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RESEARCH ARTICLE

Study on New Ratio and Product Type Estimators for Population Mean Under Ranked Set Sampling

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Abstract

Objectives: The study deals with the concept of Ranked Set Sampling (RSS) as an effective measure to find out the mean of the population when a sample of small size can be found without measuring or with other methods like ranking. The primary goal of this study is to investigate how well the proposed estimators perform compared to existing estimators. **Methods:** This study has introduced new ratio and product estimators to see the population mean under ranked set sampling and compare them with the existing ones in terms of Mean Square Error (MSE) and efficiency. **Findings:** The optimum value of the modified estimators is obtained. The proposed estimators are better than the existing estimators, such as the mean per unit estimator, ratio estimator, and product estimator in ranked set sampling. We have used bivariate normal distribution through the ranked set sampling technique, and the proposed estimators' relative efficiency is higher. **Novelty:** Two tables are used to show the comparison between the proposed estimator and the existing estimators by using bivariate normal distribution. Empirical studies using simulations further validate the effectiveness of these new estimators, suggesting their potential for broader application in statistical analysis.

Keywords: Estimators; Population; Mean Square Error; Efficiency; Ranked Set Sampling

1 Introduction

The Ranked Set Sampling approach is used in situations in which it is difficult to measure all the units that are included in the population, but it is simple and inexpensive to choose units that are included in a population of a smaller size. The unique feature of RSS is that it combines simple random sampling and increases the chance that collected samples will yield representative measurements, which are measurements that span the low, medium and high values in the population. RSS can be more cost-efficient than simple random sampling because fewer samples need to be collected and measured.

In [1] considered RSS as an example of a sampling design for which the inclusion probabilities are not equal and suggested that the sample mean is an unbiased estimator for the population mean. Estimation of population mean using exponential and product type estimator is given in [2] for ranked set sampling. Estimation of population distribution function under stratified random sampling given in [3]. In [4] RSS literature appears in two survey articles. Estimation of the population distribution function in simple random sampling using auxiliary variables is given in [5]. In case of measurement errors, it was given in [6]. Simulation and diverse applications of RSS are introduced in [7]. In [8], showed that the efficiency of the estimator depends upon the population and size of errors in ranking. A new linear combination of ratio estimators under ranked set sampling is given in [9] and [10] to estimate the mean of the population. Another design of ranked set sampling called dual sampling can be used to calculate the population distribution function given in [11]. Sinha AK [12] and Alam, R., Hanif, M., Shahbaz, S.H. et al [13] also showed that the suggested estimators are better than the ratio and product type estimators. In biological and medical research, the cost and collateral damage caused during the collection and measurement of a sample are the reasons behind a compromise on the inference with a fixed and accepted approximation error. The ranked set sampling (RSS) performs better in such scenarios, and the use of auxiliary information even enhances the performance of the estimators. In [14], two new families of estimators are proposed for estimating the finite population distribution function in the presence of non-response under simple random sampling. The proposed estimators require information on the sample distribution functions of the study and auxiliary variables, and additional information on either sample mean or ranks of the auxiliary variable. In [15], two new estimators for estimating the finite population distribution function under simple and stratified random sampling schemes using supplementary information on the distribution function, mean and ranks of the auxiliary variable. In [16], an improved estimator for assessing the mean of a population using a twofold auxiliary variable under simple random sampling. To the first order of approximation, the numerical expression of the bias and mean square error are derived. In [17], a generalized class of exponential-type estimators for estimating the finite population distribution function using dual auxiliary variables under stratified sampling. The biases and mean squared errors (MSEs) of the proposed class of estimators are derived up to the first order of approximation.

The manuscript's main points are as follows: Section 1 includes a literature review and structure of Ranked Set Sampling; Section 2 presents notations and methodology used; Section 3 includes a literature review of "Some existing estimators"; Sections 4 and 5 present two new proposed estimators based on ranked set sampling technique and discuss the comparative analysis of their MSE and variance with the existing estimators; Section 6 discusses comparison of MSE and relative efficiencies of proposed estimators and existing estimators for a particular value of n, r and m using bivariate normal distribution; Section 7 "Conclusion" presents the findings of our study.

1.1 Structure of a Ranked Set Sample

The goal of ranked set sampling is to collect observations from a population that are more likely to span the full range of values in the population than the same number of observations obtained via simple random sampling. To obtain a ranked set sample (RSS) of t observations from a population, we proceed as follows. First, an initial simple random sample (SRS) of t units is selected from the population and rank offered on the attribute of interest. A variety of mechanisms can be used to obtain this ranking, including visual comparisons, expert opinion, or the use of auxiliary variables, but it cannot involve actual measurements of the attribute of interest on the selected items. The unit that is judged to be the smallest in this ranking is included as the first item in RSS and is denoted by $S_{[1]}$. The remaining (t – 1) units in our initial SRS are not considered further in making inferences about the population, their role was solely to assist in the selection of the smallest ranked units for measurement.

Following the selection of $S_{[1]}$, a second SRS of size t is selected from the population. From this, we select the item ranked as the second smallest of the t units and add its attribute measurement, $S_{[2]}$, to the RSS. The entire process results in the t measured observations $S_{[1]}$, ……, $S_{[t]}$ and is called a cycle. The measured observations $S_{[1]}$, ……, $S_{[t]}$ constitute a balanced ranked set sample of size t where the term 'balance' refers to the fact that we have collected one judgment order statistic for each of the ranks 1,2, …., t.

To obtain a balanced RSS with a desired total number of measured observations of size n = tm, we repeat the entire process for m independent cycles, yielding the following balanced RSS of size n

Cycle 1 $S_{[1]1}$ $S_{[2]1}$ … … … ${S_{[t]}}_1$
Cycle 2 $S_{[1]2}$ $S_{[2]2}$ … … … ${S_{[t]}}_2$
.
.
Cycle m $S_{[1]m}$ $S_{[2]m}$ … … … ${S_{[t]}}_m$

2 Methodology

Let $(y_{j[1]},x_{j[1]})$, $(y_{j[2]},x_{j[2]})$,….., $(y_{j[m]},x_{j[m]})$; j = 1, 2, ..., r be the ranked set sample obtained from the j^{th} replication process, where $y_{j[i]}$ denotes the i^{th} judgment order statistics for the variable y. We perform a simulation study to verify the results of the proposed estimators. A total number of 1,000 samples are drawn from bivariate normal distribution BVN (200, 100, 4, 4, ρ) for ρ = -0.7, -0.5, -0.3, 0.3, 0.5, 0.7 under RSS.

Some notations and basic results are as follows: Assume that μ_x and μ_y be the population means of auxiliary and study variables respectively, $\bar{x}$ and $\bar{y}$ denotes the sample mean of auxiliary and study variables variable, correspondingly σ_x and σ_y denotes the population standard deviation. The mean per unit ranked set sample estimator $\overline{y}_{rss}$ is simply the average of the sample observation, given by the formula

$\overline{y}_{rss}= \sum_{j=1}^{r}\sum_{i=1}^{m}\frac{y_{j(i)}}{rm}$

And its variance is given by

V $(\overline{y}_{rss})$ = $\frac{\sigma_y{}^2}{rm}$ - $\frac{1}{r}\frac{1}{m^2}\sum_{i=1}^{m}(\mu_{y(i)}-\mu_y)^2$

Also, we are having the following notations.

$\mu_{x(i)}$ = E $(x_{j[i]})$ $\mu_{y(i)}$ = E $(y_{j[i]})$

$\sigma^2_{x(i)}$ = V $(x_{j[i]})$ = E $(x_{j[i]}-\mu_{x(i)})^2\sigma^2_{y(i)}$ = V $(y_{j[i]})$ = E $(y_{j[i]}-\mu_{y(i)})^2$

$\tau_{x(i)} = \mu_{x(i)}$ - $\mu_x\tau_{y(i)} = \mu_{y(i)}$ - μ_y

$\tau_{xy(i)} = \tau_{x(i)}\tau_{y(i)}\sigma_{xy[i]}$ = E $(x_{j[i]}-\mu_{x(i)})\,(y_{j[i]}-\mu_{y(i)})$

Also, we can easily verify the following results

$\sum_{i=1}^{m}\mu_{x(i)}$ = m$\mu_x\sum_{i=1}^{m}\mu_{y(i)}$ = mμ_y

$\sum_{i=1}^{m}\tau_{x(i)}$ = 0 $\sum_{i=1}^{m}\tau_{y(i)}$ = 0

$\sum_{i=1}^{m}\sigma^2_{x(i)}$ = mσ^2_x - $\sum_{i=1}^{m}\tau^2_{x(i)}\sum_{i=1}^{m}\sigma^2_{y(i)}$ = mσ^2_y - $\sum_{i=1}^{m}\tau^2_{y(i)}$

$\sum_{i=1}^{m}\sigma^2_{xy(i)}$ = mσ^2_{xy} - $\sum_{i=1}^{m}\tau^2_{xy(i)}$

3 Existing Estimators

The existing estimators in RSS are ratio type estimator and product type estimator.

3.1 Ratio Estimator Under RSS

The ratio estimator under RSS is defined as

$\overline{y}_{Rrss} = \overline{y}_{rss}\,\frac{\mu_x}{\overline{x}_{rss}}$

its MSE is given by

MSE $(\overline{y}_{Rrss})$ = $\frac{1}{rm}$ $(\sigma_y{}^2$+$R^2\sigma_x{}^2$-2R$\sigma_{xy})$ - $\frac{1}{r}\frac{1}{m^2}\sum_{i=1}^{m}(\tau_{y(i)}-R\tau_{x(i)})^2$

3.2 Product Estimator Under RSS

The product estimator under RSS is defined as

$\overline{y}_{Prss} = \overline{y}_{rss}\,\frac{\overline{x}_{rss}}{\mu_x}$

its MSE is given by

MSE $(\overline{y}_{Prss})$ = $\frac{1}{rm}$ $(\sigma_y{}^2$+$R^2\sigma_x{}^2$+2R$\sigma_{xy})$ - $\frac{1}{r}\frac{1}{m^2}\sum_{i=1}^{m}(\tau_{y(i)}+R\tau_{x(i)})^2$

4 Proposed Estimator I

$\overline{y}_{prop1}$ = $\frac{\overline{y}_{rss}}{2}\left[\left(\frac{\overline{x}_{rss}}{\mu_x}\right)^{1+\alpha_0} + \left(\frac{\overline{x}_{rss}}{\mu_x}\right)^{-1+\alpha_0}\right]$

Where α_0 is a real constant, $\overline{y}_{rss}$ and $\bar{x}_{rss}$ are sample means of study and auxiliary variables respectively under RSS.

To obtain the approximate expression for bias and MSE of the proposed estimator, we will write $\overline{y}_{rss}$ and $\bar{x}_{rss}$ in terms of δ's as

$\overline{y}_{rss} = \mu_y(1+\delta_1)\;\bar{x}_{rss} = \mu_x(1+\delta_2)$

$\overline{y}_{prop1}$ = $\frac{\mu_y(1+\delta_1)}{2}\left[\left(\frac{\mu_x(1+\delta_2)}{\mu_x}\right)^{1+\alpha_0} + \left(\frac{\mu_x(1+\delta_2)}{\mu_x}\right)^{\alpha_0-1}\right]$

= $\frac{\mu_y(1+\delta_1)}{2}$ $[(1+\delta_2)^{1+\alpha_0}$ +$((1+\delta_2)^{(\alpha_0-1)}$]

= $\frac{\mu_y(1+\delta_1)}{2}$ $[(1+\delta_2)^{1+\alpha_0} + (1+\delta_2)^{-1+\alpha_0}]$

After solving and including the terms up to degree two in δ_1 and δ_2, we have

$$\overline{y}_{prop1} = \mu_y\,(1+\delta_1+\alpha_0\,\delta_2+\frac{(\alpha_0^2-\alpha_0+1)\delta_2^2}{2}+\alpha_0\,\delta_1\delta_2)$$

$$\overline{y}_{prop1}-\mu_y = \mu_y\,(\delta_1+\alpha_0\,\delta_2+\frac{(\alpha_0^2-\alpha_0+1)\delta_2^2}{2}+\alpha_0\,\delta_1\delta_2) \tag{1}$$

After taking the expectation on both sides, we can obtain the expression for bias up to the first-order approximation as

Bias $(\overline{y}_{prop1})$ = $\frac{1}{\mu_y}$ $[\frac{(\alpha_0{}^2-\alpha_0+1)}{2}R^2$ V $(\bar{x}_{rss})$ + α_0 R Cov $(\overline{y}_{rss}, \bar{x}_{rss})]$

Where R = $\frac{\mu_y}{\mu_x}$

Now substituting the values of V $(\bar{x}_{rss})$ and Cov $(\overline{y}_{rss}, \bar{x}_{rss})$, we get

Bias $(\overline{y}_{prop1})$ = $\frac{1}{rm}\frac{1}{\mu_y}$ $[\frac{(\alpha_0{}^2-\alpha_0+1)}{2}R^2\sigma_x{}^2$ + α_0 Rσ_{xy}] - $\frac{1}{r}\frac{1}{m^2}\frac{1}{\mu_y}$ $[\frac{(\alpha_0{}^2-\alpha_0+1)}{2}R^2\sum_{i=1}^{m}\tau_{x(i)}^2$ + α_0 R $\sum_{i=1}^{m}\tau_{xy(i)}]$

To find out the approximate expression for MSE, first, take square on both sides of Equation (1) and consider the terms of δ's up to degree two

$(\overline{y}_{prop1}-\mu_y)^2$ = $\mu_y{}^2(\delta_1{}^2$ + $\alpha_0{}^2\delta_2{}^2$+ $2\alpha_0\,\delta_1\delta_2)$

Taking expectation on both sides, we get

MSE $(\overline{y}_{prop1})$ = V$(\overline{y}_{rss})$ + $\alpha_0{}^2R^2$ V $(\bar{x}_{rss})$ + 2R α_0 Cov $(\overline{y}_{rss}, \bar{x}_{rss})$

= $\frac{1}{rm}$ $(\sigma_y{}^2$+$\alpha_0{}^2R^2\sigma_x{}^2$+2R$\alpha_0\,\sigma_{xy})$ - $\frac{1}{r}\frac{1}{m^2}\sum_{i=1}^{m}(\tau_{y(i)}+\alpha_0\,R\tau_{x(i)})^2$

Which is the required approximate expression for MSE

To find the minimum value of MSE, we will solve the equation

$\frac{\partial MSE(\overline{y}_{prop1})}{\partial\alpha_0}$ = 0

After solving we get the optimum value of α_0 as

Optimal (α_0) = $\frac{-Cov(\overline{y}_{rss}, \bar{x}_{rss})}{RV\,(\bar{x}_{rss})}$

Now substituting this value in the expression of MSE, we get

Minimum [MSE $(\overline{y}_{prop1})$] = [1 - $\rho^2(\overline{y}_{rss}, \overline{x}_{rss})$] V $(\overline{y}_{rss})$

Where $\rho^2(\overline{y}_{rss}, \overline{x}_{rss})$ = $\frac{Cov^2(\overline{y}_{rss}, \overline{x}_{rss})}{V\,(\overline{x}_{rss})V(\overline{y}_{rss})}$

Case 1: When α_0 = 1

$\overline{y}_{prop1}\ \frac{\overline{y}_{rss}}{2}\left[\left(\frac{\overline{x}_{rss}}{\mu_x}\right)^2+1\right]$

Which is the average of the mean per unit and quadratic product type estimator in the RSS

Case 2: when $\alpha_0 = 0$

$\overline{y}_{prop1}=\overline{y}_{rss}\left[\frac{\overline{x}_{rss}}{\mu_x}+\frac{\mu_x}{\overline{x}_{rss}}\right]$

Which is the average of product and ratio type estimator in RSS

Case 3: When $\alpha_0 = -1$

$\overline{y}_{prop1}=\frac{\overline{y}_{rss}}{2}\left[1+\left(\frac{\mu_x}{\bar{x}_{rss}}\right)^2\right]$

Which is the average of the mean per unit and quadratic ratio type estimator in the RSS

Comparison with $\overline{y}_{Rrss}$

MSE $(\overline{y}_{Rrss})$ – Minimum [MSE $(\overline{y}_{prop1})$] = $[(\rho\,(\overline{y}_{rss}, \overline{x}_{rss})\sqrt{V\,(\overline{y}_{rss})}-R\sqrt{V\,(\overline{x}_{rss})}]^2\geq 0$

Which shows that $\overline{y}_{prop1}$ is always more efficient than $\overline{y}_{Rrss}$

Comparison with $\overline{y}_{Prss}$

MSE $(\overline{y}_{Prss})$ – Minimum [MSE $(\overline{y}_{prop1})$] = $[(\rho(\overline{y}_{rss}, \overline{x}_{rss})\sqrt{V\,(\overline{y}_{rss})}+R\sqrt{V\,(\overline{x}_{rss})}]^2\geq 0$

Which shows that $\overline{y}_{prop1}$ is always more efficient than $\overline{y}_{Prss}$

5 Proposed Estimator II

$\overline{y}_{prop2} = \frac{\overline{y}_{rss}}{2}\left[\left(\frac{\mu_x}{\bar{x}_{rss}}\right)^{1+\alpha_0} + \left(\frac{\mu_x}{\bar{x}_{rss}}\right)^{-1+\alpha_0}\right]$

Where α_0 is a real constant, $\overline{y}_{rss}$ and $\bar{x}_{rss}$ are sample means of study and auxiliary variables respectively under RSS.

To obtain the approximate expression for bias and MSE of the proposed estimator, we will write $\overline{y}_{rss}$ and $\bar{x}_{rss}$ in terms of δ's as

$\overline{y}_{rss} = \mu_y(1+\delta_1)$ $\bar{x}_{rss} = \mu_x(1+\delta_2)$

$\overline{y}_{prop2} = \frac{\mu_y(1+\delta_1)}{2}\left[\left(\frac{\mu_x}{\mu_x(1+\delta_2)}\right)^{1+\alpha_0} + \left(\frac{\mu_x}{\mu_x(1+\delta_2)}\right)^{\alpha_0-1}\right]$

$= \frac{\mu_y(1+\delta_1)}{2}\ [(1+\delta_2)^{-1-\alpha_0} + ((1+\delta_2)^{-\alpha_0+1}\ ]$

After solving and including the terms up to degree two in δ_1 and δ_2, we have

$$\overline{y}_{prop2} = \mu_y\left(1+\delta_1 - \alpha_0\,\delta_2 + \frac{(\alpha_0^2+\alpha_0+1)\delta_2^2}{2} - \alpha_0\delta_1\delta_2\right.$$

$$\overline{y}_{prop2} - \mu_y = \mu_y\left(\delta_1 - \alpha_0\,\delta_2 + \frac{(\alpha_0^2+\alpha_0+1)\delta_2^2}{2} - \alpha_0\delta_1\delta_2\right. \tag{2}$$

After taking the expectation on both sides, we can obtain the expression for bias up to the first-order approximation as

Bias $(\overline{y}_{prop2}) = \frac{1}{\mu_y}\ [\frac{({\alpha_0}^2+\alpha_0+1)}{2}R^2\ V\ (\bar{x}_{rss})$ - α_0 R Cov $(\overline{y}_{rss}\ ,\bar{x}_{rss})]$

Where R = $\frac{\mu_y}{\mu_x}$

Now substituting the values of V $(\bar{x}_{rss})$ and Cov$(\overline{y}_{rss}\ ,\bar{x}_{rss})$, we get

Bias $(\overline{y}_{prop2}) = \frac{1}{rm}\frac{1}{\mu_y}\ [\frac{({\alpha_0}^2+\alpha_0+1)}{2}R^2{\sigma_x}^2$ - α_0 Rσ_{xy}] - $\frac{1}{r}\frac{1}{m^2}\frac{1}{\mu_y}$ $[\frac{({\alpha_0}^2+\alpha_0+1)}{2}R^2\sum_{i=1}^{m}\tau_{x(i)}^2$ - α_0 R $\sum_{i=1}^{m}\tau_{xy(i)}$]

To find out the approximate expression for MSE, first take square on both sides of Equation (2) and consider the terms of δ's up to degree two

$(\overline{y}_{prop2} - \mu_y)^2 = {\mu_y}^2({\delta_1}^2 + {\alpha_0}^2{\delta_2}^2 - 2\alpha_0\,\delta_1\delta_2)$

Taking expectation on both sides, we get

MSE $(\overline{y}_{prop2})$ = V$(\overline{y}_{rss})$ + ${\alpha_0}^2R^2$ V $(\bar{x}_{rss})$ - 2R α_0 Cov$(\overline{y}_{rss}\ ,\bar{x}_{rss})$

$= \frac{1}{rm}\ ({\sigma_y}^2+{\alpha_0}^2R^2{\sigma_x}^2$-2R$\alpha_0\ \sigma_{xy})$ - $\frac{1}{r}\frac{1}{m^2}\sum_{i=1}^{m}(\tau_{y(i)} - \alpha_0\ R\tau_{x(i)})^2$

Which is the required approximate expression for MSE

To find the minimum value of MSE, we will solve the equation

$\frac{\partial MSE(\overline{y}_{prop2})}{\alpha_0} = 0$

After solving we get the optimum value of α_0 as

Optimal $(\alpha_0) = \frac{Cov(\overline{y}_{rss}\ ,\overline{x}_{rss})}{RV\ (\overline{x}_{rss})}$

Now substituting this value in the expression of MSE, we get

Minimum [MSE $(\overline{y}_{prop2})$] = [1 - $\rho^2(\overline{y}_{rss}\ ,\overline{x}_{rss})$] V $(\overline{y}_{rss})$

Where $\rho^2(\overline{y}_{rss}\ ,\overline{x}_{rss}) = \frac{Cov^2(\overline{y}_{rss}\ ,\overline{x}_{rss})}{V\ (\overline{x}_{rss})V(\overline{y}_{rss})}$

Case 1: When α_0 = 1

$\overline{y}_{prop2} = \frac{\overline{y}_{rss}}{2}\left[\left(\frac{\mu_x}{\bar{x}_{rss}}\right)^2 + 1\right]$

Which is the average of the mean per unit and quadratic ratio type estimator in the RSS

Case 2: when $\alpha_0 = 0$

$\overline{y}_{prop2} = \overline{y}_{rss}\left[\frac{\bar{x}_{rss}}{\mu_x} + \frac{\mu_x}{\bar{x}_{rss}}\right]$

Which is the average of product and ratio type estimator in RSS

Case 3: When $\alpha_0 = -1$

$\overline{y}_{prop2} = \frac{\overline{y}_{rss}}{2}\left[1 + \left(\frac{\bar{x}_{rss}}{\mu_x}\right)^2\right]$

Which is the average of the mean per unit and quadratic product type estimator in the RSS

Comparison with $\overline{y}_{Rrss}$

MSE ($\overline{y}_{Rrss}$) – Minimum [MSE ($\overline{y}_{prop2}$)] = $[(\rho\left(\overline{y}_{rss}, \overline{x}_{rss}\right)\sqrt{V\left(\overline{y}_{rss}\right)} - R\sqrt{V\ \left(\overline{x}_{rss}\right)}]^2 \geq 0$

Which shows that $\overline{y}_{prop2}$ is always more efficient than $\overline{y}_{Rrss}$

Comparison with $\overline{y}_{Prss}$

MSE ($\overline{y}_{Prss}$) – Minimum [MSE ($\overline{y}_{prop2}$)] = $[(\rho(\overline{y}_{rss}, \overline{x}_{rss})\sqrt{V\left(\overline{y}_{rss}\right)} + R\sqrt{V\ \left(\overline{x}_{rss}\right)}]^2 \geq 0$

Which shows that $\overline{y}_{prop2}$ is always more efficient than $\overline{y}_{Prss}$

6 Numerical Study

The relative efficiency of an estimator $\triangle$ with respect to $\overline{y}_{rss}$ to estimate the population mean is

Efficiency ($\triangle$) = $\frac{MSE(\overline{y}_{rss})}{MSE(\triangle)}$

The MSE values of $\overline{y}_{rss}$, $\overline{y}_{Rrss}$, $\overline{y}_{Prss}$, $\overline{y}_{prop1}$, $\overline{y}_{prop2}$ are obtained for different values of n, m, and r in Table 1.

The relative efficiencies of $\overline{y}_{Rrss}$, $\overline{y}_{Prss}$, $\overline{y}_{prop1}$, $\overline{y}_{prop2}$ with respect to $\overline{y}_{rss}$ for different values of ρ and n are shown in Table 2.

Table 1. MSE (n =12, m = 4, r = 3)

ρ	**MSE ($\overline{y}_{rss}$)**	**MSE ($\overline{y}_{Rrss}$)**	**MSE ($\overline{y}_{Prss}$)**	**MSE ($\overline{y}_{prop1}$)**	**MSE ($\overline{y}_{prop2}$)**
-0.7	0.33475	1.48903	0.70457	0.09962	0.11618
-0.5	0.33475	1.56394	1.00284	0.12259	0.14332
-0.3	0.33475	1.57367	1.23878	0.13417	0.15696
0.3	0.33475	1.23961	1.57318	0.13428	0.15716
0.5	0.33475	1.00288	1.56033	0.12277	0.14350
0.7	0.33475	0.70402	1.48545	0.09982	0.11643

Table 2. Relative Efficiencies (n = 12, m = 4, r = 3)

ρ	**Eff ($\overline{y}_{rss}$)**	**Eff ($\overline{y}_{Rrss}$)**	**Eff ($\overline{y}_{Prss}$)**	**Eff ($\overline{y}_{prop1}$)**	**Eff ($\overline{y}_{prop2}$)**
-0.7	1	0.22481	0.47512	3.36036	2.88125
-0.5	1	0.21405	0.33381	2.73071	2.33571
-0.3	1	0.21236	0.27023	2.49507	2.13270
0.3	1	0.27005	0.21279	2.49289	2.12999
0.5	1	0.33379	0.21454	2.72672	2.33272
0.7	1	0.47549	0.22536	3.35365	2.87525

Some results based on Tables 1 and 2 are as follows:

1. $\overline{y}_{prop1}$ and $\overline{y}_{prop2}$ are always more efficient than the other estimators.
2. The efficiency of all the estimators increases as the value of ρ increases.
3. For this particular population, all the estimators are approximately unbiased estimators of population mean.

7 Conclusion

This study deals with the MSE and efficiencies of the proposed estimators for different values of n, r, and m. But in all the cases the proposed estimators are much better than the existing estimators. The proposed estimator (1) $\overline{y}_{prop1}$ and proposed estimator (2) $\overline{y}_{prop2}$ is found to be more statistically efficient than the mean per unit estimator, product estimator, and ratio estimator in RSS.

It is clear from Tables 1 and 2 that the proposed estimators have less mean square error and higher efficiency than the existing estimators. It is also seen that for different values of α_0 the proposed estimators reduce to average of some existing estimators. Also, the optimum value of α_0 give minimum mean square error which is the same in both the proposed estimators. Our findings indicate that the proposed estimators provide more efficient estimates under certain conditions, particularly when sample sizes are small, and rankings are either perfect or imperfect.

As the sample size is increased, the efficiency of the proposed estimator was found to be higher. In the same way as ρ increases, the efficiency of the proposed estimator also increases. Hence the use of the proposed estimator of population mean would be more beneficial than the existing ones.

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