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Optimal Design of a 2-DOF Parallel Manipulator Considering Kinematics and Natural Frequency

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Abstract

Objectives: A Two Degree of Freedom parallel manipulator is used in many industries due its high payload to weight ratio, high dynamic capabilities and structural rigidity. The natural frequency is determined based on stiffness dynamic model**. Methods:** The stiffness matrix and mass matrix was derived, and the structural dynamics is modeled. A MATLAB code has been developed to obtain stiffness modelling and dynamic modelling equations. Design and simulation of a two DOF parallel manipulator is done by using CATIA V5 R20 software. Modal analysis is performed in ANSYS 18.1 to find the natural frequencies and mode shapes. Convergence test is made to get accurate modal frequency results. Harmonic response analysis has been made for steady-state response of a model subjected to a load that changes harmonically. **Findings:** From results, it is observed that the natural frequency of the circular trajectory increases as the radius increases. **Novelty/Improvement:** A global frequency index is taken as objective for finding the optimal dimensions of the parallel manipulator considering circular trajectory of the end effector. Modal analysis and Harmonic analysis is done using ANSYS. Using Modal analysis mode shapes and natural frequency of the parallel manipulator is obtained. Harmonic analysis is carried out to find the frequency of the parallel manipulator at the maximum deformation. The machine should run at below this frequency to avoid the high vibrations due to resonance.

Keywords: Parallel manipulator; Global frequency; Natural frequency; Modal analysis; Harmonic analysis

1 Introduction

Nowadays parallel manipulators are mostly used in the automation industry due to their high accuracy and precision and also having high stiffness and low degrees of freedom joints. The advantages of parallel robots compared to serial robots are high stiffness, high rigidity, and less deflection of the links. Now a days parallel robots with single DOF joints are commonly using in Industries because of their advantage of solving kinematic and dynamic equations. However parallel robots suffer with less workspace compared to serial robots. Some researchers that studied and observed parallel manipulators are Choudhury R and Singh Y[1] and presented review on various planar parallel manipulators, which offer three degrees of freedom (X, Y, and rotation about Z-axis). It is observed from this review that these planar parallel manipulators have high stiffness, agility, and payload capacity, they are increasingly used in fields like manufacturing, medicine, and surveillance. Chenglin Dong et al.,[2] presented dynamic modeling and design method for a novel 5-DOF hybrid machining robot using workspace efficiency, lateral stiffness, and dynamic compliance. In this study, it analyses how design variables affect performance, formulates a design strategy for the robot's parallel mechanism, and validates the approach through simulations and experiments.

Parallel manipulators are to be optimized for finding the best optimal dimensions so that the weight of the manipulators can be reduced to be minimum. Some researchers that studied different optimization methods are Wu M and Corves B[3], proposed a new parameter optimization method for parallel mechanism-based industrial robots using Stackelberg game theory and Gaussian process regression. In this approach the design variables and objectives are divided into leader and follower groups. This simplifies the complex multi-objective optimization problem. Ganesh, S. et al.,[4] studied about 4-DOF parallel gripper. In this actuator positions, velocities, and accelerations are determined using kinematics equations. Using Energy Manipulation Index (EMI), optimal dimensions of the gripper are determined using Genetic Algorithm. Ren, J. et al.,[5] proposed two optimal design methods for 3-PSS flexible parallel micromanipulator. The two optimal design methods are progressive optimization and synchronous optimization, the first one maximizes the workspace while ensuring minimum dexterity, then minimizes driving forces. And the second one focuses on maximizing the first-order natural frequency by adjusting only the scale parameters, with constraints on workspace size, actuator stroke, and hinge displacement.

Vibration study is very important in the design of parallel manipulators since vibrations affect the performance like machining and positioning of the mechanisms. Some researchers that studied the effect of vibrations Gao X et.al.,[6], proposed a novel isolator based on 2-RPC/2-SPC parallel mechanism with magneto-rheological dampers. Kinematics and dynamics analysis of the isolator was carried using Lagrange method. Grey relation analysis was used to determine the geometric parameters on natural frequency. In this analysis it was observed that the first order natural frequency of the isolator is affected by the length of the fixed platform most significantly. Jiang S et.al.,[7] studied design of 3-DOF parallel stabilization platform. It is designed to offset disturbances in both vertical and horizontal directions, aiming for high vibration isolation on challenging terrain. In this design it includes mechanical optimization, motion modeling, and a force-position composite control strategy based on active compliance for impact cushioning. Ganesh, S. et.al.,[8] studied kinematic, workspace, and dynamic analysis of a 2-DOF translational parallel robot with four links, combining revolute and prismatic joints. Using stiffness and mass matrices, with the first natural frequency used as a performance metric, termed the Global Natural Frequency Index (GNFI). Using GNFI, optimal dimensions of the parallel robot are determined using Genetic Algorithm. Ma et. Al.,[9] studied about vibration issues in robotic machining caused by low stiffness. Using the Finite Element Method (FEM) a dynamic model for a 5 -DOF parallel robot is developed. Also, sensitivity analysis was carried to identify key design parameters affecting Global Vibration Modes (GVMs). Multi objective optimization was carried using Gas to optimize stiffness uniformity and the first-order GVM frequency across the workspace. From this study it was observed improved dynamic performance and reduced vibration sensitivity. Gong et. Al.,[10] proposed an efficient elastodynamic modeling method for a 3-PRS parallel manipulator using independent displacement coordinates and substructure synthesis. In this connecting rods are divided into elements and dynamic equations are derived for each element. Using multipoint constraint theory and singularity analysis, globally independent coordinates are identified. Due to this design approach natural frequencies and dynamic characteristics are evaluated efficiently. Optimization of robots is carried out in order to find the optimal dimensions of the manipulator. In this objective function subjected to constraints is used for optimization. In literature[5],[8] several objective functions like Global Condition Index (GCI), Global Stiffness Index (GSI) are considered for optimization. Optimization is a way of getting the optimum results under the given conditions. It is used in the field of construction, design and maintains every engineering system to take decisions at several stages. The aim of the optimization is to increase the benefit or reduce the effort. Optimization can be explained by the process of finding the maximum or minimum values of a function.

In this paper method used for kinematic analysis is inverse kinematic analysis for finding the actuator positions for known end-effector positions. The vibration model is designed consists of mass inertia matrix and stiffness matrix which are useful for finding natural frequency. Using virtual work principle and lagrangian dynamics, stiffness matrix and inertia matrix is obtained. A global frequency index is proposed to obtain the optimal dimensions of the parallel manipulator using Genetic Algorithm. Finally, vibration analysis is carried out using ANSYS for finding the mode shapes and natural frequencies of the manipulator.

2 Kinematic analysis

Kinematic analysis of 2-DOF parallel manipulator is carried out to find the actuator positions for the known end-effector positions. The Schematic arrangement and CATIA model of 2-DOF parallel manipulator is shown in Figure 1. It is composed of a gantry frame, a moving plat form, three sliders and three kinematic chains, one of which consists of a parallelogram structure. Sliders B1, B2, B3 and B4 drive links A1B1, A2B2, A3B3 and A4B4 when they slide along the vertical guide ways, and the sliders are driven by the servomotors via lead screw. The moving platform possesses a 2-DOFpurely translational moving capability in a plane. Thus, the manipulator is redundantly actuated since it has three actuators and only a 2-DOF output.

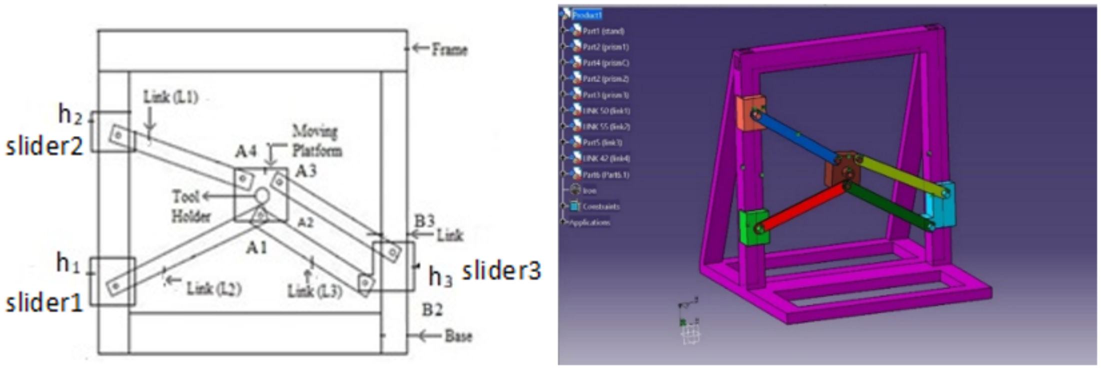

Fig 1. Schematic arrangement and CATIA model of 2-DOF parallel manipulator

2.1 Inverse kinematics

O - xy is fixed to base, x-axis is horizontal and y-axis is vertical

O' - *xy* is attached to the moving platform

Let r_{ai} and r_{bi} be position vectors of joint vectors A_i and B_i respectively.

The coordinate system is defined as $r_{O'} = [x \quad y]^T$

The kinematic constrained equation can be expressed as

$$r_{ai} - r_{bi} = l.n_i \tag{1}$$

Where (*i*=1,2,3,4), *l*is length and n_i is unit vector of link A_iB_i The inverse solutions of the kinematics are

$$h_1 = y - d_3 - \sqrt{l^2 - x^2} \tag{2}$$

$$h_2 = y - d_3 - \sqrt{l^2 - (x - d_1)^2} \tag{3}$$

$$h_3 = y + d_3 + \sqrt{l^2 - (x - d_2)^2} \tag{4}$$

Taking the time derivative of **equations (2) - (4)** the Jacobian matrix can be obtained as

$$\dot{h_1} = \dot{y} + \frac{x}{\sqrt{l^2 - x^2}}\dot{x}$$

$$\dot{h_2} = \dot{y} + \frac{x - d_1}{\sqrt{l^2 - (x - d_1)^2}}\dot{x}$$

$$\dot{h_3} = \dot{y} + \frac{-x + d_2}{\sqrt{l^2 - (x - d_2)^2}}\dot{x}$$

$$\begin{bmatrix} \dot{h}_1 \\ \dot{h}_2 \\ \dot{h}_3 \end{bmatrix} = \begin{bmatrix} \frac{x}{\sqrt{l^2-x^2}} & 1 \\ \frac{x-d_1}{\sqrt{l^2-(x-d_1)^2}} & 1 \\ \frac{-x+d_2}{\sqrt{l^2-(x-d_2)^2}} & 1 \end{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix}$$

q_1, q_2 and $q_3$are the height of the slider S_1, S_2, S_3 and S_4. d_1 is the distance from the point S_2, d_2 and d_3 are the width and height of the moving platform.

$$J = \begin{bmatrix} \frac{x}{\sqrt{l^2-x^2}} & \frac{x-d_1}{\sqrt{l^2-(x-d_1)^2}} & \frac{-x+d_2}{\sqrt{l^2-(x-d_2)^2}} \\ 1 & 1 & 1 \end{bmatrix}^T \tag{5}$$

Using the kinematic equations the 2-DOF parallel manipulator is simulated in MATLAB environment in order validate the working of the manipulator for circular trajectory as shown in Figure 2.

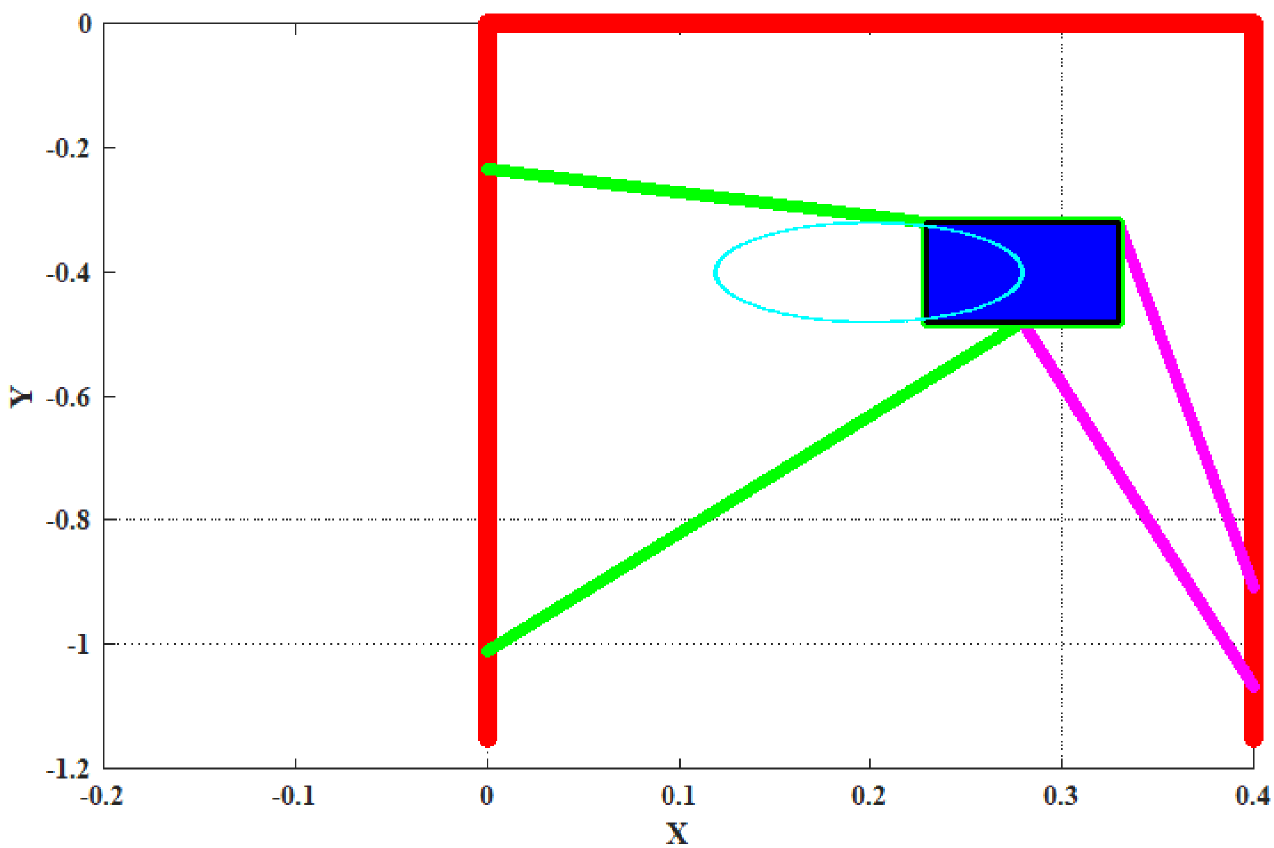

Fig 2. Circular trajectory of 2-DOF parallel manipulator

2.2 Structural dynamic model

Generalized stiffness matrix under the wrench $\begin{bmatrix} F^T & T^T \end{bmatrix}^T$ tested on the moving platform. The axial stiffness of links will make the platform to have a twist $\begin{bmatrix} \Delta_p{}^T & \theta_p \end{bmatrix}^T$

Virtual work principle has the form

$$\begin{bmatrix}\delta\Delta_p{}^T & \delta\theta_p\end{bmatrix}^T \begin{bmatrix}F\\T\end{bmatrix} = \sum_{i=1}^{4}\delta\Delta L_i n_i{}^T f_i \tag{6}$$

where ΔL_i is the deformation and f_i is the axial force of the i^{th} link, $f_i = k_p \Delta L_i n_i$

Here $\frac{1}{k_p} = \frac{1}{k_1} + \frac{2}{k_2}$

Where k_1 and k_2 are the stiffness of link and joint respectively then **equation (6)** can be rewritten as

$$\begin{bmatrix}\delta\Delta_p{}^T & \delta\theta_p\end{bmatrix}^T \begin{bmatrix}F\\T\end{bmatrix} = k_p \delta\Delta L^T \Delta L \tag{7}$$

where $\Delta L = \begin{bmatrix}\Delta L_1 & \Delta L_2 & \Delta L_3 & \Delta L_4\end{bmatrix}^T$

equation (1) can be rewritten as

$$r_{ai} = r_{bi} + l.n_i = r_{o'} + r'_{ai} \tag{8}$$

r'_{ai} is the position vector of point A_i in the moving coordinate system

Small perturbation (force caused by an outside influence) to both sides on both sides of **equation (8)** implementing linearization leads to

$$\Delta_p + \theta_p \times r'_{ai} = \Delta L_i n_i + l\theta_i \times n_i \qquad (i = 1,2,3,4) \tag{9}$$

Where θ_i is the angle deformation of link A_iB_i

Taking dot product with n_i^T on both sides of **equation (9)** leads to

$$\Delta L_i = \begin{bmatrix}n_i{}^T & (r'_{ai} \times n_i)^T\end{bmatrix} \begin{bmatrix}\Delta_p\\ \theta_p\end{bmatrix} \qquad (i = 1,2,3,4) \tag{10}$$

Equation (10) can be rearranged in matrix form as

$$\Delta L = J_p \begin{bmatrix}\Delta_p\\ \theta_p\end{bmatrix} \tag{11}$$

where $J_p = \begin{bmatrix} n_1 & n_2 & n_3 & n_4 \\ (r'_{ai} \times n_1) & (r'_{ai} \times n_2) & (r'_{ai} \times n_3) & (r'_{ai} \times n_4) \end{bmatrix}$

substituting **equation (11)** in **equation (7)** yields the stiffness of the manipulator as

$$\begin{bmatrix}F\\T\end{bmatrix} = K_s \begin{bmatrix}\Delta_p\\ \theta_p\end{bmatrix} \tag{12}$$

Where $K_s = k_p J_p^T J_p$

Introducing a homogenous transformation matrix

$$T_1 = \begin{bmatrix}1 & 0 & 0 & 0 & 0 & 0\\ 0 & 1 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 1\end{bmatrix}^T$$

The stiffness matrix of the redundant parallel manipulator is determined by

$$K = T_1^T K_s T_1 \tag{13}$$

2.3 Generalized mass matrix

The kinetic energy of moving platform can be expressed as

$$E_p = \frac{1}{2} m_p \left(\dot{x}^2 + \dot{y}^2\right) + \frac{1}{2} I_p \alpha^2 \tag{14}$$

where m_p is the mass of moving platform

I_p moment of inertia of the moving platform

Kinetic energy of slider, lead-screw and counterweight can be written as

$$E_s = \frac{1}{2} m_1 \dot{q}_1^2 + \frac{1}{2} m_2 \dot{q}_2^2 + \frac{1}{2} m_3 \dot{q}_3^2 \tag{15}$$

Where m_1 is the mass of slider B_1, lead-screw and counterweight , m_2 is the mass of slider and m_3 is the mass of slider B_4, m is mass of the link $A_i B_{i.}$

The kinetic energy of link $A_1 B_1$ can be expressed as

$$E_{11} = \frac{1}{6} m \left[(\dot{x} - \alpha d_3)^2 + \dot{y}^2 + \dot{q}_1^2 + \dot{y}\dot{q}_1 \right] \tag{16}$$

The kinetic energy of the links $A_2 B_2$, $A_3 B_3$ and $A_4 B_4$

$$E_{12} = \frac{1}{6} m \left[(\dot{x} - \alpha d_3)^2 + \dot{y}^2 + \dot{q}_2^2 + \dot{y}\dot{q}_2 \right] \tag{17}$$

$$E_{13} = \frac{1}{6} m \left[(\dot{x} - \alpha d_3)^2 + (\dot{y} - \alpha d_2)^2 \dot{y}^2 + \dot{q}_2^2 + (\dot{y} - \alpha d_2)\dot{q}_2 \right] \tag{18}$$

$$E_{14} = \frac{1}{6} m \left[(\dot{x} - \alpha d_3)^2 + (\dot{y} + \alpha d_2)^2 \dot{y}^2 + \dot{q}_3^2 + (\dot{y} - \alpha d_2)\dot{q}_3 \right] \tag{19}$$

The total kinetic energy can be written as

$$E = E_{11} + E_{12} + E_{13} + E_{14} + E_s + E_p \tag{20}$$

Equation (20) rewritten as

$$E = \frac{1}{2} \dot{X}^T M \dot{X} \tag{21}$$

where $X = \begin{bmatrix} x & y & \alpha \end{bmatrix}^T$, M is generalized mass matrix and

$$M = \begin{bmatrix} M_{11} & M_{12} & M_{13} \\ M_{21} & M_{22} & 0 \\ M_{31} & 0 & M_{33} \end{bmatrix}$$

$$M_{11} = \frac{4 + (e_1^2 + 2e_2^2 + e_3^2)}{6} m + \frac{1}{2} m_p + \frac{1}{2} m_1 e_1^2 + \frac{1}{2} m_2 e_2^2 + \frac{1}{2} m_3 e_3^2$$

$$M_{12} = M_{21} = \frac{1}{4} m (e_1 + 2e_2 + e_3) + \frac{1}{2} (m_1 e_1 + m_2 e_2 + m_3 e_3)$$

$$M_{12} = M_{21} = \frac{1}{4} m (e_1 + 2e_2 + e_3) + \frac{1}{2} (m_1 e_1 + m_2 e_2 + m_3 e_3)$$

$$M_{22} = 2m + \frac{1}{2}m_p + \frac{1}{2}(m_1 + m_2 + m_3)$$

$M_{33} = \frac{2}{3}\mathrm{md}_3^2 + \frac{1}{2}I_p$ where
$e_1 = \frac{x}{\sqrt{l^2 - x^2}}$,
$e_2 = \frac{d_1\text{-x}}{\sqrt{l^2 - (d_1\text{-x})^2}}$,

$$e_3 = \frac{\text{x-d}_2}{\sqrt{l^2 - (\text{x} - d_2^2)^2}},$$

2.4 Vibration model

Based on the mass matrix and stiffness matrix, the dynamic equation of the parallel manipulator can be expressed as

$$M\ddot{X} + C\dot{X} + \mathrm{KX} = F \tag{22}$$

where C is the damping matrix and F is the external force, neglecting the damp term in **equation (22)** leads to

$$M\ddot{X} + \mathrm{KX} = 0 \tag{23}$$

From **equation (23)**, we get

$$\det(-\omega^2 M + K) = 0 \tag{24}$$

where ω denotes the natural frequency

3 Optimization using MATLAB

In this, optimal dimensions of the 2 DOF parallel manipulator are obtained by considering the following objectives using Genetic Algorithm(GA). The objective function of the proposed GA is to find global frequency. Considering design variables as coordinates (x, y) and radius using MATLAB to obtain the global frequency of the manipulator. A mathematical model contains single objective function.

3.1 Single objective optimization using genetic algorithm (ga)

The optimal design vector [X, Y, R] found as a single objective optimization problem in which the design parameters are lower boundary [0.2 -0.55 0.01] upper boundary [0.4 -0.1 0.08] and some GA parameters are to be selected, i.e., best fitness value. The objective function is evaluated, and the results are shown in Figure 2. The objective function near to the global frequency is 4308.17 and the best fitness value is 4138.13.

3.2 Global frequency

Using genetic algorithm method in MATLAB, values of lower and upper boundary conditions are given, and optimization is done. At different coordinates and different frequencies are obtained.

3.2.1. Modal analysis:
Results obtained by FEM modal analysis are shown from Figure 5 to Figure 7. First six mode shapes are only taken into account because operating speed of machine is only 3000 rpm. Hence higher modes will be irrelevant because excitation frequency will never reach there.

4 Results and Discussion

Mode shapes and natural frequency were obtained by using Modal analysis in ANSYS 18.1. Harmonic analysis with a load of 500N is applies at tool holder of the parallel manipulator in y- direction.it was observed that maximum amplitude of 0.29667 with a natural frequency of 1207.2 Hz.

MATLAB code has been developed to minimize objective function and global frequency; best fitness value is 4138.1332

In the process of optimization at specified coordinates natural frequency increases with increase in the radius of the circular trajectory.

Table 1. Variation of global frequency along the coordinates

Coordinates		Frequency (F1)	
		Minimum	Maximum
X=0.1	Y= -0.2	1.4688	93.3058
X=0.1	Y= -0.4	1.1426	133.668
X=0.1	Y= -0.6	0.7005	179.4006
X=0.1	Y= -0.8	0.7197	233.1722
X=0.2	Y= -0.2	0.2278	135.0774
X=0.2	Y= -0.4	1.5539	156.2153
X=0.2	Y= -0.6	0.0347	191.0709
X=0.2	Y= -0.8	0.4113	229.6715
X=0.3	Y= -0.2	0.8555	177.9418
X=0.3	Y= -0.4	1.3713	201.0167
X=0.3	Y= -0.6	1.0392	234.079
X=0.3	Y= -0.8	0.9317	270.225

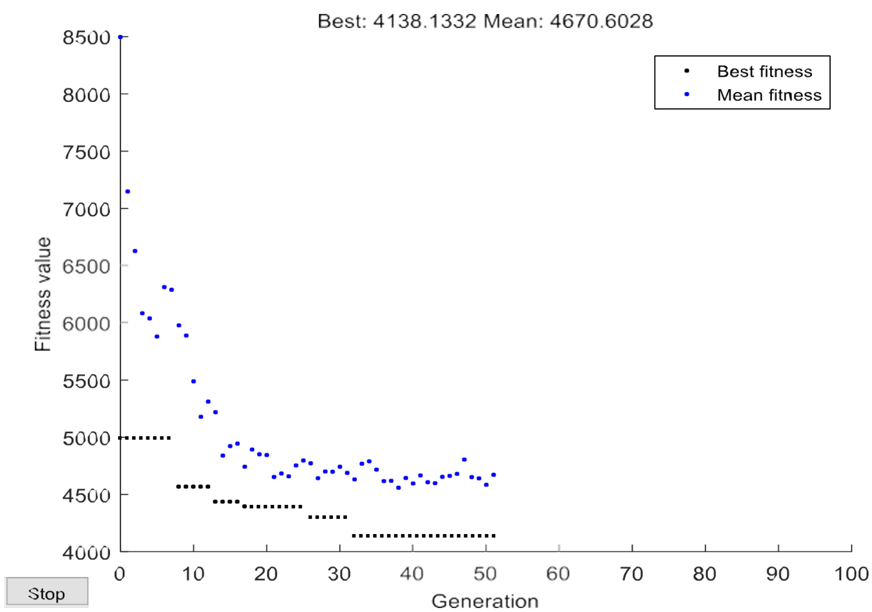

Fig 3. Best fitness value of global frequency of 2-DOF Parallel Manipulator

Table 2. Element type

Element type	Tetrahedron
Number of Elements	89234
Number of Nodes	683158

Table 3. Natural frequencies at different modes

Mode	Natural Frequency (Hz)	Mode	Natural Frequency (Hz)
1	246.56	4	1294.7
2	774.09	5	1596.1
3	1204.0	6	1728.3

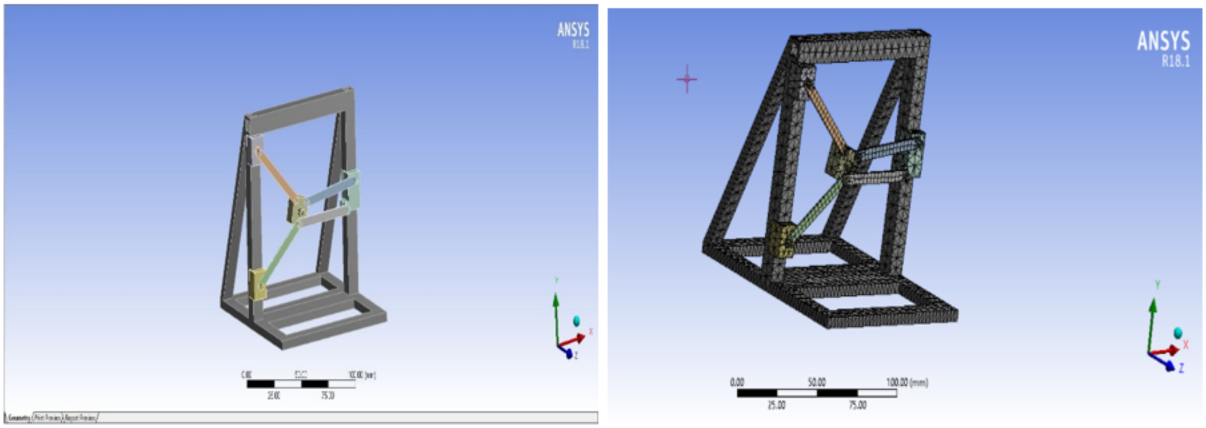

Fig 4. ANSYS Modal and Mesh Model 2-DOF Parallel Manipulator

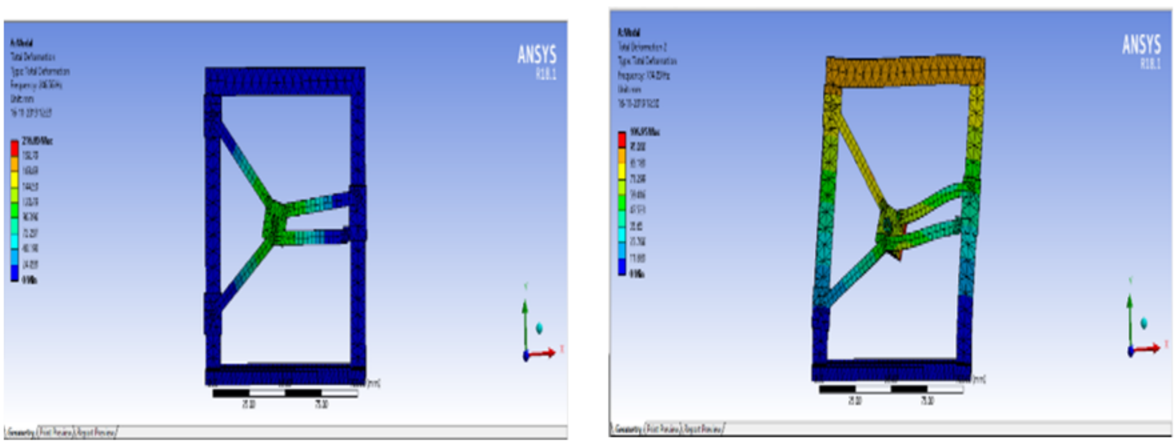

Fig 5. Mode shape 1 at 246.56Hz and Mode shape 2 at 774.09 Hz

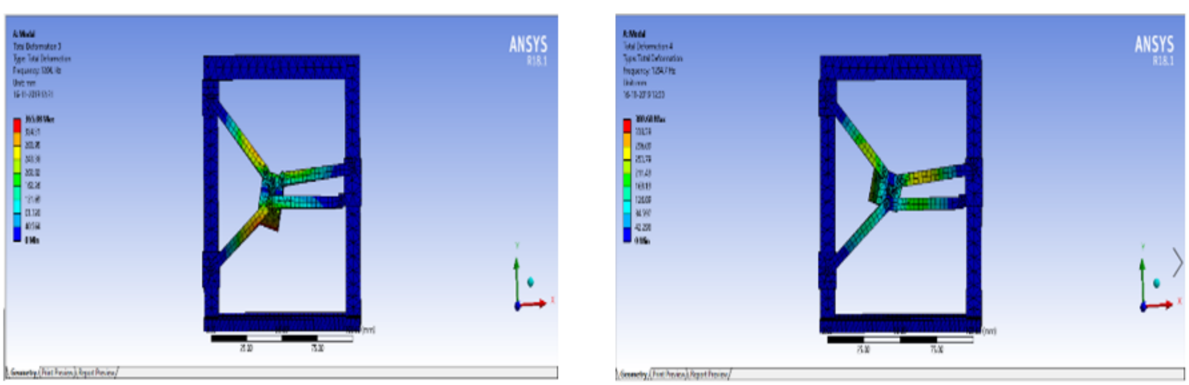

Fig 6. Mode shape 3 at 1204.0 Hz and Mode shape 4 at 1294.7 Hz

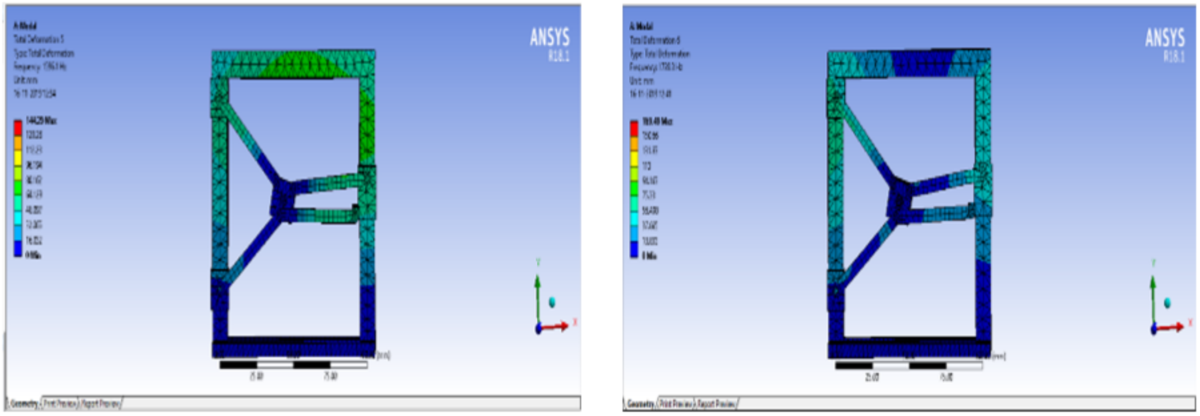

Fig 7. Mode shape 5 at 1596.1Hz and Mode shape 6 at 1728.3Hz

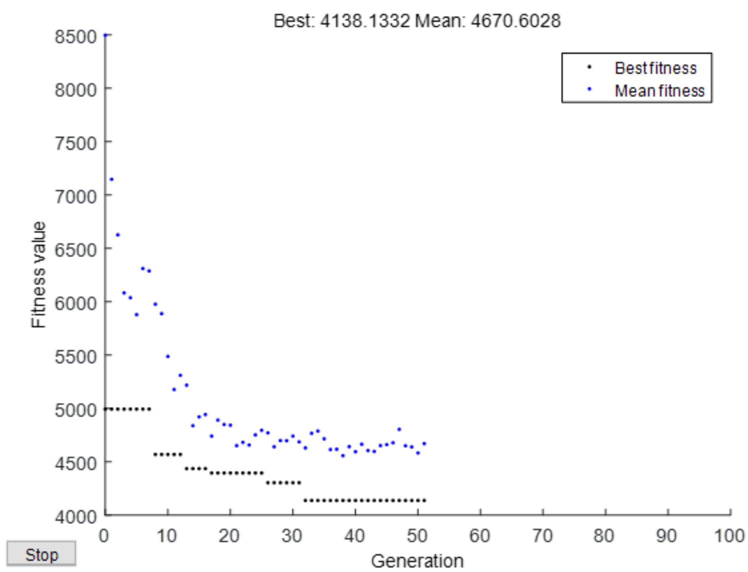

Fig 8. Best fitness value of global frequency

4.1 Harmonic analysis

Harmonic analysis is performed with load of 500N at tool end in y – direction. From the results it is observed that there is a maximum deformation in y – direction is at a natural frequency of 1207.02Hz with an amplitude of 0.2966mm. So, the machine should run at below this frequency to avoid the high vibrations due to resonance. Frequency response results obtained from harmonic analysis are shown in Table 4.

The results of the paper are compared with the literature (Jun Wu et.al.,11]). In addition to this work, a new performance index like a global frequency index is taken as objective for finding the optimal dimensions of the parallel manipulator considering circular trajectory of the end effector. In the literature there is no work presented on Modal analysis and Harmonic analysis using ANSYS for this current Two Degree of Freedom parallel manipulator.

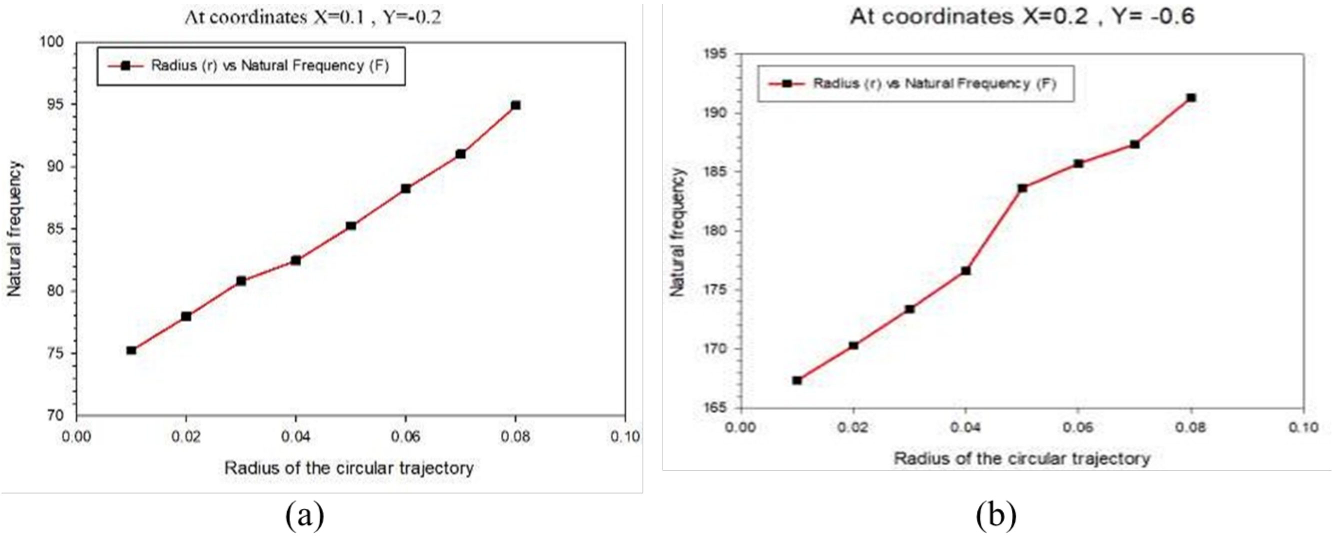

Fig 9. (a) and (b) Variation of natural frequency at different radius of the circular trajectory

Table 4. Harmonic Analysis results

Natural Frequency (Hz)	Harmonic Response
255.6	3.2215×10^{-3}
754.8	2.508×10^{-3}
1207.2	0.29667
1597	0.23173
1722	2.6145×10^{-2}

5 Conclusion

In this paper kinematics and dynamics of Two Degree of Freedom parallel manipulator is done. Using MATLAB code graphics is created and it is simulated in order to know the movement of the mechanism in the workspace. A global frequency index is proposed to obtain the optimal dimensions of the parallel manipulator using Genetic Algorithm. The best fitness value obtained for various circular trajectories in the workspace is 4138.13. Modal analysis is carried out in order to determine the inherent dynamic characteristics of a structure, specifically its natural frequencies and mode shapes. Six mode shapes are extracted for the parallel manipulator. Out of six mode shapes, mode shape1obtained minimum natural frequency 246.6 Hz at the end effector which shows that the end effector has less vibration at minimum natural frequency and at mode shape 6 obtained maximum natural frequency is 1728.3 Hz it indicates where the coupled deformation of legs and end effector. Also, Harmonic analysis is carried out to determine the frequencies at which resonance occurs. The maximum natural frequency obtained in this analysis is 1207.02Hz with an amplitude of 0.2966 mm. So, the machine should run at below this frequency to avoid the high vibrations due to resonance. Harmonic analysis is performed with load of 500N at tool end in y – direction. From the results it is observed that there is a maximum deformation in y – direction is at a natural frequency of 1207.02Hz with an amplitude of 0.2966mm. So, the machine should run at below this frequency to avoid the high vibrations due to resonance.

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