Equation (23)** in **Equation (7)** we get the displacement vector as follows,

$$\beta^2(t)=\frac{4}{3}\left[\frac{s(2p+s)}{t^2}+\frac{1}{m_2^2 t^{2s}}-\frac{\emptyset(t)+\sqrt{\emptyset^2(t)-4(1-\omega_0)D_0}}{2(1-\omega_0)}\right] \tag{28}$$

We observed that, $\beta^2(t)$ is a decreasing function of time (t).

If we put $D_0=0$ i.e. in the absence of Chaplygin gas **Equation (28)** reduces to,

$$\beta^2(t)=\frac{4}{3}\left[\frac{s(2p+s)}{t^2}+\frac{1}{m_2^2 t^{2s}}-\frac{\emptyset(t)}{(1-\omega_0)}\right] \tag{29}$$

and if we put $\omega_0=0$ i.e. in the absence of dark energy then **Equation (28)** becomes,

$$\beta^2(t)=\frac{4}{3}\left[\frac{s(2p+s)}{t^2}+\frac{1}{m_2^2 t^{2s}}-\frac{\emptyset(t)+\sqrt{\emptyset^2(t)-4D_0}}{2}\right] \tag{30}$$

We find the expression for bulk viscosity as follows,

$$\xi = \frac{3\xi_0 t + \xi_1 (p+2s)}{3t} \tag{31}$$

where ξ_0 and ξ_1 are constants.

From the **Equation (6)** and **Equation (23)**, we analysed the expression for stability which is given by,

$$C_s^2 = \frac{dp}{d\rho} = \omega_0 + \frac{4D_0(1-\omega_0)^2}{\left[\emptyset(t) + \sqrt{\emptyset^2(t) - 4(1-\omega_0)D_0}\right]^2} \tag{32}$$

Also, we observed the Statefinder parameters by using **Equation (17)** and **Equation (19)** as follows,

$$r = \frac{(p+2s-3)(p+2s-6)}{(p+2s)^2} \;\&\; s = \frac{2}{p+2s} \tag{33}$$

We observed from **Equation (23)** the density in the absence of Lyra's geometry as follows,

$$\rho = \frac{s(2p+s)}{t^2} + \frac{1}{m_2^2 t^{2s}} \tag{34}$$

Also, in the absence of Chaplygin gas, we calculated pressure as,

$$p = \omega_0 \left[\frac{s(2p+s)}{t^2} + \frac{1}{m_2^2 t^{2s}}\right] \tag{35}$$

The Hubble parameter and expansion scalar exhibit a decreasing trend with time *t*. These observations provide valuable insights into the evolving nature of our universe, suggesting that the expansion rate is gradually slowing down.

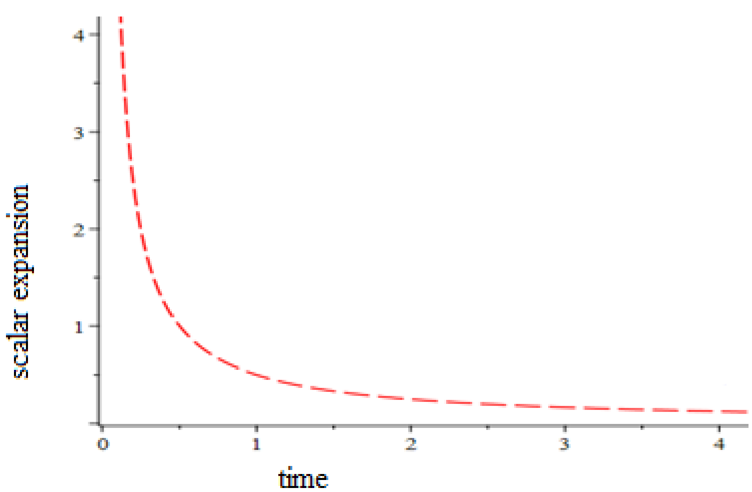

Fig 1. Scalar expansion θ versus time t

It has been observed that the expansion scalar (θ) & density (ρ) are decreasing functions of time which can be seen in Figure 1 and Figure 2 whereas the cosmological parameter i.e. the stability of the model which is positive, which can be seen in Figure 3.

3.3 Key Observations

It is observed that the volume rises over time, indicating that the model is expanding. As time *t* increases, the physical parameters such as Hubble parameter (H), and Shear scalar (σ^2) decreases. The stability C_s^2 is positive throughout the region for the suitable values of constant, hence the model is stable. Also, we investigated the outcomes in the absence of Chaplygin gas and dark energy. The results in this paper agree with the observational feature of the universe.

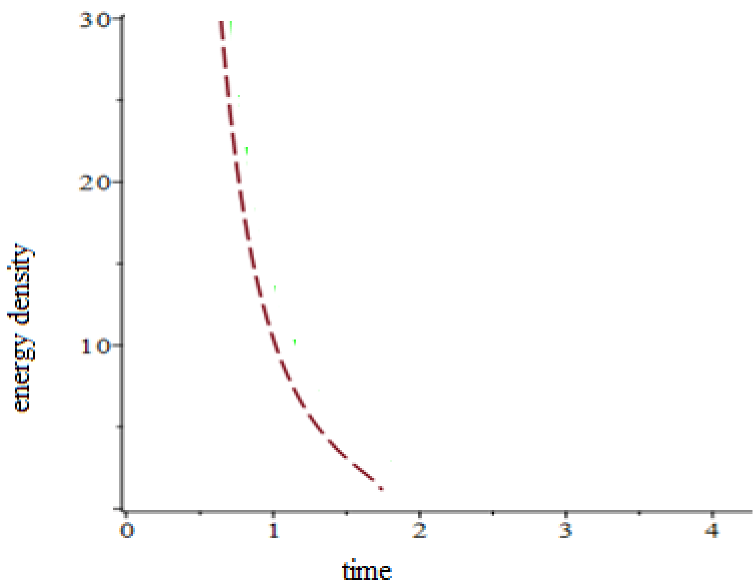

Fig 2. Energy density ρ versus time t

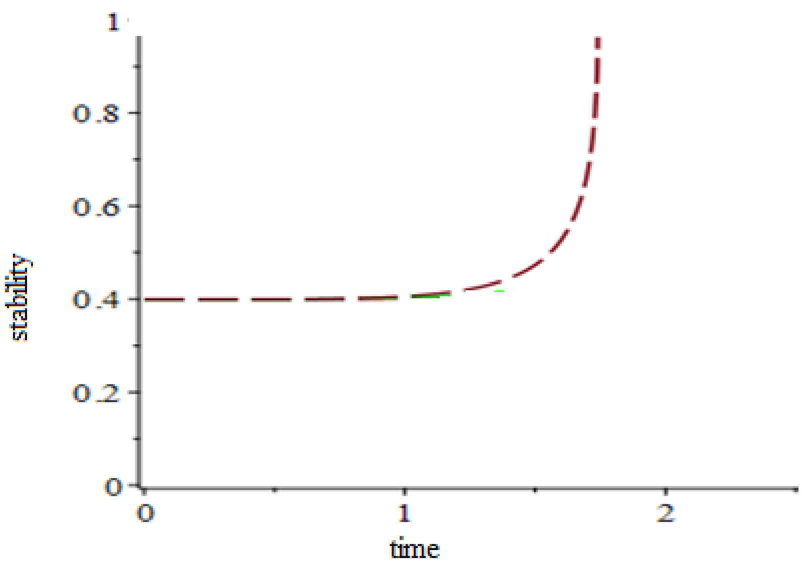

Fig 3. Stability $\mathbf{C_s^2}$ versus time t

4 Conclusion

This study presents the Kantowski-Sachs cosmological model with Modified Chaplygin Gas in Lyra's geometry. The physical and kinematical parameters, which are very important in the description of cosmological model has been obtained and discussed. For this model we get, $\frac{\sigma}{\theta} \neq 0$. Hence, model shows anisotropic nature. The results obtained are consistent with the recent observation representing the present universe. For p =1 & s =1, the shear scalar (σ^2), anisotropy parameter (Δ), deceleration parameter $q(t)$ and jerk parameter $j(t)$ vanishes. Also, the Statefinder pair $(r \;\&\; s)$ disappears. The dark energy model under consideration provides deep insight to understand biggest cosmological mysteries i.e. Dark Energy which is driving force behind accelerated expansion of universe.

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