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Mathematical Analysis of Cross Fluid Flow Over Exponentially Stretching Sheet with Heat and Mass Transfer

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Abstract

Objective: To examine the heat and mass transfer of cross-fluid flow over exponentially stretchable sheet for the flow, temperature, and concentration profile. **Method:** This work examines the heat and mass transfer rate for the various profiles viz. velocity, temperature, and concentration for different values of the skin friction constraint, Nusselt quantity, and Sherwood quantity. The equations of momentum, flow, thermal, and concentration aspects have been governed as per the physical aspects considered for this research. Suitable similarity variables are applied to simplify the momentum equation and to alter the remaining governing equations to non-linear ordinary differential equations for solution. The R-K Felhberg (Runge -Kutta Fifth order) method evaluates the complex system of equations based on the shooting technique. **Findings:** The heat transfer rate exhibits a 40% increase with enhancements in the Prandtl number values (1, 5, 10). Additionally, the stretching parameter directly influences velocity, leading to a 56% improvement in the heat transfer rate for the corresponding parameter values (1.0, 1.5, and 2.0). **Novelty:** This research presents a pioneering approach to fluid mechanics by investigating non-Newtonian fluids on stretching surfaces, offering crucial insights for engineering advancements. Notably, it introduces the Cross-fluid model to analyze heat and mass transfer rates over an exponentially stretching sheet, employing the Runge-Kutta Fehlberg method for precise numerical evaluation. This novel framework enhances existing literature by addressing complex rheological behaviours and improving predictive accuracy in thermal and mass transport applications.

Keywords: Exponentially stretching sheet; Skin friction coefficient; Cross fluid; Heat and mass transfer; Shooting technique

1 Introduction

MHD boundary layer flow and heat transfer are discussed over an exponentially stretching sheet incorporating the effects of a heat source sink[(1), (2)]. The study of cross-fluid flow over an exponentially stretching sheet with heat and mass transfer plays a crucial role in various engineering and industrial applications, such as polymer processing, aerodynamic heating, and electronic cooling[3]. In this phenomenon, a fluid flows over a sheet that stretches exponentially, important to multipart variations in velocity, temperature, and concentration. Heat transfer is inclined by aspects such as the Prandtl number, and thermal radiation, while mass transfer depends on the potential chemical reactions and Schmidt number[4]. It also plays a vital role in thermal lining coatings, nuclear reactor cooling, and solar energy harvesting, where efficient heat and mass transfer are important for performance enhancement[5]. In chemical engineering, this phenomenon applies to the drying of paints and coatings, separation processes, and catalytic reactions, where regulatory temperature and concentration gradients are perilous for efficiency. Moreover, cross-fluid applications in progressive cooling technologies, such as heat exchangers and microchannel cooling systems, depend on amended thermal conduction to optimize performance[6].

The investigation of cross-fluid flow over an exponentially stretching sheet with heat and mass transfer offers numerous research gaps, mainly concerning the properties of Prandtl number, temperature, and concentration fields. Current studies chiefly focus on fluids with a constant Prandtl number, but the effect of varying or temperature-dependent Prandtl numbers on heat transfer dynamics remains underexplored. Thermal radiation is often modeled using a linear approximation, whereas real-world applications involve nonlinear radiative heat transfer, which significantly impacts temperature distribution, especially in high-temperature environments. Additionally, the effect of viscous dissipation, particularly in high-speed and high-viscosity flows, is not extensively studied in combination with cross-fluid behavior and exponentially stretching surfaces. The interaction between temperature-dependent viscosity and thermal conductivity is another aspect that requires deeper analysis to understand its role in modifying the thermal boundary layer. Many existing models assume uniform or constant concentration fields, whereas, in practical applications, non-uniform concentration profiles, species diffusion, and chemical reactions significantly alter mass transfer rates. Addressing these research gaps will contribute to the development of more comprehensive and practically relevant models that can improve industrial and engineering applications such as polymer processing, thermal coating, and biofluid transport.

To simplify the analysis, similarity conversions are used to change the governing partial differential equations into ordinary differential equations, which are numerically solved by different methods like Runge-Kutta or shooting techniques. The findings offer valuable insights into velocity distribution, temperature variations, and concentration profiles, aiding in the optimization of industrial processes and improving heat and mass transfer efficiency.

2 Problem Formulation

The problem involves the steady, two-dimensional, incompressible flow of a cross fluid over an exponentially stretching sheet. The sheet is endangered to thermal radiation, and adjustable heat and mass transfer conditions, considering temperature and concentration-dependent dispersion effects. The governing equations embrace the Navier-Stokes equations for momentum and, the energy equation for heat transfer.

Governing Equations[7], [8]

Continuity Equation (Incompressible fluid assumption):

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

Momentum Equation (Considering cross fluid model and stretching sheet):

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\vartheta\frac{\partial^2 u}{\partial y^2}\left[1+\Gamma^2\left(\frac{\partial u}{\partial y}\right)^2\right]^n \tag{2}$$

Energy Equation:

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k}{\rho c_p}\frac{\partial^2 T}{\partial y^2} \tag{3}$$

Concentration Equation:

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial Y}=D_B\frac{\partial^2 C}{\partial y^2} \tag{4}$$

Boundary Conditions

$$u=u_w=be^{\frac{x}{2l}},\ v=o, T=T_w,\ C=C_w\ \ at\, y=0 \tag{5}$$

$$u \to U = ae^{\frac{x}{2l}},\ T \to T_\infty,\ C \to C_\infty\ \ when\ y \to \infty \tag{6}$$

3 Methodology

The governing equations, including the equation of continuity, momentum, energy, and concentration, are initially formulated while accounting for the belongings of the Prandtl number, thermal radiation, and concentration. These partial differential equations (PDEs) are then converted into a system of ordinary differential equations (ODEs) by suitable similarity alterations and simplifying the problem. The shooting method[(9)],[(10)],[(11)] and the R-K Fehlberg method are used to solve the equations numerically. A comprehensive parametric study is directed to observe the effect of key dimensionless parameters on velocity, temperature, and concentration distributions. The results are confirmed against existing literature, and graphical representations are employed to illustrate the physical behavior, highlighting the significance of heat transfer and mass diffusion effects. This methodology offers a vigorous outline for investigating heat and mass transfer in cross-fluid flows, supporting in the optimization of industrial and engineering applications

Similarity Variables[(12)],[(13)]

$$\eta = y\sqrt{\frac{\mu}{2l\vartheta}}e^{\frac{x}{2l}};\psi = \sqrt{2l\mu}\,e^{\frac{x}{2l}}\ f(\eta);v = -\sqrt{\frac{2b\vartheta}{n+1}}\left[\frac{n+1}{2}x^{\frac{n-1}{2}}f(\eta) + yx^{n-1}\left(\frac{n-1}{2}\right)f'\sqrt{\frac{b(n+1)}{2\vartheta}}\right] \tag{7}$$

$$\theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \qquad \phi(\eta) = \frac{C - C_\infty}{C_w - C_\infty} \tag{8}$$

Converted system

$$2f'^2 - ff'' - f'''\left[1 + (We)^2 f''^2\right]^n = 0 \tag{9}$$

$$\theta'' + Pr\,(f\theta' - 4f'\theta) = 0 \tag{10}$$

$$\emptyset'' + Sc\,(f\emptyset' - 4\emptyset f') = 0 \tag{11}$$

Reduced boundary conditions are

$$f(0) = 0, f'(0) = 1, \theta(0) = 1, \phi(0) = 1 \tag{12}$$

$$f'(\infty) = \lambda,\ \theta(\infty) = 0,\ \phi(\infty) = 0 \tag{13}$$

4 Results and Discussion

The mathematical analysis of cross-fluid flow over an exponentially stretching sheet with heat and mass transfer offers valuable insights into the influence of Prandtl number, thermal radiation, temperature, and concentration effects on the flow behavior. The results are analyzed through velocity, temperature, and concentration profiles under varying physical parameters, and their implications are discussed in detail. Prandtl Number (Pr), temperature-dependent properties, and Schmidt Number (Sc) govern the relative thickness of momentum and thermal boundary layers, variation in viscosity, and cross-diffusion phenomena respectively.

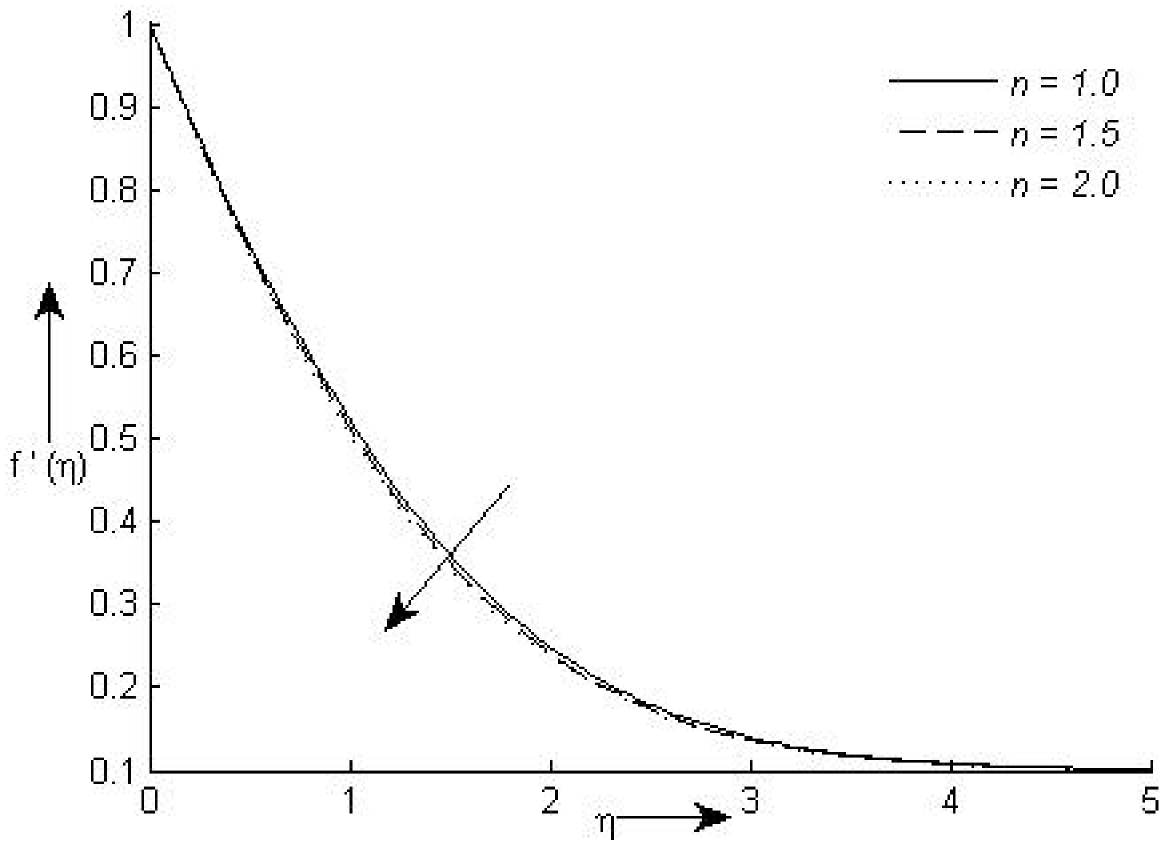

Fig 1. Velocity vs stretching parameter

4.1 Velocity parameters

Figure 1 illustrates the velocity profile $f'(\eta)$ as a function of the similarity variable η for different values of parameter n (1.0, 1.5, and 2.0). The velocity starts at its maximum near the stretching surface ($\eta = 0$) and decreases as η increases, marking boundary layer development. Higher n values lead to a faster velocity decline, indicating a thinner boundary layer and stronger shear-thinning effects in non-Newtonian fluids. This trend, typical of power-law fluids, highlights reduced viscosity and accelerated momentum dispersion, which is crucial for applications like polymer processing and coating technologies.

4.2 Velocity vs Weissenberg number

Figure 2 illustrates the velocity profile $f'(\eta)$ as a function of the similarity variable η for three different values of the Weissenberg number (We), which represents the elasticity of the fluid. The curves resemble We=0.5, We=1.0, and We = 1.5, showing how growing viscoelastic affects the velocity distribution. The velocity primarily decreases rapidly before steadying as η increases, indicating the development of the boundary layer. As We increase, the velocity profile shifts downward, indicating that stronger elastic forces resist the stretching motion, resulting in reduced velocity. This effect is predominantly evident in the far-field region, where higher We values cause a delayed momentum diffusion. The results highlight the impact of viscoelasticity on boundary layer formation, which is crucial in polymer processing, biofluid mechanics, and industrial applications involving non-Newtonian fluids.

4.3 Velocity with Cross fluid index

Figure 3 presents the velocity profile $f'(\eta)$ as a function of η for different values of m (1.0, 1.2, and 1.4). The velocity peaks near the stretching surface ($\eta = 0$) and gradually decreases, illustrating boundary layer development. Higher mm values cause a sharper velocity decline, resulting in a thinner boundary layer and enhanced momentum diffusion. This behavior is crucial for optimizing fluid flow in applications like polymer extrusion, heat-treated materials, and coating processes.

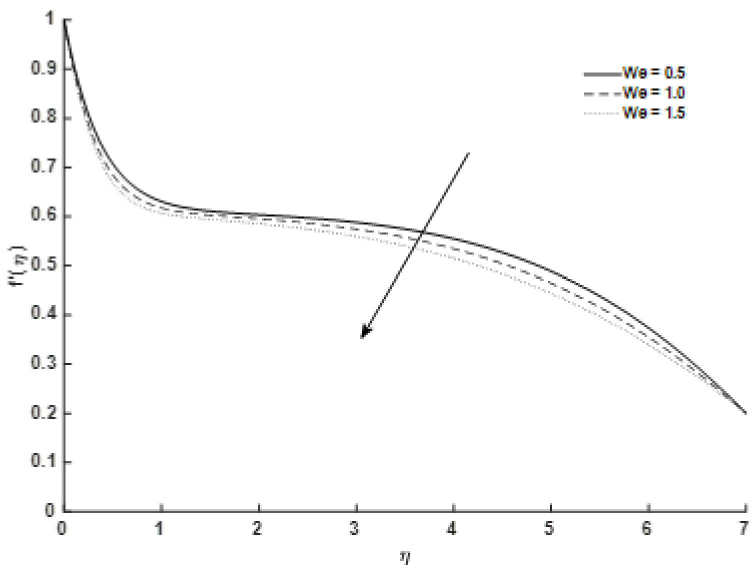

Fig 2. Comparison of Velocity profile and Weissenberg number

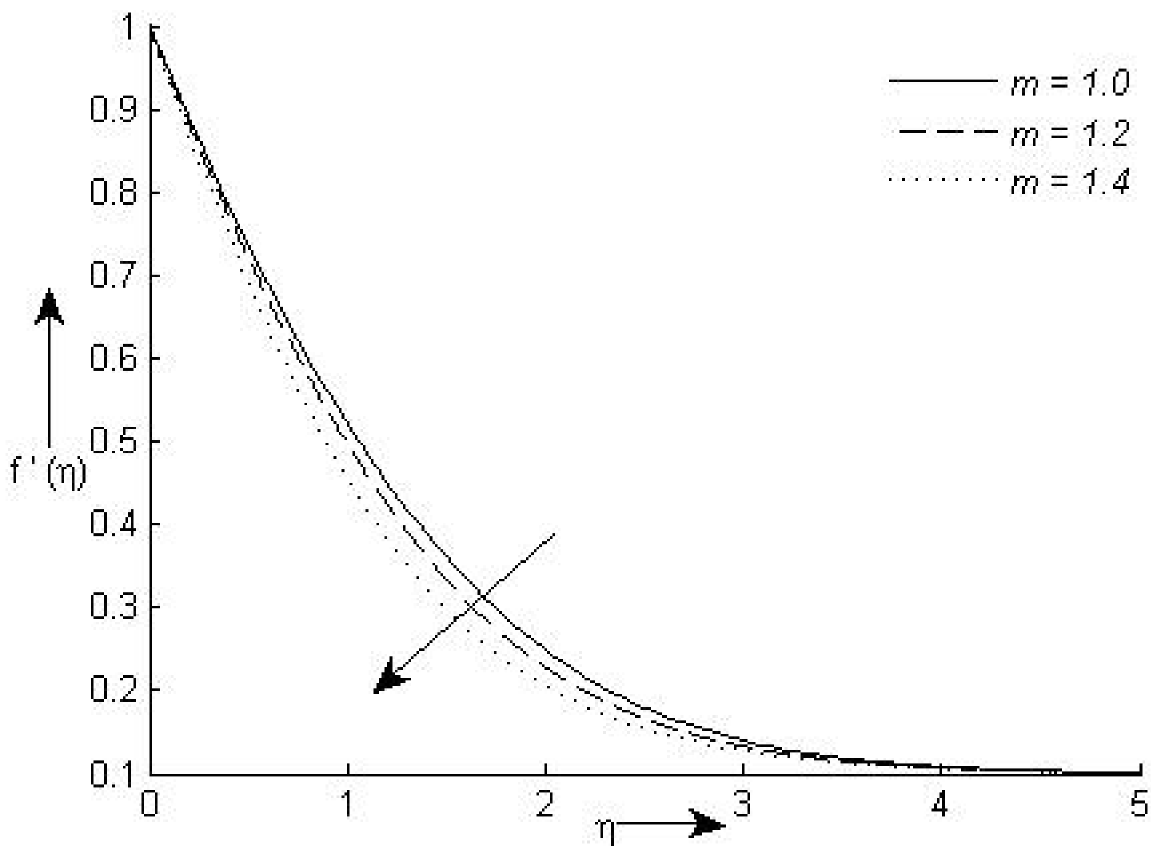

Fig 3. Velocity with Cross fluid index

4.4 Temperature vs Prandtl Number

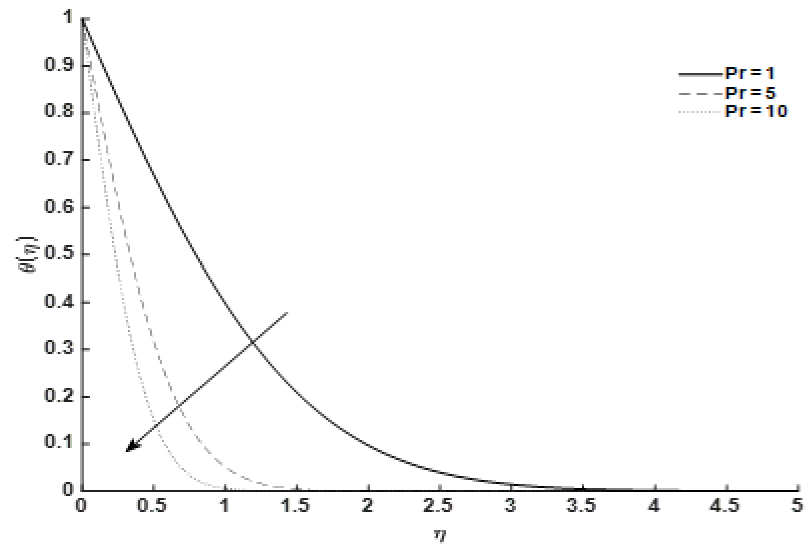

Fig 4. Temperature vs Prandtl number

Table 1. Comparison of the outcomes of the temperature profile to the varying values of the dimensionless parameter

Dimensionless number (Pr)	Temperature Profile $[-\theta'(0)]$				
	Awais [14]	Residual error	Sharma [15]	Residual error	Present result
1	1.1315	0.4441	1.1287	0.4413	0.6874
5	1.9886	0.3636	1.7998	0.1748	1.6250
10	2.6038	0.2563	2.6182	0.2707	2.3475

The findings of this work are more prominent than the previous as compared in the above table with the work of the Awais [14] and Sharma [15] corresponding to the different values of the dimensionless constant. Much better results for heat transfer rate are achieved using the latest method for solving such complex problems. The findings are one of a kind due to the physical aspects considered and analyzed according to the different parameters. In future, this work can be extended to more complex three-dimensional surfaces to study the effects of the physical quantities over the heat and mass transfer rate.

4.5 Contour expression of temperature to cross-fluid

This research is focused on analyzing the mathematical and substantial changes of cross fluid model on a nonlinear variable sheet with heat and mass transit. The investigation is made for some physical properties of the problem like velocity, concentration, and temperature profile in the occurrence of cross-fluid parameters, nonlinear parameters, Weissenberg numbers, Prandtl numbers, etc.

5 Conclusion

This study discusses the cross-fluid on an exponential stretching sheet to analyze the physical aspects of the problem in correspondence to the cross-fluid index (m), stretching parameter (n), Weissenberg number (We), Schmidt Number (Sc), Prandtl Number (Pr). The problem is decorated with a set of highly complex partial differential leading equations with appropriate boundary settings and converted into a structure of higher-order differential equations employing the transformation variables. The problem is then solved by the well-known mathematical software MATLAB with the differential system solver bvp4c and the results in the form of numerical data and graphs as considered in the research.

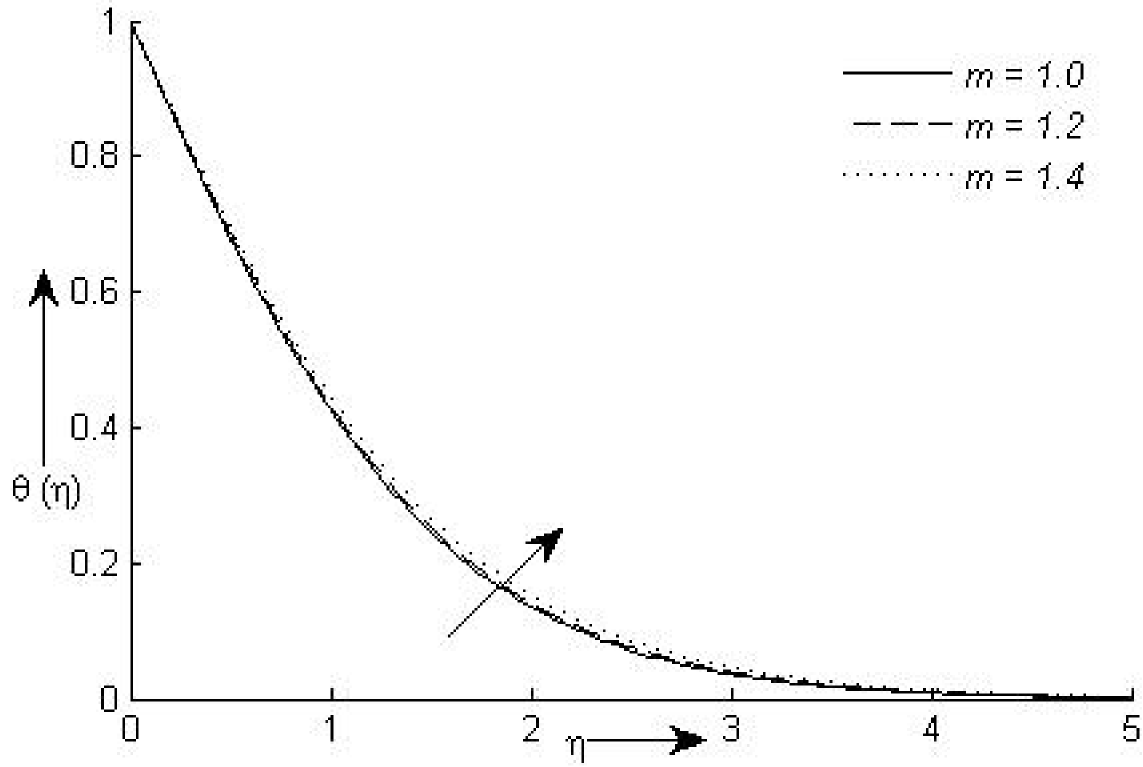

Fig 5. Contour expression of temperature to cross fluid index

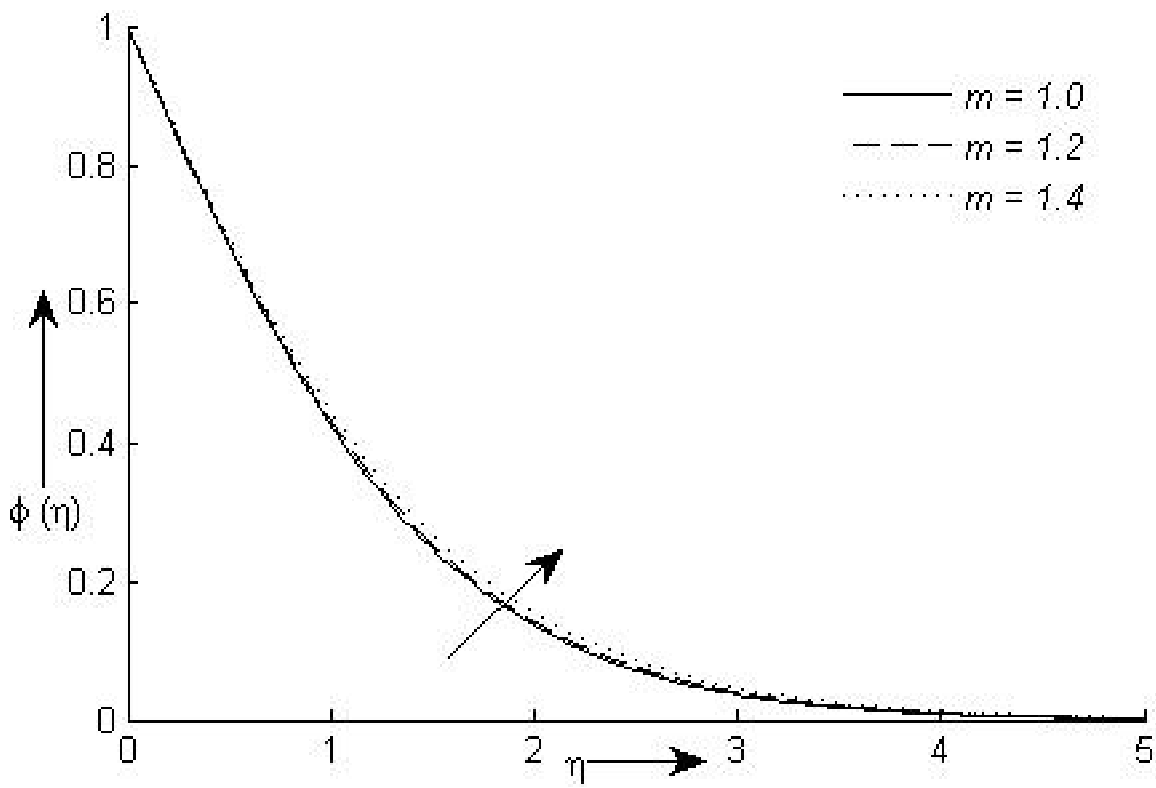

Fig 6. Contour expression of concentration to cross fluid index

Table 2. The explanation of the physical problem of cross-fluid flow past an exponential expanding sheet is expected in the form of Nusselt quantity, Skin friction, and as the Sherwood integer in the existence of the various parameters with distinct values as given below

Pr	m	n	Sc	We	$-f''(0)$	$-\theta'(0)$	$-\emptyset'(0)$
1	1	1	0.2	0.5	1.0423	0.6874	0.6874
5					1.0423	1.6250	0.6874
10					1.0423	2.3475	0.6874
	1				0.5744	0.6588	0.6588
	1.2				0.6048	0.6496	0.6496
	1.4				0.6728	0.6331	0.6331
		1			1.0423	0.6874	0.6874
		1.5			0.5932	0.6549	0.6549
		2.0			0.6046	0.6525	0.6525
			0.2		1.0423	0.6874	0.6874
			0.4		0.8285	0.7153	0.7153
			0.6		0.6728	0.6331	0.6331
				0.5	1.0423	0.6874	0.6874
				1.0	1.1532	0.6799	0.6799
				1.5	1.1630	0.6749	0.6749

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