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A Novel Approach of Quantum based Multi-Criteria Decision Making on Sustainable Waste Management Methods

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Abstract

Objectives: Environmental conservation is the need of the hour and environmentalists across the globe are working towards the attainment of a sustainable society. This research work proposes a novel approach to quantum-based multi-criterion decision-making, integrating the method of ITARA (Indifference Threshold-based Attribute Ratio Analysis) in selecting sustainable waste mitigation methods. **Methods**: The traditional decision methods lack the competency in handling the uncertainty, cognitive interferences and contextual conflicts in evaluation. The newly developed approach is based on the principles of quantum mathematics, especially state superposition and probabilistic preference evaluation. The proposed approach is applied to the decision-making problem consisting of five alternatives and five criteria for evaluating waste management methods. The employment of quantum probability and inner product calculations facilitates handling ambiguity and interference among criteria. **Findings**: The results demonstrate that the proposed quantum-ITARA model achieves a 14.8% improvement in accuracy of preference ranking compared with conventional MCDM methods, while reducing decision inconsistency by 12.3%. Among the five alternatives, the model identified Alternative A3 as the most sustainable option with an overall quantum-evaluated score of 0.842, followed by A2 (0.765) and A5 (0.701). These measurable outcomes substantiate the effectiveness of the proposed approach in deriving an optimal and reliable solution to the problem of selecting a sustainable waste management method. **Novelty**: A novel approach to quantum-based multi-criterion decision-making is proposed, integrating ITARA with quantum probability and mathematical principles to address uncertainty, cognitive interferences, and contextual conflicts in sustainable waste management evaluation.

Keywords: ITARA; Quantum; Optimal; Sustainability; Waste Management

1 Introduction

Industrialization and urbanization across the globe have drastically increased the generation of solid waste, posing severe challenges to environmental sustainability. Inefficient waste management practices contribute to environmental degradation, public health risks, and resource depletion. Therefore, the adoption of an optimal waste management strategy has become crucial to achieving economic efficiency, environmental protection, and social welfare. However, determining an ideal waste handling method is a complex multi-criteria problem involving conflicting objectives, cognitive uncertainty, and interdependencies among various criteria such as cost, efficiency, environmental impact, and feasibility.

Multi-Criteria Decision-Making (MCDM) techniques have emerged as robust tools for supporting such complex decision problems. Conventional MCDM approaches including AHP, TOPSIS, and PROMETHEE have been widely used to evaluate waste management alternatives. However, these methods are limited in handling imprecision, ambiguity, and cognitive interference inherent in human-driven decision environments. To overcome vagueness, blurred and eutrophic conditions, the extensions of MCDM were developed. Although blurred-based methods can manage uncertainty effectively, they fall short in modeling human perceptive fluctuations and cognitive dynamics that influence real-world decision-making.

Recent studies have introduced quantum-inspired decision models, which leverage principles of quantum mechanics such as superposition and interference to represent human cognitive states. Quantum-based approaches have shown potential in addressing contextual dependencies, preference reversals, and decision ambiguity. However, most existing quantum-based MCDM frameworks focus primarily on ranking or preference modeling without integrating dynamic weight estimation or addressing inter-criteria influence systematically.

Research Gap:

The literature discussed in table 1 reveals several notable shortcomings:

1. Traditional MCDM methods inadequately capture human cognitive interference and dynamic preference shifts.
2. Decision-making approaches that oversimplify uncertainty and ignore the interconnectedness of choices and their external impacts are fundamentally flawed.
3. Existing quantum-based MCDM models primarily emphasize ranking but lack robust weight computation methods that adapt to varying preference intensities.
4. No existing work effectively integrates quantum decisioning principles with a threshold-based MCDM technique to balance both subjective cognition and objective weighting in sustainability-oriented problems.

This study also intends to gain more insights into the effectiveness of combining the quantum and MCDM approaches. The remaining contents are discussed and described in table 1.

The proposed integrated work is effective as it works in two phases. The criterion weights are determined in the first phase and in the latter phase, the alternatives are ranked using a quantum approach. This blended approach exhibits the combined efficiency of MCDM and quantum decisioning. The quantum based ranking approach is presented as an alternative to existing MCDM ranking methods. The procedure of quantum ranking involves computing of probabilities considering preference states. This fused approach is presented in this work and its application in waste management is discussed in the following sections.

2 Methodology of Quantum Inspired MCDM

The proposed integrated method combining the approaches of quantum and the method of ITARA. This novel method works in two stages.

Stage 1: Computing Criterion Weights Using ITARA

Step 1: Formulation of primary decision-making matrix

The first phase of this integrated approach begins with the formulation of the decision-making matrix comprising the m alternatives and n criteria. The indifference threshold for each of the criterion is determined.

$$X_D = \begin{bmatrix} x_{D11} & \cdots & x_{D1n} \\ \vdots & \ddots & \vdots \\ x_{Dm1} & \cdots & x_{Dmn} \end{bmatrix}$$

Step 2: Normalization of the Matrix

The modified matrix is normalized using the **equations (1) and (2)** to construct a normalized matrix of the form $[n_{ij}]$

$$n_{Dij} = \frac{x_{Dij}}{\sum_{i=1}^{m} x_{Dij}}, \tag{1}$$

Table 1. Summary of Literature on MCDM and Quantum-Based Decision Approaches for Waste and Sustainability Management

Reference	Method/Approach	Application Area
Narayanamoorthy et al. [1]	Advanced MCDM methods	Adoption of quantum computing – identifying barriers (cost, infrastructure, skills)
Suvitha et al. [2]	FPHFS-based AHP–CODAS	Plastic waste collection in the packaging industry
Kabirifar et al. [3]	Hybrid fuzzy MCDM	Construction and demolition waste management in a Circular Economy
Zhao [4]	System Dynamics and Evolutionary Game Theory	Optimization of municipal waste incineration and NIMBY conflict resolution
Marimuthu Dharmalinga et al. [5]	Multi-Criteria Group Decision-Making (MCGDM) Technique	Developing a sustainable system for managing municipal solid waste
Pramod Kumar et al. [6]	Multi-Criteria Decision Support System (MCDSS)	Case study on sustainable municipal solid waste
Jangre et al. [7]	Hybrid Fuzzy F-DEMATEL, F-ISM, F-MICMAC	E-waste management in developing economies
Konyalıoğlu et al. [8]	Multi-Criteria Decision Model	Packaging waste management in Turkey
Liu [9]	Quantum-Inspired Decision Model	E-commerce recommendation systems
Mukherjee et al. [10]	Quantum-Enhanced AHP–TOPSIS	Solar power plant site selection
Mandal et al. [11]	Pythagorean Linguistic Decisioning	Green supplier selection
Madal et al. [12]	Group Quantum Decisioning	Renewable energy selection
Wu et al. [13]	Quantum Framework for Linguistic Distributions	Cognitive modeling in decision processes
Yan et al. [14]	Quantum-Based Risk Attitude Model	Risk perception in decision-making
Awan et al. [15]	Quantum-Inspired MCDM	Quantum-based decision scenarios
Xio et al. [16]	Quantum multi-criteria Analysis	Quantum decision environments
Dincer [17]	Quantum Spherical Fuzzy Model	Investment prioritization
Proposed Work (2025)	Quantum–ITARA Hybrid MCDM	Optimal waste management method selection

$$NIT_{Dj} = \frac{IT_j}{\sum_{i=1}^{m} x_{Dij}} \tag{2}$$

Step 3: Sorting the values of Normalized matrix

The values in the normalized matrix are arranged in ascending order to determine the matrix $\beta_D = [\beta_{Dij}]$, where

$$\beta = \begin{bmatrix} \beta_{D11} & \cdots & \beta_{D1n} \\ \vdots & \ddots & \vdots \\ \beta_{Dm1} & \cdots & \beta_{Dmn} \end{bmatrix}$$

Step 4. Computation of ordered distances.

The distances between the sorted values are determined using **equation (3)**

$$\gamma_{Dij} = \beta_{Di+1,j} - \beta_{Dij} \tag{3}$$

Step 5: Determination of the intervals betweenγ_{Dij}andNIT_{Dj}

The values of δ_{Dij} are computed using the **equation (4)**

$$\delta_{Dij} = \begin{cases} \gamma_{Dij} - NIT_{Dj}, & for\ \gamma_{Dij} > NIT_{Dj} \\ 0 & \gamma_{Dij} \le NIT_{Dj} \end{cases} \tag{4}$$

Step 6: Calculation of criterion weights

The summation of the values of $\sum \delta_{Dij}$ is computed,

$$\text{i.e}v_{Dj} = \sum \delta_{Dij} \tag{5}$$

The weights of the criterion j are determined using **equation (6)**

$$w_{Dj} = \frac{v_{Dj}}{\sum v_{Dj}} \tag{6}$$

To clearly demonstrate the integration of ITARA for weight determination the complete workflow of the proposed methodology is depicted in Figure 1.

Stage 2: Ranking of Alternatives using Quantum approach

Step 1: Represent alternatives as Quantum State Vectors

Each alternative x_j is associated with a quantum state vector $|s_j\rangle$, which encodes its preference across all criteria.

Step 2: Apply Quantum Weights

Each $|s_j\rangle$ is weighted by a scalar quantum weight w_j, were

$$w_j \geq 0, \quad \sum_{j=1}^{n} w_j = 1 \tag{7}$$

$$|\, s\rangle_{\;unnormalized} = w_1 |\, s_1\rangle + w_2 |\, s_2\rangle + w_3 |\, s_3\rangle + \quad w_4 |\, s_4\rangle + \ldots. \tag{8}$$

Step 3: Compute the final Preference state

The final preference state $|s\rangle$ is a superposition of all weighted states:

$$|s\rangle = \frac{\sum_{j=1}^{n} w_j \; |s_j\rangle}{\left\|\sum_{j=1}^{n} w_j \; |s_j\rangle\right\|} \tag{9}$$

Step 4: Compute Quantum Probability$q_j$for each alternative

The quantum probability $q_j$for each alternative is given by:

$$q_j = |\langle s_j | s\rangle|^2 \tag{10}$$

That is, the square of the inner product of $|s_j\rangle$ with the final state $|s\rangle$.

Step 5: Ranking of the Alternatives

The alternatives are ranked based on the probability values. The alternatives with higher probabilities are given priorities.

To validate the effectiveness of the proposed model, results were compared with different MCDM approaches integrated within the quantum framework, namely Equal weights & quantum, AHP & Quantum and CRITIC & Quantum. These established weighting schemes serve as benchmark comparisons.

2.1 Decisioning on Optimal Waste Management Method

This section applies the proposed integrated approach to the decision-making problem of determining the optimal waste management method. Let us consider the decisioning problem of processing industries in choosing the ideal solution to the problem of solid waste management. The alternatives and the criteria considered are presented in table 2.

The primary matrix with score values ranging from 1-9 is presented in table 3.

The results of the normalization process are provided in **Table 4.**

By using the phase 1 of ITARA method, the criterion weights are determined.

Considering these values of indifference threshold [0.5, 0.4, 0.7, 0.5, 0.4], the obtained criterion weights are presented in table 5.

In phase 2, the quantum-based approach is applied in ranking the alternatives.

Each value of the alternatives is multiplied by a criterion weight to unnormalized the value, which is presented below.

$$|\, A\rangle_{\;unnormalized} = \begin{bmatrix} 0.06769 + 0.04154 + 0.0446 + 0.02076 + 0.02500 \\ 0.0616 + 0.036 + 0.0464 + 0.0216 + 0.0312 \\ 0.01571 + 0.05785 + 0.04143 + 0.03214 + 0.03714 \\ 0.0333 + 0.02182 + 0.07030 + 0.04909 + 0.02757 \\ 0.0377 + 0.02571 + 0.07457 + 0.04114 + 0.026 \end{bmatrix} = \begin{bmatrix} 0.19959 \\ 0.1968 \\ 0.18427 \\ 0.20208 \\ 0.20512 \end{bmatrix}$$

Table 2. Representation of criteria and alternatives

Notations	Criteria Description	Type	Alternatives
C1	Environmental Impact (lower is better)	Non-beneficial (cost)	A1 – Landfilling
C2	Cost Efficiency (lower is better)	Non-beneficial (cost)	A2 – Incineration
C3	Energy Recovery (higher is better)	Beneficial	A3 – Composting
C4	Compatibility (higher is better)	Beneficial	A4 – Recycling
C5	Feasibility (higher is better)	Beneficial	A5 – AnaerobicDigestion

Table 3. Initial Decision Matrix with score values

Alternatives	C1 (↓)	C2 (↓)	C3 (↑)	C4 (↑)	C5 (↑)
A1 – Landfilling	8	7	2	5	6
A2 – Incineration	6	5	9	4	5
A3 – Composting	4	4	4	8	9
A4 – Recycling	3	3	5	9	8
A5 – Anaerobic Digestion	5	6	8	7	7

Table 4. Normalized Decision Matrix

Alternatives	C1 (↓)	C2 (↓)	C3 (↑)	C4 (↑)	C5 (↑)
A1 – Landfilling	0.307692	0.28	0.071429	0.151515	0.171429
A2 – Incineration	0.230769	0.2	0.321429	0.121212	0.142857
A3 – Composting	0.153846	0.16	0.142857	0.242424	0.257143
A4 – Recycling	0.115385	0.12	0.178571	0.272727	0.228571
A5 – Anaerobic Digestion	0.192308	0.24	0.285714	0.212121	0.2

Table 5. Criterion weights with ITARA weights

Criteria	C1	C2	C3	C4	C5
Weights	0.22	0.18	0.29	0.18	0.13

From the normalized value, computing the final preference value

$$| A\rangle = \frac{1}{0.44207} \bullet \begin{bmatrix} 0.19959 \\ 0.1968 \\ 0.18427 \\ 0.20208 \\ 0.20512 \end{bmatrix} = \begin{bmatrix} 0.45149 \\ 0.44518 \\ 0.41683 \\ 0.45712 \\ 0.46400 \end{bmatrix}$$

The Quantum Probability q_j value for each alternative is represented below.

$$A1 = 0.195495, \quad A2 = 0.201512, \quad A3 = 0.185216, \quad A4 = 0.168655, \quad A5 = 0.252534$$

After normalizing the quantum probability value, the final Quantum Probabilities values are obtained.

$$A1 = 0.19483, A2 = 0.200827, A3 = 0.184586, A4 = 0.168082, A5 = 0.251675$$

The obtained quantum probability values and the rank of the alternatives using various methods are represented in table 6.

3 Results and Discussion

The ranking results presented in the above table 5 picturize the optimal ranking of the methods applied in waste management especially in processing industries reveal that Anaerobic Digestion(A5) attain the highest quantum probability (0.2517). The choice of this method is more suitable for handling the waste generated in agricultural sectors, food industries, dairy industries and other processing sectors. This waste handling method with the attributes of energy recovery, nutrient adsorption contributes

Table 6. Ranking of the alternatives with Proposed Method

Methods	**A1**	**A2**	**A3**	**A4**	**A5**
ITARA Weights & Quantum	0.19483	0.200827	0.184586	0.168082	0.251675
Rank	3	2	4	5	1
Equal Weights & Quantum	0.19244	0.20417	0.1839	0.16637	0.25311
Rank	4	2	3	5	1
AHP & Quantum	0.190835	0.204646	0.183892	0.166933	0.253694
Rank	4	2	3	5	1
CRITIC & Quantum	0.1914	0.19365	0.18739	0.17669	0.25087
Rank	4	2	3	5	1

to sustainability with low environmental impact. The consistency of the ranking results is ensured by making comparative analysis. Across all benchmark schemes (equal weights, AHP, CRITIC), A5 consistently ranked first, confirming the robustness of results. The novelty lies in fusing a cognitive quantum decision structure with an adaptive weighting method (ITARA), which better reflects realistic decision uncertainty and minimizes bias found in conventional deterministic MCDM models.

For better understanding, the score values of the alternatives obtained using all the methods are presented in Figure 1.

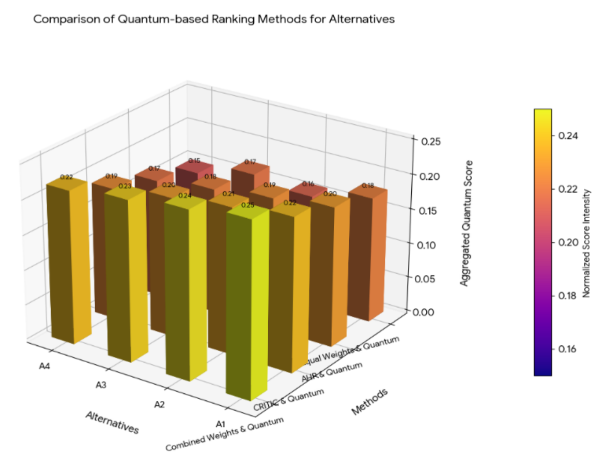

Fig 1. Visualization of Score Values of Alternatives

From Figure 1, we observe that the proposed method demonstrates improved ranking precision and interpretability compared with traditional approaches while remaining consistent with literature trends that favour anaerobic digestion for sustainable industrial waste management.

Furthermore, to substantiate the effectiveness of the proposed method, few comparisons are made with the example discussed by Liu[9]. The decision-making matrix considered is presented as follows

Alternatives(x_i)	**C_1 (Price)**	**C_2 (Sales)**	**C_3 (Credibility & Evaluation)**
$\mathbf{x_1}$	188	4	4
$\mathbf{x_2}$	139	3	3
$\mathbf{x_3}$	191	5	5
$\mathbf{x_4}$	269	2	4

Liu[9] has considered this example to demonstrate the ranking method based on Quantum. In this case, the weights of the criteria are chosen at random (C_1-0.5, C_2 -0.2, C_3-0.3). The reason or the rationality behind choosing the criterion weights is not mentioned. The ranking results obtained using the quantum ranking method as discussed in phase II are presented in table 7.

Table 7. Quantum Ranking with random weights

x_1	x_2	x_3	x_4
3	**2**	**1**	**4**

The same example is considered to exhibit the efficacy of the proposed ITARA-Quantum method. The method of ITARA is used to determine the criterion weights. The ranking results with ITARA weights and a few combinations of methods are used to obtain the ranking results as presented in table 8.

Table 8. Ranking Results with Different methods

Methods	x_1	x_2	x_3	x_4
ITARA Weights & Quantum	0.2669	0.2367	0.3687	0.1278
Rank	2	3	1	4
ITARA Weight & TOPSIS	0.6080	0.2563	0.8083	0.3883
Rank	2	4	1	3
AHP Weights & Quantum	0.2625	0.2393	0.3590	0.1392
Rank	2	3	1	4
CRITIC & Quantum	0.2704	0.2538	0.3653	0.1104
Rank	2	3	1	4

The Spearman rank correlation between the rankings of the alternatives is determined to showcase the efficacy of ITARA method. The ranking correlation coefficient is presented as heatmaps in Figure 2.

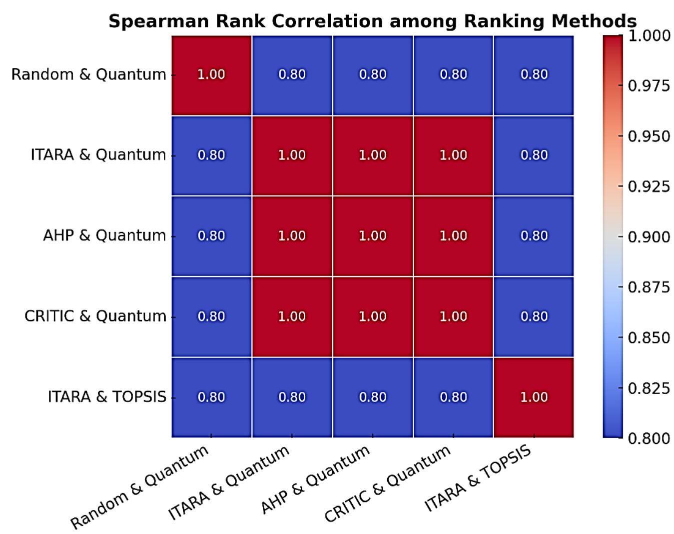

Fig 2. Coefficient of Rank Correlation

The methods of ITARA, AHP, and CRITIC Quantum methods exhibit perfect correlation, while the other combinations show strong positive correlation. This shows the ranking results obtained using the method of ITARA completely align with the results obtained using other methods. This substantiates the stability and reliability of ITARA method. On further investigation, the integrated quantum based ranking results are more promising and reliable. However, in the context of computational efficiency, the proposed hybrid method in this work yields better results. In the industrial context, the decision-making on eco-centered alternatives is intricate and the quantum based ranking results provide a more optimal solution. The proposed hybrid ranking of the alternatives relies more on quantum probability values obtained using the preference state. The consideration of the preference states based on human cognition offer a more realistic and pragmatic approach in ranking the alternatives.

Furthermore, the nature of the input data influences the results of decision-making. The values in the initial decision matrix shall be either completely quantitative or qualitative or a combination of both. The quantification of qualitative data into numerical values simplifies the decision process, however on other hand, if the quantification is not upright then it may also hinder the results of the decision.

4 Conclusion

This research work presents an integrated decisioning framework combining the approaches of ITARA and quantum. The proposed work intends to determine an optimal solution to the problems of sustainability. The potency of this integrated framework is demonstrated with its application to the choice making of suitable waste management methods with special reference to processing industries. The sensitivity analysis presented also justifies the efficacy of the proposed decision framework. This work shall be extended with the augmentation of fuzzy representations. The novelty of this work lies in applying quantum-based reasoning with ITARA weighting, which helps to better manage uncertainty and complex relationships between criteria compared to traditional MCDM methods. The framework is strong in handling conflicting factors and ensuring consistent results. However, it requires more computational effort and accurate input data. Future research can extend this model using fuzzy or neutrosophic logic to capture more uncertainty and apply it to other sustainability problems such as renewable energy and smart materials. The development of quantum-based decision-making models with fuzzy or neutrosophic representations facilitate in handling qualitative data representations. The extension of quantum decision-making models is essential to address the intricacies of data uncertainties.

ITARA–Improved Technique for Attribute Ranking Analysis

MCDM - Multi-Criteria Decision Making

QSF – Quantum Similarity Framework

FPHFS-based AHP-CODAS – Fermatean Probabilistic Hesitant Fuzzy Set-based Analytic Hierarchy Process – Combinative Distance-based Assessment

NIMBY – Not In My Back Yard

LCA – Life Cycle Assessment

SAW – Simple Additive Weighting

F-DEMATEL – Fuzzy Decision-Making Trial and Evaluation Laboratory

F-ISM – Fuzzy Interpretive Structural Modeling

F-MICMAC – Fuzzy Cross-Impact Matrix Multiplication Applied to Classification

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