INDIAN JOURNAL OF SCIENCE AND TECHNOLOGY

ORIGINAL ARTICLE

OPEN ACCESS

Received: 30/07/2025

Accepted: 28/08/2025

Published: 19/09/2025

Citation: Bhosale AA, Goswami VS (2025) Observational Constraints on Fractal Gravity with Specific Form of Deceleration Parameter. Indian Journal of Science and Technology 18(35): 2827-2836. https://doi.org/10.17485/IJST/v18i35.1336

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Funding: None

Competing Interests: None

Published By Indian Society for Education and Environment (iSee)

ISSN
Print: 0974-6846
Electronic: 0974-5645

Observational Constraints on Fractal Gravity with Specific Form of Deceleration Parameter

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Abstract

Objectives: In the framework of fractal gravity theory for the Bianchi Type V model, the phase transition and accelerated expansion of the universe are investigated using a specific form of the deceleration parameter. The field equations are solved by assuming nonzero fractal parameter and without cosmological constant. **Methods:** This study examines various physical parameters including density, pressure, and deceleration parameter. Based on the signature flip property of the deceleration parameter, we have derived the scale factor and the Hubble parameter. Observational data from the Hubble $H(z)$ and Pantheon datasets are utilized to constrain the model parameters. The optimal parameter values were determined by minimizing the Root Mean Squared Error (RMSE) criterion. **Findings:** Cosmographic analysis reveals that the model's behavior exhibits a close correspondence with that of the Λ CDM model. The best-fit values derived from the Hubble dataset are $\gamma = 53.00^{+2.4}_{-2.6}$, $\eta = 0.542^{+0.019}_{-0.021}$, $l_1 = -0.878^{+0.057}_{-0.054}$. For the Pantheon dataset, the corresponding best-fit are $\gamma = 55.3^{+2.4}_{-2.6}$, $\eta = 0.487^{+0.020}_{-0.024}$, $l_1 = -0.896^{+0.067}_{-0.053}$ with R^2 values 0.93 and 0.9967 for Hubble & Pantheon data sets respectively. The current values of q obtained at $z = 0$ are $-$ 0.24 and $-$ 0.35 for the Hubble and Pantheon dataset. **Novelty:** The deceleration parameter used here shows a transition from decelerating phase to an accelerating phase of expansion. As a result, the cosmological model that we have studied in fractal gravity is capable of explaining late time universe expansion without any dark energy component present in energy momentum tensor.

Keywords: Bianchi Type-V (BV) space time; Fractal gravity; Barotropic fluid; Dark Energy; Root Means Square Error (RMSE)

1 Introduction

A distinctive form of energy known as Dark Energy (DE), is responsible for driving the observed cosmic acceleration. Dark energy is characterized by negative pressure and its nature remains unidentified. One of the key parameters representing Dark Energy is the cosmological constant (Λ), which is a fundamental component of the ΛCDM model.

However, this model fails to adequately explain structure formation at smaller cosmological scales. Modified gravity theories have gained a lot of attention for explaining the present cosmic acceleration.

Fractal gravity has emerged as a significant area of research in recent years. Fractal cosmology (FC) is a framework for describing quantum gravity in which the couplings exhibit running scaling dimensions. This behavior enables the system to flow from an effective two-dimensional phase in the ultraviolet regime to a conventional field theory in the infrared limit. Numerous studies on fractal gravity have been published in the literature. Pawer et al.[(1)] have investigated the fractal FRW cosmological model, which incorporates two forms of dark energy. Mamon[(2)] have studied the Tsallis holographic dark energy model within the framework of FC and found that the universe has undergone a smooth transition from a decelerating to an accelerating expansion phase in the recent past. Pawar et al.[(3)] have studied the flat FRW model with observational constraints in fractal gravity. Zhang et al.[(4)] investigated Renyi holographic dark energy models within the context of FC. Manaon et al.[(5)] have examined the evolution of a fractal universe by incorporating holographic dark energy, as described by Barrow entropy, alongside the influence of dark matter.

Initially, it was hypothesized that the distribution of matter in the universe is isotropic and homogeneous, with the universe exhibiting spherical symmetry. However, recent observations into cosmological evolution indicate that near the Big -Bang singularity the assumption of isotropy may not hold. Many researchers have examined anisotropic and homogeneous Bianchi type-V (BV) space-time to describe initial stages of the universe and its evolution. The BV cosmological models are important in understanding the role of anisotropy during the early stages of the universe. Narasimharao and Neelima[(6)] have studied BV metric and constructed Tsallis holographic dark energy models with interacting and non-interacting terms. In f(R, T) theory of gravity Mahanta et al.[(7)] have investigated BV space time with quadratic equation of state. Admitting energy conditions in f (T) gravity Sahiba et al.[(8)] classified exact BV cosmological solutions. Daba and Shanti[(9)] examined an anisotropic and homogeneous BV space-time within the context of general relativity and Lyra's geometry. Warbhe et al.[(10)] have investigated BV universe in f(R, T) gravity with bulk viscous fluid.

In this paper, we investigate the anisotropic and homogeneous Bianchi Type V cosmological model within the context of fractal gravity. To solve the modified field equations, we assume particular form of fractal function and deceleration parameter, which allows cosmic expansion from deceleration to acceleration and phase transition.

2 Methodology

We use nonzero fractal parameter (ω) and a specific form of deceleration parameter $q = \eta - \frac{\gamma}{H}$ to solve the modified field equations. Using the relation between scale factor and redshift, we derived Hubble parameter, deceleration parameter, pressure, density in terms of red shift. To effectively constrain the values of parameters η, γ and l_1, we utilize the latest Observational Hubble Dataset (OHD), which comprises 57 data points derived through Differential Age (DA) and Baryon Acoustic Oscillation (BAO) methods. In addition, the Pantheon dataset is used to improve statistical analysis. Here, our aim is to determine the best fit values of η, γ and l_1 applying observational constraints from the Hubble parameter as a function of the redshift, $H(z)$, in conjunction with the Pantheon data set. The constrained values are utilized for cosmographic analysis, which is helpful to check the accelerated expansion of the universe. The methodology used in this model bridges the theoretical observations with testable cosmology.

2.1 Metric and solution of Field Equations

We consider the following anisotropic Bianchi Type-V universe

$$ds^2 = -dt^2 + A^2 dx^2 + e^{2x}\left(B^2 dy^2 + C^2\, dz^2\right) \tag{1}$$

Where $A,\ B\ \&\ C$ are the functions of cosmic time (t) only

$$S = \frac{1}{2M^2}\int \sqrt{-g}\,\left(R - 2\Lambda - \omega\,\partial_\mu\, v\,\partial^\mu\, v\right) d\xi(x) + \int \sqrt{-g}\ \mathcal{L}_m\, d\xi(x) \tag{2}$$

Where $v\ and\ \omega$ be the fractal function and parameter respectively. In fractal cosmology, the fractal function v explains how spacetime operates under non-integer (fractal) dimensionality, particularly at quantum scales. This concept expands general relativity and quantum field theory to space times that are fractal-like rather than smooth and continuous. The fractal parameter ω controls the kinetic coupling of the fractal function. In fractal cosmology, both time and space coordinates scale isotopically. Mahnaz and Ahmad[(11)], Pawar et al.[(12)], Bhoyar[(13)] are some of the authors who studied various cosmological models in fractal cosmology.

The variation of the metric tensor $g_{\zeta\tau}$ yields the standard field equations within the framework of fractal theory,

$$R_{\zeta\tau}-\frac{1}{2}g_{\zeta\tau}(R-2\Lambda)+g_{\zeta\tau}+\frac{\Box v}{v}-\frac{\nabla_\zeta\nabla_\tau v}{v}+\omega\left(\frac{1}{2}g_{\zeta\tau}\partial_\xi v\partial^\xi v-\partial_\zeta v\partial_\tau v\right)=M^2T_{\zeta\tau} \tag{3}$$

The matter content of the universe is typically modeled as a barotropic fluid with magnetic field, which can be expressed,

$$T_\zeta^\tau=(\rho+p)u_\zeta u^\tau+pg_\zeta^\tau+E_\zeta^\tau \tag{4}$$

Where E_ζ^τ is the magnetic field given by

$$E_\zeta^\tau=\overline{\mu}\left[\left|h^2\right|\left(u_\zeta u^\tau+\frac{1}{2}g_\zeta^\tau\right)-h_\zeta h^\tau\right] \tag{5}$$

$$\text{Here, } h_\zeta=\frac{\sqrt{-g}}{2\overline{\mu}}\epsilon_{\zeta jkl}F^{kl}u^j \tag{6}$$

Where $\overline{\mu},\ F^{kl}$ & $\epsilon_{\zeta jkl}$ are magnetic permeability, dual magnetic field tensor & Levi-Civita tensor density respectively. Here we take magnetic field along X-axis, hence $h_1\neq 0,\ h_2=h_3=h_4=0$. The non-vanishing component of electromagnetic field tensor is $F_{23}=K$, where K is constant.

In co-moving coordinate system, the non-vanishing component of energy momentum tensor (**Eq.4**) are,

$$T_1^1=T_2^2=T_3^3=P-\frac{K^2}{2B^2C^2}\text{ and }T_4^4=-\rho+\frac{K^2}{2B^2C^2} \tag{7}$$

Assuming $M=1,\ \Lambda=0$ and the time-like fractal Profile $v=t^{-\beta}$, Where β be the fractal dimension. In the UV-regime ($\beta=2$), the field **equations (3**) for the metric (**Eq. 1**) takes the form,

$$\frac{-1}{A^2}+\frac{\ddot{B}}{B}+\frac{\ddot{C}}{C}+\frac{\dot{B}\dot{C}}{BC}+\frac{2}{t}\left(\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right)-\frac{6}{t^2}-\frac{2\omega}{t^6}=p-\frac{K^2}{2B^2C^2} \tag{8}$$

$$\frac{-1}{A^2}+\frac{\ddot{A}}{A}+\frac{\ddot{C}}{C}+\frac{\dot{A}\dot{C}}{AC}+\frac{2}{t}\left(\frac{\dot{A}}{A}+\frac{\dot{C}}{C}\right)-\frac{6}{t^2}-\frac{2\omega}{t^6}=p-\frac{K^2}{2B^2C^2} \tag{9}$$

$$\frac{-1}{A^2}+\frac{\ddot{A}}{A}+\frac{\ddot{B}}{B}+\frac{\dot{A}\dot{B}}{AB}+\frac{2}{t}\left(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}\right)-\frac{6}{t^2}-\frac{2\omega}{t^6}=p-\frac{K^2}{2B^2C^2} \tag{10}$$

$$\frac{-3}{A^2}+\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{A}\dot{C}}{AC}+\frac{\dot{B}\dot{C}}{BC}+\frac{2}{t}\left(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right)-\frac{6\omega}{t^6}=-\rho+\frac{K^2}{2B^2C^2} \tag{11}$$

$$\frac{\dot{B}}{B}-\frac{\dot{C}}{C}=0 \tag{12}$$

The field **equations (8)** - **(11)** represent four independent equations with six unknowns. To derive a solution set for this system, additional constraints or conditions are required. We consider

(i) The fractal parameter $\omega\neq 0$

(ii) Considering the transition in the phase of cosmic expansion from deceleration to acceleration, we consider an ansatz for the temporal evolution of the deceleration parameter q

It is given by[14]

$$q=\eta-\frac{\gamma}{H} \tag{13}$$

Where $H = \frac{\dot{a}}{a}$ is the Hubble parameter, γ & η are constant.

The motivation for considering a variable deceleration parameter arises from the observed transition of the universe's expansion from a decelerating phase in the past to the current accelerating phase. The value of this parameter dictates the overall evolution of the cosmic expansion over time, providing insight into the temporal dynamics of the universe's expansion. A negative value of the deceleration parameter signifies an accelerating universe, whereas a positive value corresponds to a decelerating universe

$$a = \left(\frac{1+\eta}{\gamma C_1} e^{\gamma t} + l_1\right)^{\frac{1}{1+\eta}} \tag{14}$$

Where c_1 & l_1 are the constants.

For the metric (**Eq. 1**), the average scale factor is

$$a = (ABC)^{\frac{1}{3}} \tag{15}$$

From **Eq. (12)** we get $B = nC$, where $n \neq 1$

Using (ii) and **Eqs. (14) & (15)**, we get

$$C = n^{\frac{-2}{3}} \left(\frac{1+\eta}{\gamma C_1} e^{\gamma t} + l_1\right)^{\frac{1}{1+\eta}} \tag{16}$$

$$A = B = n^{\frac{1}{3}} \left(\frac{1+\eta}{\gamma C_1} e^{\gamma t} + l_1\right)^{\frac{1}{1+\eta}} \tag{17}$$

The Hubble parameter is obtained as

$$H = \frac{e^{\gamma t}}{C_1} \left(\frac{1+\eta}{\gamma C_1} e^{\gamma t} + l_1\right)^{-1} \tag{18}$$

From **Equations (9) and (10)**, the pressure and density are obtained as

$$p = \left[-n^{\frac{-2}{3}} + \frac{K^2 n^{\frac{2}{3}}}{2}\right] D^{\frac{-2}{1+\eta}} + (1-2\eta)\frac{e^{2\gamma t}}{C_1^2} D^{-2} + \left(2\gamma + \frac{4}{t}\right)\frac{e^{\gamma t}}{C_1} D^{-1} - \frac{6}{t^2} + \frac{2\omega}{t^6} \tag{19}$$

$$\rho = \left[-n^{\frac{-2}{3}} + \frac{K^2 n^{\frac{2}{3}}}{2}\right] D^{\frac{-2}{1+\eta}} - \frac{e^{2\gamma t}}{C_1^2} D^{-2} - \frac{6e^{\gamma t}}{tC_1} D^{-1} + \frac{6\omega}{t^6} \tag{20}$$

Where $D = \frac{1+\eta}{\gamma C_1} e^{\gamma t} + l_1$

The deceleration parameter (q) reads as $q = -1 + l_1 c_1 \gamma e^{-\gamma t}$ (21)

3 Results and Discussion

In this section we discuss the results obtained in previous section.

3.1 Behavior of the Cosmological parameter in terms of Redshift

In this section, we have calculated and discussed cosmological quantities in relation with redshift. The following equation establishes the relationship between the scale factor (a) and the redshift (z)

$a = \frac{1}{1+z}$

In terms of redshift The Hubble parameter is given by

$$H = \frac{\gamma}{1+\eta}\left[1 - l_1 (1+z)^{1+\eta}\right] \tag{22}$$

3.2 Observational Constraints on Model Parameters

Empirical data derived from multiple astrophysical observations, including Type Ia Supernovae (SNeIa), SDSS, BAO and CMB radiation, offer critical constraints on cosmological models describing the evolution of the universe. The Λ CDM model, characterized by its concordance with these observational datasets, presently stands as the predominant cosmological paradigm.

3.3 Observational Hubble Datasets (OHD)

The Hubble parameter is related to the differential redshift as

$$H(z) = -\frac{1}{1+z}\frac{dz}{dt} \tag{23}$$

In the context of cosmological observations, (dz) is derived from spectroscopic surveys, while (dt) measurement enables an independent determination of the Hubble parameter as a function of redshift, $H(z)$ without reliance on any specific cosmological model assumptions. The Hubble parameter, as a function of redshift (z), provides crucial insights into the expansion rate of the universe across different cosmic epochs, illuminating its dynamic evolution over time.

We use R^2-test to obtain the best-fit values.

$$R^2 = 1 - \frac{\sum_{i=1}^{57}\left[(H_i)_{obs} - (H_i)_{th}\right]^2}{\sum_{i=1}^{57}\left[(H_i)_{obs} - (H_i)_{mean}\right]^2} \tag{24}$$

Where $(H_i)_{obs}$ represents the observed values of $H(z)$, while $(H_i)_{th}$ corresponds to the theoretical values obtained from the best-fit curve.

3.4 Apparent Magnitude and Luminosity Distance

The luminosity distance, denoted as (dL), is a cosmological distance measure that quantifies the total flux of electromagnetic radiation received from an astrophysical source. It is defined as:

$$d_L(z) = (1+Z)\int_0^z \frac{H_0}{H(z^*)}dz^* \tag{25}$$

To determine the best-fit values of η, γ and l_1, the apparent magnitude m is expressed as function of luminosity distance:

$$m(z) = M + 5\log_{10} d_L - 5\log_{10}\left(\frac{H_0}{Mpc}\right) + 25 \tag{26}$$

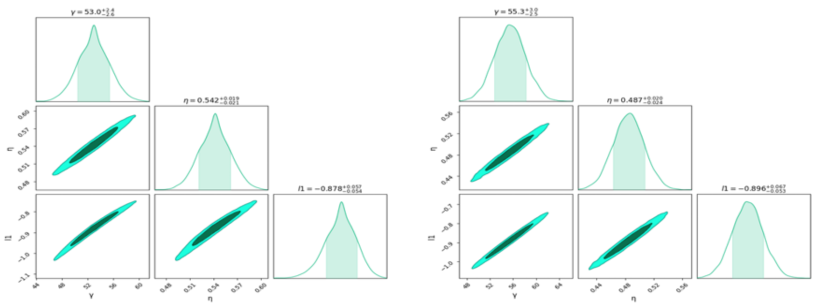
Where, for all Type Ia supernovae, M is constant.

The distance modulus μ (z) is defined as

$$\mu(z) = 5\log_{10} d_L - 5\log_{10}\left(\frac{H_0}{Mpc}\right) + 25 \tag{27}$$

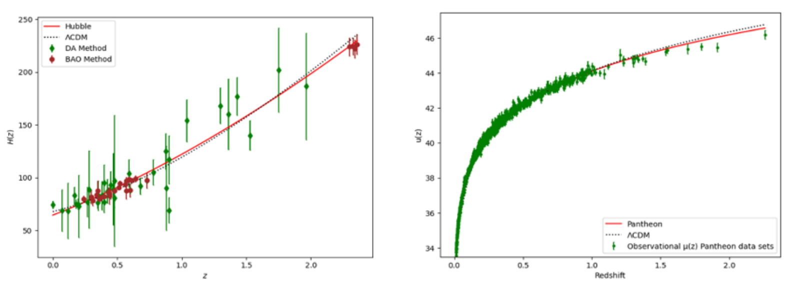
To derive the best-fit curve for the apparent magnitude , we use the Pantheon sample dataset, which comprises 1048 distance modulus data points of Type Ia supernovae within the redshift range $0.01 \leq z_i \leq 2.26$. The statistically significant value of M is -19.30. We use R^2-test for the best fit curve between theoretical and observed results. We also impose the constraints $-1 < z$ and $0 < H_0$ to ensure a valid parametric space. The maximum likelihood contours for the parameters η, γ and

l_1, including 1σ and 2σ error bounds, are displayed in figures 1a, 1b. The error bar plots for the 57 Hubble dataset points and the 1048 Pantheon dataset points are presented in figures 2a and 2b. These plots utilize the constrained model parameters $\gamma = 53.00^{+2.4}_{-2.6}$, $\eta = 0.542^{+0.019}_{-0.021}$, $l_1 = -0.878^{+0.057}_{-0.054}$ for Hubble and $\gamma = 55.3^{+2.4}_{-2.6}$, $\eta = 0.487^{+0.020}_{-0.024}$ and $l_1 = -0.896^{+0.067}_{-0.053}$ for Pantheon datasets, with R^2 values 0.93 and 0.9967 for Hubble & Pantheon data sets respectively. Root Mean Square Error (RMSE) measures the average magnitude of errors between observed and predicted values in a model by taking the square root of the mean of the squared differences. It provides a scale-dependent, single value that indicates the typical difference between actual and predicted data points. The smaller RMSE values signifies better model accuracy. Both datasets are compared against the well-established ΛCDM model (depicted by the dotted black line), employing the cosmological parameters $H_0 = 67.8$ km/s/Mpc, $\Omega_{\Lambda 0}$ = 0.7, and Ω_{m0} = 0.3.



(a) The Hubble dataset $H(z)$ (b) The Pantheon dataset

Fig 1. Contour Plots



(a) Error bar plot for 57 Hubble dataset Points with best-fit curves (b) Error bar plot for 1048 Pantheon Dataset points with best-fit curves

Fig 2. Error Bar Plots

3.5 Deceleration Parameter (q)

The deceleration parameter in terms of red shift is given by,

$$q = -1 + l_1 (1+\eta)\left[(1+z)^{-(1+\eta)} - l_1\right] \tag{28}$$

The figure 3 shows the evolution of the deceleration parameter $q(z)$ as a function of the redshift z for the Hubble and Pantheon data sets, respectively, using best-fit values of model parameters. The plot illustrates that the deceleration parameter $q(z)$ shows the transition from a decelerating phase to the current epoch of accelerated expansion of the universe. The transition redshifts from a decelerating to an accelerating phase occur at $z_{tr} = 0.65$ for the Hubble data set and $z_{tr} = 0.73$ for the Pantheon data set. The recent observational data supports this value of q. Recently using SN Ia + BAO/CMB (Planck) + H (z) data Santos et al.[15] obtained $z_{tr} = 0.66$. The current values of q at $z = 0$ are -0.24 and -0.35 for the Hubble and Pantheon datasets, respectively. The values obtained for q_0 and z_{tr} are in close agreement with the recent observations.[16], [17]

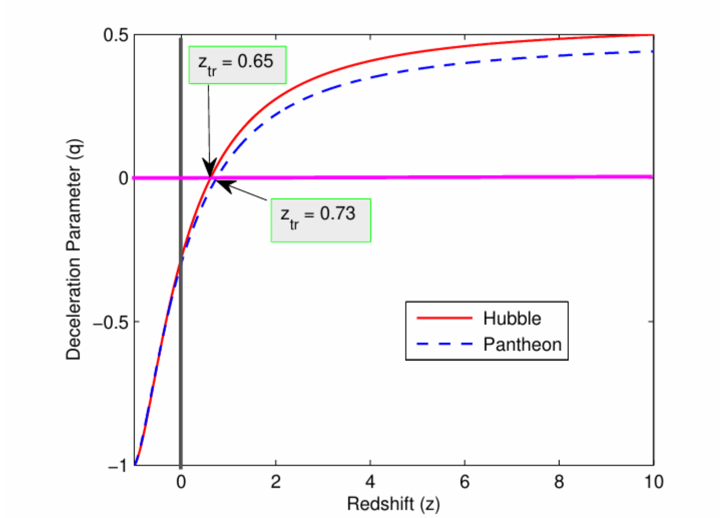


Fig 3. Deceleration parameter vs. redshift for the Hubble & Pantheon dataset

3.6 Pressure (P) and Density (P)

Pressure and energy density in terms of redshift are respectively obtained as,

$$p = \left[-n^{\frac{-2}{3}} + \frac{K^2}{2} n^{\frac{2}{3}}\right](1+z)^2 + \frac{(1-2\eta)\gamma^2}{(1+\eta)^2} L_1^2 (1+z)^{2(1+\eta)} + \frac{2\gamma^2}{1+\eta}(1+z)^{(1+\eta)} L_1$$

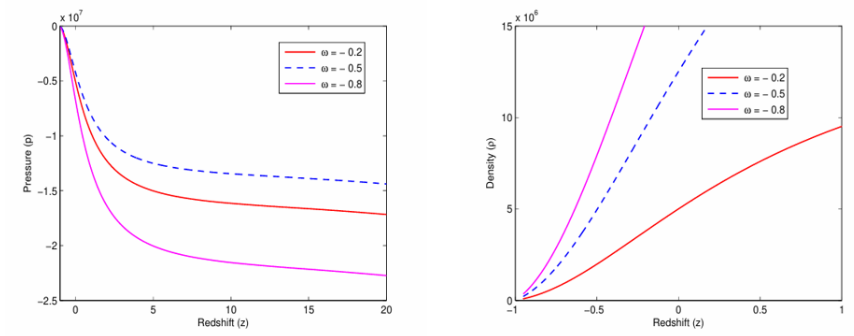
$$\left[1+2ln\left(\frac{\gamma c_1}{1+\eta}L_1\right)^{-1}\right] - 6\gamma^2 \ln\left(\frac{\gamma c_1}{1+\eta}L_1\right)^{-2} + 2\omega\gamma^6 \ln\left(\frac{\gamma c_1}{1+\eta}L_1\right)^{-6} \tag{29}$$

$$\rho = \left[-n^{\frac{-2}{3}} + \frac{K^2}{2} n^{\frac{2}{3}}\right](1+z)^2 - \frac{2\gamma^2}{(1+\eta)^2}(1+z)^{2(1+\eta)} - 6\gamma^2 \ln\left(\frac{\gamma c_1}{1+\eta}L_1\right)^{-1}$$

$$+6\omega\gamma^6 \ln\left(\frac{\gamma c_1}{1+\eta}L_1\right)^{-6} \tag{30}$$

Where $L_1 = (1+z)^{-1(1+z)} - l_1$

Figures 4a illustrate the variation of pressure as a function of redshift, based on the constrained parameter values $\gamma = 53$, $\eta = 0.542$,$l_1 = 0.878$ for the along with $\eta = 0.2$ for different values of fractal parameter (ω). It is observed that the pressure decreases with redshift and remains negative throughout the entire cosmic evolution, which is according to current observations of accelerating universe. **Figures 4b** illustrate energy density plots with constrained parameters for Hubble data. It is observed that the energy density is positive and increases with redshift and continues to evolve as the universe expands. It is important to note that we get same behavior of pressure and density for Pantheon data also.



(a) $P(z)$ Vs. z for Hubble Dataset

(b) $\rho(z)$ Vs. z for Hubble Dataset

Fig 4. Plot of Pressure and Density for Hubble Dataset.

3.7 Cosmographic Analysis

In this section, we discuss the cosmographic analysis of the model. We consider the jerk (j), snap (s) and lerk (l) parameters. These are dimensionless quantities that depend on the scale factor (a) and its third, fourth, and fifth-order time derivatives. These parameters enable a more precise determination of the rate of cosmic expansion through a model-independent analysis of the universe's evolution. In the field of cosmology, the precise value of the jerk parameter holds significant importance, as it plays a crucial role in understanding the nature of dark energy. The dark energy is a mysterious force believed to drive the accelerated expansion of the Universe. A positive jerk indicates that the expansion is accelerating at an increasing rate, while a negative jerk implies a deceleration in the acceleration. The lerk parameter provides the most accurate representation of the universe's expansion, detecting minute variations in both acceleration and deceleration rates. These parameters at any time are defined as,

$$j = \frac{\dddot{a}}{aH^3}, s = \frac{a^{(4)}}{aH^4} \text{ and } l = \frac{a^{(5)}}{aH^5} \tag{31}$$

Where $a^{(4)}$ and $a^{(5)}$ represent the fourth-order and fifth-order derivatives of the scale factor, respectively.

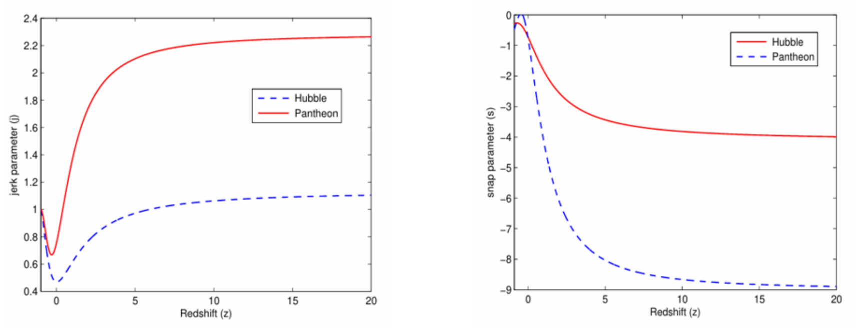
In terms of redshift, jerk, snap and lerk parameters reads as

$$j = \eta(1+2\eta) - 3\eta(1+\eta)L_1^{-1}(1+z)^{-1(1+\eta)} + (1+\eta)^2 L_1^{-2}(1+z)^{-2(1+\eta)} \tag{32}$$

$$\begin{aligned} s =& -\eta(1+2\eta)(2+3\eta) + 6\eta(1+\eta)(1+2\eta)\, L_1^{-1}(1+z)^{-1(1+\eta)} \\ & -7\eta(1+\eta)^2 L_1^{-2}(1+z)^{-2(1+\eta)} + (1+\eta)^3 L_1^{-3}(1+z)^{-3(1+\eta)} \end{aligned} \tag{33}$$

$$\begin{aligned} l =& \eta(1+2\eta)(2+3\eta)(3+4\eta) - 10\eta(1+2\eta)(1+3\eta)\, L_1^{-1}(1+z)^{-1(1+\eta)} \\ & +25\eta(1+\eta)^2 + 2\eta)L_1^{-2}(1+z)^{-2(1+\eta)} - 15(1+\eta)^3 L_1^{-3}(1+z)^{-3(1+\eta)} \\ & +(1+\eta)^4 L_1^{-4}(1+z)^{-4(1+\eta)} \end{aligned} \tag{34}$$

From figure 5a, it is evident that the jerk parameter maintains positive value throughout cosmic evolution for both data sets. Conversely, the snap parameter initially assumes a negative value, but transitions to a positive value at later cosmic epochs, as shown in figure 5b. This transition signifies an accelerated expansion of the universe.



(a) j Vs. z (b) s Vs. z

Fig 5. Jerk and Snap Parameter vs. redshift for the Hubble & Pantheon datasets

In Figure 5a, we clearly observe that the jerk parameter (j) remains positive throughout the entire evolution of the universe and aligns with the behavior predicted by ΛCDM model for the given values of η. Notably the model closely approximates the ΛCDM model in this respect.

4 Conclusion

In this study, we have explored the anisotropic and homogeneous Bianchi Type V space time within the framework of fractal gravity. Using the signature flip property of the deceleration parameter $q = \eta - \frac{\gamma}{H}$, where η & γ are constants, we have derived expressions for the scale factor and the Hubble parameter. Using observational data sets from Hubble H (z) measurements and the Pantheon sample, we constrained the key parameters. The best-fit values derived from the Hubble dataset are $\gamma = 53.00^{+2.4}_{-2.6}$, $\eta = 0.542^{+0.019}_{-0.021}$, and $l_1 = -0.878^{+0.057}_{-0.054}$ .For the Pantheon dataset, the corresponding best-fit values are $\gamma = 55.3^{+2.4}_{-2.6}$, $\eta = 0.487^{+0.020}_{-0.024}$ and $l_1 = -0.896^{+0.067}_{-0.053}$ with R^2 values 0.93 and 0.9967 for Hubble & Pantheon data sets respectively. The deceleration parameter, as shown in figure 3, exhibits a signature flip, indicating a transition from deceleration to acceleration phase of expansion. The transition redshift values for our model are z_{tr} = 0.65 for the Hubble dataset and z_{tr} =0.73 for the Pantheon dataset. The values obtained for z_{tr} are in close agreement with the recent observations [16], [17]

Furthermore, we examined pressure p energy density ρ as functions of redshift z for the best fit value of key parameter set from for Hubble and Pantheon data set. The analysis was carried out for a fixed value $n = 0.2$. For both data sets, the energy density maintains a positive value throughout the entire cosmic evolution, thereby confirming the physical viability of the model. The pressure remains negative, the presence of negative pressure indicates dark energy, which is responsible for accelerated expansion of the universe. Additionally, we perform the cosmographic analysis. The analysis of the jerk parameter reveals alignment with the standard ΛCDM $(j = 1)$ model, whereas the examination of the snap parameter indicates an accelerating universe. The analysis of the model is carried out without cosmological constant $\Lambda = 0$, associated with dark energy. The fractal gravity model describes late-time cosmic acceleration without invoking dark energy in the matter sector, allowing a transition from deceleration to acceleration. It is important to note that, based on these findings, it is possible to conclude that bartropic fluid may be the source of dark energy for Bianchi type-V space -time in fractal theory of gravity.

4.1 Data availability statement

This manuscript does not contain associated data, nor will the data be deposited. [Authors' comment: This is a theoretical study, and no experimental data were collected.]

4.2 Details of Ethical clearance

The authors declare that they have no known competing financial interest or personal relationship that could have appeared to influence the work reported in this paper

5 Acknowledgment

The author would like to thank anonymous referee for his/her valuable suggestions that have significantly improved the quality of the article and its presentation.

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