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Mathematical Study on Immigration and Migration of Prey-Predator Eco System with Unlimited Resources for the Predator

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Abstract

Objectives: To study the stability of two species in a prey-predator eco-system, immigration and migration with unlimited resources for the predator. **Method:** The system consists of a prey (S_1), a predator (S_2) that migrates and immigrates to subsist off of S_1, and a prey (S_1). In this instance, the second species' limitless resources are quantified by the corresponding carrying capacity. A collection of first-order non-linear ordinary differential equations make up the system's model equations. **Findings:** The stability of each of the three equilibrium points is examined and determined. If all of the characteristic roots are negative (if they are real) and have negative real parts (if they are complex), the system would be stable. To determine the stability criteria, the linearized equations for the perturbations over the equilibrium points are analysed. The global stability of linearized equations can be discussed by constructing a suitable Liaupnov's function. Additionally, the Runge-Kutta fourth order technique is used to compute the numerical solutions for the growth rate equations. **Novelty:** The connection where one species benefits at the other's expense is called predation. In this mutualistic relationship, one creature takes up the other's biomass. We refer to this as a predator. Because of their tight relationship, one organism in this interaction consumes the other. A tiger and a deer are two examples of predator and prey in the real world. The tiger feeds on other organisms, and the deer are its prey.

Keywords: Equilibrium State; Neutrally Stable; Predator; Prey; Stable; Trajectories; Unstable

1 Introduction

A real-world scenario can be mathematically described by a model. Therefore, by properly analysing the model with the use of the relevant mathematical tools, we can gain a deeper knowledge of the system if a mathematical model can reflect or replicate the behaviour of a real-life situation. Additionally, as the model is being constructed, we find a number of characteristics that influence the system, are crucial to it, and show the relationships between its many components. It is impossible to overlook the significance of mathematical modelling in the fields of physics, chemistry, biology, economics, and even industry. In the basic sciences, mathematical modelling is becoming more and more common, particularly in the fields of economics, industrial challenges, and biology. Research and development in the field of mathematical modelling is quite strong. In recent years, testable experimental predictions have resulted from the building and testing of mathematical models that have been used to support hypotheses derived from experimental data. There are remarkable examples where mathematical models have shed new light on issues related to biological, physical, and economical systems as well as decision-making, spatial modelling, industry, and economy. Important developments in the field of computational mathematics are intimately linked to the advancement of mathematical modelling. provides a mathematical model for analysing obesity and overweight in a population and how it affects the rise in the number of diabetes[(1)]. He considers social issues and the relationships between various societal elements when building the model. A thorough overview of the state of the art in using mathematical models to examine how human population growth and population pressure affect forest biomass and wildlife that depends on forests can be found in[(2)]. Ecological Systems method, a novel bio-inspired evolutionary search method, was mathematically modelled by[(3)]. The dynamics of a diffusive three-species Lotka–Volterra system with delays were investigated in[(4)]. A partial differential equation system was approximated mathematically using delay differential equations and the Galerkin Method, which yields semi-analytical solutions.

Ecology is a subfield of evolutionary biology that studies living things that cohabit in the physical world and get their sustenance from shared resources. It is a well-known fact that species that are similar to one another cannot coexist peacefully in isolation from one another.[(5)] has discussed important studies in the field of theoretical ecology. Numerous mathematicians and ecologists made contributions to the development of this field of study. Autecology and Synecology are the two primary subdisciplines of mathematical ecology. An ecosystem made up of two or more different species is known as a syn-ecology. In one way or another, species interact with one another. There are other ways to categorise ecological interactions, including mutualism, competition, commensalism, ammensalism, parasitism, and so forth. faster-reproducing animals are more likely to exhibit high-dimensional, nonlinear dynamics, as demonstrated by[(6)], which is consistent with earlier ecological theories.[(7)] investigate the stability of an ecosystem model with three species and three couples; one pair performs two roles—that of a host and an opponent with a monod response. a fractional-order predator-prey model that takes into account predator species' anti-predator and prey refuge behaviours. He provides evidence for the existence, singularity, non-negativity, and infinite nature of solutions. The evolution of discrete time models from the Malthus model to contemporary population models that consider a wide range of variables impacting the dynamics and structure is discussed. The authors' most significant and intriguing findings from their investigation of biological systems using recurrent equations[(8)]. Numerous scholar[(9)] and mathematician[(10)] have contributed significant viewpoints, worthwhile tenets, and intriguing applications to the field of mathematical ecology that allow for the analysis of the behaviours of various ecological models. In intercultural aspects, Hari Prasad[(11)] investigated the local and global stability of numerous mathematical model of syn-ecology. Using nonlinear differential equations,[(12)] developed and examined a comprehensive mathematical model that described the dynamics of a prey-predator scenario under the effect of an infectious disease known as SIS. An eco-epidemiological model of prey–predator with infectious diseases in the prey was proposed by[(13)]. Next, examine the ODE model and diffusive model utilising the Dirichlet and homogeneous Neumann boundary conditions.[(14)] suggested a mathematical model for eco-epidemiology to explain how migration affects a prey-predator population's dynamics. Using the Lyapunov principle and the Routh–Hurwitz criterion, respectively, numerous thresholds are calculated and applied to examine the local and global stability outcomes.[(15)] considers a Prey-Ammensal, a Predator, and an Enemy to the Prey-Ammensal in order to investigate a significant ecology and The study of the global stability of an enemy ecological model with immigration for ammensal species at a constant rate and a mathematical model of mortal ammensal is clarified by[(16)].

The creation of mathematical models that tackle difficult real-world problems, particularly in ecological systems and public health, has greatly enhanced the area of computational mathematics. But there are still a number of important holes that need to be investigated further:

- Integration of Multidisciplinary Factors: Isolated components within a system are frequently the focus of current models. For example, ecological models concentrate on species interactions and environmental factors, whereas models of obesity and diabetes mainly take biological and behavioural factors into account. There aren't many integrated models that take into account biological, environmental, and socioeconomic aspects at the same time to provide us a comprehensive picture of these intricate systems.
- Including stochastic elements: A lot of current models are deterministic and don't take into consideration the inherent unpredictability and randomness seen in real-world systems. Adding stochastic processes to models can improve their

realism and predictive ability, especially in ecological systems where population dynamics can be greatly impacted by random events.

- Spatial and Temporal Dynamics: More thorough models that take into account both temporal evolution and spatial heterogeneity are required, even if some models only address spatial or temporal elements. Given that species distribution and environmental conditions change over time and space, this is particularly relevant in ecological modelling.
- Data Integration and Validation: The usability of mathematical models depends on their validation against actual data. To validate and improve forecasts, there is a lack of models that successfully incorporate several datasets, including socioeconomic indicators, environmental measurements, and medical records.
- Advanced Computational Techniques: When developing and analysing intricate mathematical models, the use of advanced computational techniques, such as machine learning and high-performance computing, is still underutilised. By utilising these methods, model efficiency, accuracy, and capacity to manage large-scale systems can all be enhanced.
- Applications in Policy and Decision-Making: Models that are both practically useful and theoretically sound are required for resource management and policy-making. Creating intuitive tools and interfaces based on these models can help stakeholders make well-informed choices about ecological conservation and public health initiatives.

2 Methodology

Notation Adopted:

N_1(t) : The population strength of prey species (S_1)
N_2(t) : The population strength of predator species (S_2)
t : Time instant
a_i : Natural growth rates of S_i, i = 1, 2
a_{11} : Self inhibition coefficient of S_1
$a_{12,}$ a_{21} : Interaction coefficients of S_1 due to S_2 and S_2 due to S_1
M, I : Rate of migration and immigration of S_1 and S_2

Further the variables N_1, N_2 are non-negative and the model parameters a_1, a_2, a_{11}, a_{12}, a_{21}, M, I are assumed to be non-negative constants.

2.1 Basic Equations of the Model

The basic equation for the model is given by the following system of non-linear ordinary differential equations.

Equation for the growth rate of prey species (S_1):

$$\frac{dN_1}{dt} = a_1 N_1 - a_{11} {N_1}^2 - a_{12} N_1 N_2 - M N_1 \tag{1}$$

Equation for the growth rate of predator species (S_2):

$$\frac{dN_2}{dt} = a_2 N_2 + a_{21} N_1 N_2 + I N_2 \tag{2}$$

2.2 Equilibrium States

The system under investigation has three equilibrium states given by

$\frac{dN_i}{dt} = 0; i = 1, 2$

Fully washed out state:$E_1 : \overline{N_1} = 0, \overline{N_2} = 0$

Predator washed out stat:$E_2 : \overline{N_1} = \frac{(a_1 - M)}{a_{11}} = k_1 - \frac{M}{a_{11}}, \overline{N_2} = 0$

The co-existent state (or) normal steady state:$E_3 : \overline{N_1} = -\alpha_1, \overline{N_2} = \alpha_2 - \frac{M}{a_{12}}$

Where

$$\alpha_1 = \frac{(a_2 + I)}{a_{21}} > 0 \text{ and } \alpha_2 = \frac{a_1 + \alpha_1 a_{11}}{a_{12}} > 0 \tag{3}$$

2.3 Stability of Equilibrium States

Let $N_i(t) = (N_1, N_2) = \overline{N_i} + U_i(t); i = 1,2$ Where $U_i(t)$ is a small perturbation over the equilibrium point $N = (\overline{N_1}, \overline{N_2})$. The basic equations are quasi lineralized to obtain the equations for the perturbed state as

$$\frac{du_1}{dt} = (a_1 - 2a_{11}\overline{N_1} - a_{12}\overline{N_2} - M)u_1 - a_{12}\overline{N_1}u_2 \tag{4}$$

$$\frac{du_2}{dt} = a_{21}\overline{N_2}u_1 + (a_2 + a_{21}\overline{N_1} + I)u_2 \tag{5}$$

The characteristic equation is

$$|A - \lambda I| = 0 \tag{6}$$

Where

$$A = \begin{pmatrix} a_1 - 2a_{11}\overline{N_1} - a_{12}\overline{N_2} - M & -a_{12}\overline{N_1} \\ a_{21}\overline{N_2} & a_2 + a_{21}\overline{N_1} + I \end{pmatrix} \tag{7}$$

The equilibrium state is stable, if all the roots of **equation (6)** are negative, in case they are real or have the negative real parts , in case they are complex.

3 Results and Discussion

3.1 Stability of fully washed out state

The basic equations are linearized to obtain the equations as

$$\frac{du_1}{dt} = (a_1 - M)u_1;\ \frac{du_2}{dt} = (a_2 + I)u_2 \tag{8}$$

The characteristic **equation of (8)** is

$$[\lambda - (a_1 - M)]\,[\lambda - (a_2 + I)] = 0 \tag{9}$$

The characteristic roots are $(a_1 - M)$ and $(a_2 + I)$.

Since one root is positive. Hence, E_1 is **unstable** and the solutions of the **equations (8)** are

$$u_1 = u_{10}e^{(a_1 - M)t};\ u_2 = u_{20}e^{(a_2 + I)t} \tag{10}$$

Where u_{10} and $u_{20}$are the initial values of u_1 and $u_2$respectively.

Trajectories of perturbations

The trajectories in $u_1 - u_2$ plane are $\left(\frac{u_1}{u_{10}}\right)^{(a_2+I)} = \left(\frac{u_2}{u_{20}}\right)^{(a_1-M)}$

3.2 Stability of predator washed out state

In this state, the basic equations can be linearized, We get,

$$\frac{du_1}{dt} = (M - a_1)u_1 + a_{12}\left(\frac{M - a_1}{a_{11}}\right)u_2;\ \frac{du_2}{dt} = (\beta_2 - \frac{a_{21}M}{a_{11}})u_2 \tag{11}$$

Where

$$\beta_2 = a_2 + I\ + \frac{a_1 a_{21}}{a_{11}} > 0 \tag{12}$$

The characteristic equation is

$$[\lambda - (M - a_1)]\left[\lambda - (\beta_2 - \frac{a_{21}M}{a_{11}})\right] = 0 \tag{13}$$

$M-a_1$and $\beta_2-\frac{a_{21}M}{a_{11}}$ are the characteristic roots of[12]

Situation 1: When $M>a_1, a_{11}\beta_2>a_{21}M;\ M<a_1, a_{11}\beta_2>a_{21}M;\ M>a_1, a_{11}\beta_2<a_{21}M$ $M=a_1, a_{11}\beta_2>a_{21}M;\ M>a_1, a_{11}\beta_2=a_{21}M$

In this situation, E_2 is **unstable.**

Situation 2: When $M<a_1, a_{11}\beta_2<a_{21}M$

Hence, the predator washed out state is **stable.**

Situation 3: When $M<a_1, a_{11}\beta_2=a_{21}M;\ M=a_1, a_{11}\beta_2<a_{21}M;\ M=a_1, a_{11}\beta_2=a_{21}M$

In this situation, E_2 is **neutrally stable.**

The **equations (11)** yield the solutions,

$$u_1=(1-A_1)u_{10}e^{(M-a_1)t}+A_1u_{20}e^{(\beta_2-\dfrac{a_{21}M}{a_{11}})t};\ u_2=u_{20}e^{(\beta_2-\dfrac{a_{21}M}{a_{11}})t} \tag{14}$$

Where

$$A_1=\frac{(M-a_1)a_{12}}{(a_1+\beta_2)a_{11}-(a_{11}+a_{21})M};\ (a_1+\beta_2)a_{11}\neq(a_{11}+a_{21})M;\ M\neq a_1, \beta_2\neq\frac{a_{21}M}{a_{11}} \tag{15}$$

If $M=a_1, a_{11}\beta_2\neq a_{21}M$ then the solutions are

$$u_1=u_{10} \text{ and } u_2=u_{20}e^{(\beta_2-\dfrac{a_{21}M}{a_{11}})t} \tag{16}$$

If $M\neq a_1, a_{11}\beta_2=a_{21}M$ then the solutions are given by

$$u_1=(1-A_1)e^{(M-a_1)t}+A_1u_{10};u_2=u_{20} \tag{17}$$

$$\text{If } M=a_1, a_{11}\beta_2=a_{21}M \text{ then } u_1=u_{10};u_2=u_{20} \tag{18}$$

Trajectories of perturbations

$u_1=u_{10}(1-A_1)\left(\frac{u_2}{u_{20}}\right)^{\frac{a_{11}(M-a_{11})}{a_{11}\beta_{12}-a_{21}M}}+A_1u_2$ are the trajectories.

3.3 Stability of the normal steady state

The Linearized equations are

$$\frac{du_1}{dt}=a_{11}\alpha_1u_1+a_{12}\alpha_1u_2;\ \frac{du_2}{dt}=a_{21}(\alpha_2-\frac{M}{a_{12}})u_1 \tag{19}$$

The characteristic equation is

$$\lambda^2-a_{11}\alpha_1\lambda-\alpha_1a_{21}(a_1+\alpha_1a_{11}-M)=0 \tag{20}$$

Let λ_1 and $\lambda_2$be the roots of(?)

Here, either λ_1 or λ_2 is positive. Hence, the normal steady state is **unstable.**

The solutions are

$$u_1=\left[\frac{a_{12}\alpha_1(u_{20}-u_{10}\gamma_2)}{\lambda_1-\lambda_2}\right]e^{\lambda_1t}+\left[\frac{a_{12}\alpha_1(u_{20}-\gamma_1u_{10})}{\lambda_2-\lambda_1}\right]e^{\lambda_2t} \tag{21}$$

$$u_2=\left[\frac{a_{12}\alpha_1(u_{20}-u_{10}\gamma_2)}{\lambda_1-\lambda_2}\right]\gamma_1e^{\lambda_1t}+\left[\frac{a_{12}\alpha_1(u_{20}-\gamma_1u_{10})}{\lambda_2-\lambda_1}\right]\gamma_2e^{\lambda_2t} \tag{22}$$

Where

$$\gamma_1=\frac{\lambda_1}{a_{12}\alpha_1}-\frac{a_{11}}{a_{12}} \text{ and } \gamma_2=\frac{\lambda_2}{a_{12}\alpha_1}-\frac{a_{11}}{a_{12}} \text{ with } a_{12}\lambda_1\neq\alpha_1a_{11}a_{12};\ a_{12}\lambda_2\neq\alpha_1a_{11}a_{12} \tag{23}$$

The trajectories in u_1-u_2 plane are $\frac{(u_1-v_1u_2)^{v_1}}{(u_1-v_2u_2)^{v_2}}=c_3u_2{}^{\frac{1}{c}[(1-c)v_2+(c-1)v_1]}$

Where v_1, v_2 are roots of the quadratic equation $cv^2-av-b=0$.

and $a=a_{11}\alpha_1, b=a_{12}\alpha_1, c=a_{21}(\alpha_2-\frac{M}{a_{12}}), v=\frac{u_1}{u_2}$ and c_3 is an arbitrary constant.

3.4 Liaupnov's Function for Global Stability

The linearized **equations (4) and (5)** becomes

$$\frac{du_1}{dt} = -a_{11}\bar{N}_1 u_1 - a_{12}\bar{N}_1 u_2;\ \frac{du_2}{dt} = a_{21}\bar{N}_2 u_1 \tag{24}$$

The characteristic **equation of (24)** is

$$\lambda^2 + p\lambda + q = 0 \tag{25}$$

where $p = a_{11}\overline{N_1} > 0, q = [a_{12}a_{21}]\overline{N_1 N_2} > 0$

Hence, the conditions for Liaupnov's function are satisfied.

Define, $E(u_1, u_2) = \frac{1}{2}\left[a{u_1}^2 + 2bu_1u_2 + c{u_2}^2\right]$

$$\begin{aligned} \text{where } a &= \frac{1}{D}\left[\left(a_{21}\overline{N_2}\right)^2 + \{a_{12}a_{21}\}\overline{N_1N_2}\right];\ b = \frac{1}{D}(a_{11}a_{21})\overline{N_1N_2} \\ c &= \frac{1}{D}\left[\left(a_{11}\overline{N_1}\right)^2 + \left(a_{12}\overline{N_1}\right)^2 + \{a_{12}a_{21}\}\overline{N_1N_2}\right];\ D = \left(a_{11}\overline{N_1}\right)\left([a_{12}a_{211}]\overline{N_1N_2}\right) \end{aligned}$$

It is clear that $D > 0, a > 0$.

$$\begin{aligned} \text{Also, } (ac - b^2) &= \frac{1}{D}\left[\left(a_{21}\overline{N_2}\right)^2 + \{a_{12}a_{21}\}\overline{N_1N_2}\right]\frac{1}{D}\left[\left(a_{11}\overline{N_1}\right)^2 + \left(a_{12}\overline{N_1}\right)^2 + \{a_{12}a_{21}\}\overline{N_1N_2}\right] \\ &\quad - \left(\frac{1}{D}(a_{11}a_{21})\overline{N_1N_2}\right)^2 \\ &= \frac{1}{D^2}\left\{\left[\left(a_{21}\overline{N_2}\right)^2\right]\{a_{12}a_{21}\}\overline{N_1N_2} + \left\{\left[\left(a_{11}\overline{N_1}\right)^2 + \left(a_{12}\overline{N_1}\right)^2\right]\{a_{12}a_{21}\}\overline{N_1N_2}\right\}\right\} \\ &\quad + \left[\left(a_{21}\overline{N_2}\right)^2\right]\left[\left(a_{11}\overline{N_1}\right)^2 + \left(a_{12}\overline{N_1}\right)^2\right] > 0 \Rightarrow D^2(ac - b^2) > 0 \end{aligned}$$

Therefore $E(u_1, u_2)$ is positive definite. Further, we find

$$\frac{\partial E}{\partial u_1}.\frac{du_1}{dt} + \frac{\partial E}{\partial u_2}.\frac{du_2}{dt} = (au_1 + bu_2)\left(-a_{11}\overline{N_1}u_1 - a_{12}\overline{N_1}u_2\right) + (bu_1 + cu_2)(a_{21}\overline{N_2}u_1)$$

Substituting the terms a, b, cwe have $\frac{\partial E}{\partial u_1}.\frac{du_1}{dt} + \frac{\partial E}{\partial u_2}.\frac{du_2}{dt} < 0$

we get, $\frac{\partial E}{\partial u_1}.\frac{du_1}{dt} + \frac{\partial E}{\partial u_2}.\frac{du_2}{dt} = \frac{1}{D}\left[-a_{12}a_{21}a_{11}\overline{N_1}^2\overline{N_2}{u_1}^2 - a_{12}a_{21}a_{11}\overline{N_1}^2\overline{N_2}{u_2}^2\right] = -\left[{u_1}^2 + {u_2}^2\right]$

which is negative definite. Hence, $E(u_1, u_2)$is a Liapunov's function for the linear system.

Now, to prove that $E(u_1, u_2)$is a Liapunov's function for the nonlinear system.

$$F_1(N_1, N_2) = N_1(a_1 - a_{11}N_1 - a_{12}N_2 - M) \tag{26}$$

$$F_2(N_1, N_2) = N_2(a_2 + a_{21}N_1 + I) \tag{27}$$

Letting $N_1 = \overline{N_1} + u_1, N_2 = \overline{N_2} + u_2$,

$$\begin{aligned} \text{Therefore, } \frac{du_1}{dt} &= \left(\overline{N_1} + u_1\right)\left[a_1 - a_{11}\left(\overline{N_1} + u_1\right) - a_{12}\left(\overline{N_2} + u_2\right) - M\right] \\ &= -a_{11}\overline{N_1}u_1 - a_{12}\overline{N_1}u_2 + f_1(u_1, u_2) = F_1(u_1, u_2) \text{ and} \\ \frac{du_2}{dt} &= \left(\overline{N_2} + u_2\right)\left[a_2 + a_{21}\left(\overline{N_1} + u_1\right) + I\right] \\ &= a_{21}\overline{N_2}u_1 + f_2(u_1, u_2) = F_2(u_1, u_2) \end{aligned}$$

Now, $\frac{\partial E}{\partial u_1}.F_1 + \frac{\partial E}{\partial u_2}.F_2 = \frac{\partial E}{\partial u_1}.\frac{du_1}{dt} + \frac{\partial E}{\partial u_2}.\frac{du_2}{dt}$

$= (au_1 + bu_2)\left(-a_{11}\overline{N_1}u_1 - a_{12}\overline{N_1}u_2 + f_1(u_1, u_2)\right) + +(bu_1 + cu_2)(a_{21}\overline{N_2}u_1 + f_2(u_1, u_2))$

Introducing the polar coordinates $u_1 = r\cos\theta, u_2 = r\sin\theta$ and solving we get

Therefore, $\frac{\partial E}{\partial u_1}.F_1 + \frac{\partial E}{\partial u_2}.F_2 < 0$ which is negative definite.

Thus, $E(u_1, u_2)$is positive definite function with the property that $\frac{\partial E}{\partial u_1}.F_1 + \frac{\partial E}{\partial u_2}.F_2$ is negative definite. Hence equilibrium point is asymptotically stable.

3.5 A Numerical Approach of the Growth Rate Equations

The numerical solutions of the growth rate **equations (1) and (2)** computed employing the fourth order Runge-Kutta method for specific values of the various parameters that characterize the model and the initial conditions. In this paper, we use the parameter values for the **figures from 1 to 9**.

Table 1. Parameter values of the model

S. No	a_1	a_2	a_{11}	a_{12}	a_{21}	M	I	N_1	N_2	t^*
1	0.931	0.147	0.203	0.095	0.304	0.026	0.127	0.485	0.192	3.51
2	0.791	0.565	0.116	0.052	0.039	0.144	0.225	0.392	0.113	3.62
3	1.044	0.288	0.127	0.098	0.046	0.042	0.173	0.045	0.141	2.49
4	0.920	0.036	0.268	0.080	0.280	0.110	0.160	0.750	0.750	-
5	0.961	0.049	0.065	0.042	0.036	0.023	0.788	0.832	0.148	4.12
6	0.063	0.435	0.023	0.706	0.134	0.931	0.082	0.289	0.414	0.50
7	0.170	0.230	0.270	1.230	0.330	0.330	0.300	3.330	0.320	0.98
8	0.918	0.592	0.500	0.059	0.023	0.029	0.108	0.289	0.120	3.48
9	0.035	0.014	0.726	0.170	0.003	0.691	0.007	3.840	0.060	-

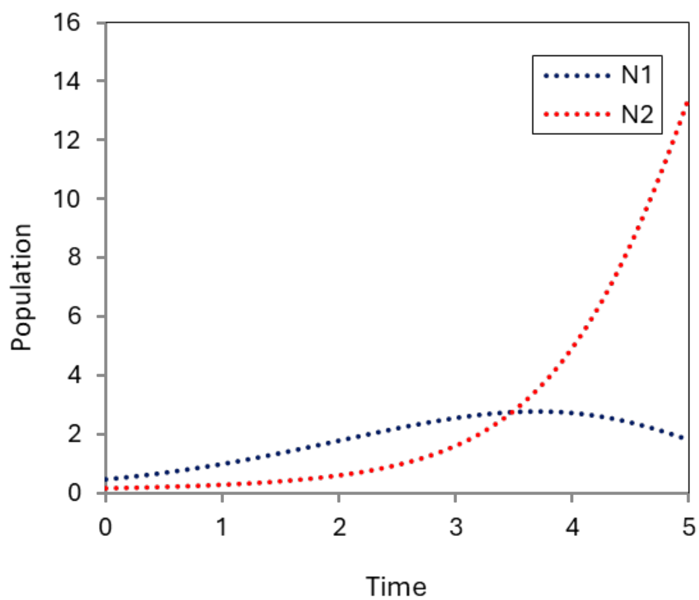

Fig 1. $N_1 = 0.485$, $N_2 = 0.192$

4 Conclusion

The research studies how predator-prey relationships function through evaluating the effects of population movement between environments with unlimited predator availability. The analysis included several relevant cases which displayed various environmental factors that determined ecosystem behavior and stability. This section presents a summary of case evidence about prey-predator interactions by showing population relationships together with the elements that alter prey and predator systems.

Case (1): The prey species dominates the predator when it shows the highest natural birth rate according to Case 1. Between times $t^* = 0$ to $t^*=3.51$ (as in Figure 1) the prey population maintained dominance but after $t^*=3.51$ the predator started to achieve higher numbers than the prey. The dominant position of prey items lasts until time $t^* = 3.51$ because even though their high reproduction produces an initial advantage their population control eventually gets surpassed by predator reproduction

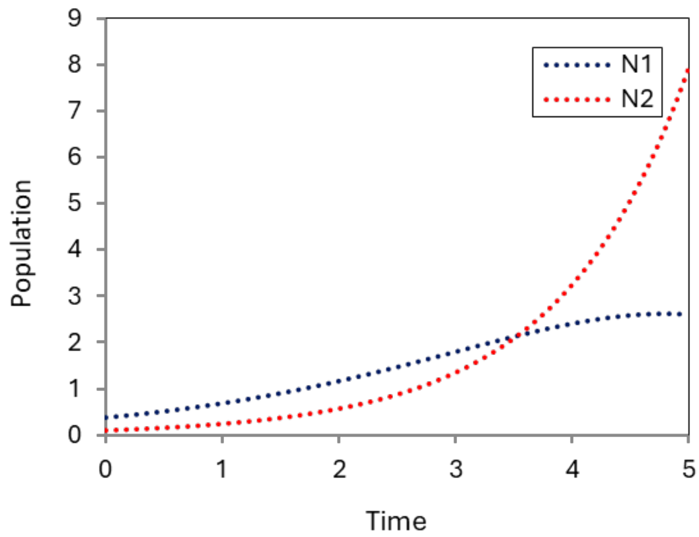

Fig 2. $N_1 = 0.392$, $N_2 = 0.113$

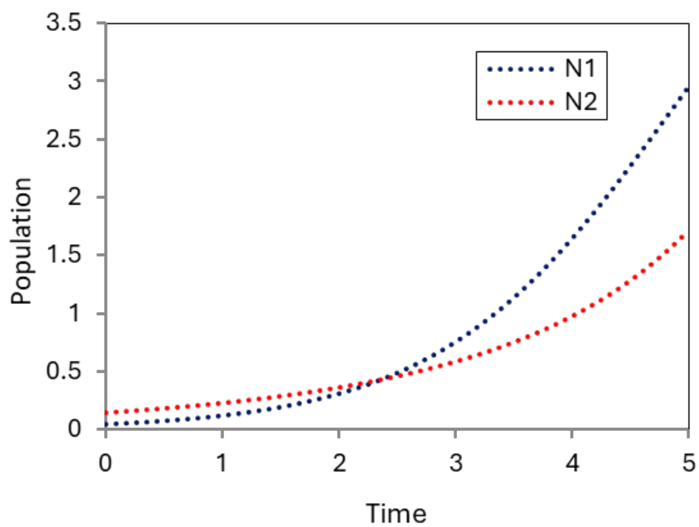

Fig 3. $N_1 = 0.045$, $N_2 = 0.141$

capabilities. This situation shows how the prey birth rate influences first population dynamics but experts agree that long-lasting stability also depends on predator population responses.

Case (2): The delayed domination of prey is caused by the extremely low coefficient of interaction between predators and prey in Case 2. Predators have a weak impact on prey population until the point $t^* = 3.62$ (as in Figure 2) when they become stronger than the prey population. This specific scenario shows that the population count of prey exceeds that of predators before the start of the experiment. A low interaction coefficient slows down the predator's ability to control the prey population because it determines the predator's impact on the prey.

Case (3): The predator population shows a low natural birth rate during this scenario until its numeric supremacy over the prey ends at $t^* = 2.49$ (as in Figure 3). Beyond this date the prey population increases to surpass the predator numbers. The predator has produced fewer descendants than the prey during the initial phase which explains why the prey assumes dominance during the cycle. The relationship between predator-prey depends significantly on the growth rate of predator populations. From the start of the predator population advantage the prey will eventually establish numerical superiority because of lower reproduction rates.

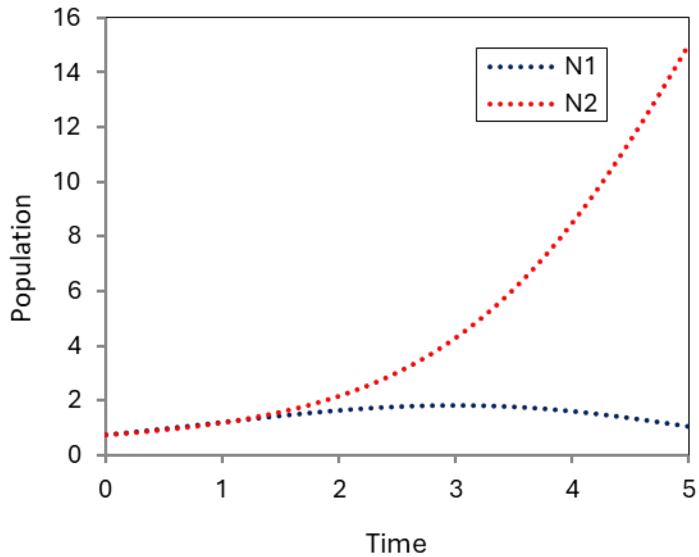

Fig 4. $N_1 = 0.750$, $N_2 = 0.750$

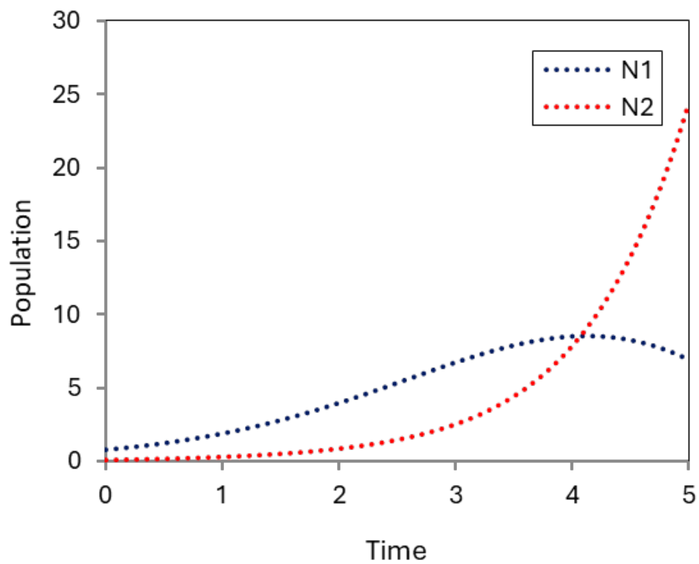

Fig 5. $N_1 = 0.832$, $N_2 = 0.148$

Case (4): When starting conditions between prey and predator populations are the same yet the predator exhibits a minimum natural birth rate this leads to an unsuccessful attempt to establish dominance over the prey. The overall migration patterns serve as a decisive factor which influences this scenario since the prey population shows lower migration numbers compared to the predator population. The interactions between migration control and birth rate determine how population levels will evolve over time according to this population simulation (Figure 4). A restricted reproduction pattern and elevated immigration ability enable the predator in the long run to reach its prey despite equal initial population levels.

Case (5): The prey in this simulation begins with higher population numbers than predators but the predator takes over dominance before time equals 4.12 (as in Figure 5). At $t^* == 4.12$ the predator population reaches dominance over the prey. The populations stay in a permanent state of variation while they avoid permanent expansion throughout the time period. The population system stabilizes into a fluctuating pattern that prevents any species from maintaining continuous population growth despite the initial prey population advantage. These role flips between predator and prey demonstrate the tricky nature

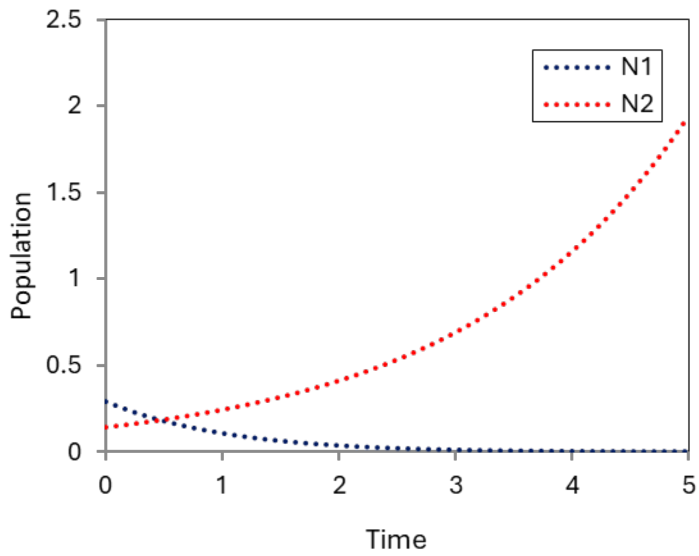

Fig 6. $N_1 = 0.289$, $N_2 = 0.414$

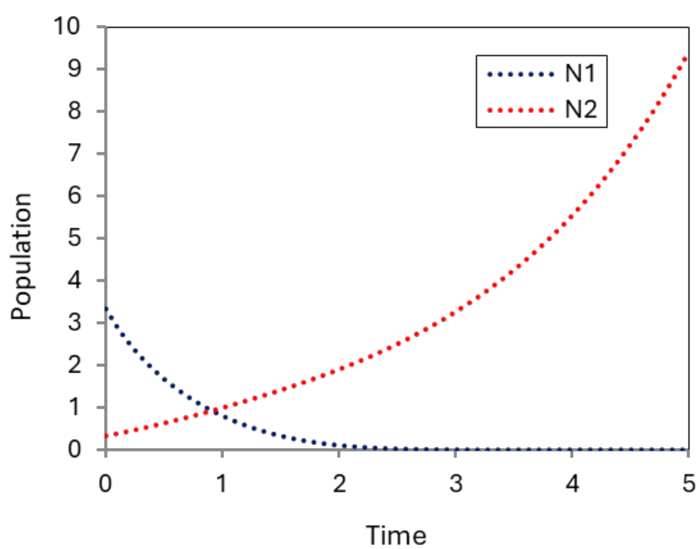

Fig 7. $N_1 = 3.330$, $N_2 = 0.320$

of predator-prey relationships while indicating that system dynamics and migration patterns and species contact rates along with starting population sizes can create unstable patterns without identifiable long-term supremacy.

Case (6): In this case both prey and predator undergo dramatic population drops until times $t^* = 0.50$ and $t = 0.98$ respectively (as in Figure 6). The populations undergo rapid growth following their initial decreases at equal speeds. System perturbing environmental conditions lead to major temporary decreases which cause both species to simultaneously increase their numbers towards restoration. This case demonstrates that environmental stressors can trigger population crashes, but the population recovery becomes possible once reproductive mechanisms remain functional.

Case (7): Both prey and predator populations experienced rapid growth up until $t^* = 3.48$ before the prey numbers began to decrease (as in Figure 7). At time $t^* = 3.48$ the prey population starts decreasing yet the predator population continues to grow. The case demonstrates how predators succeed in maintaining their numbers during periods when prey numbers decrease to low levels. Certain circumstances allow the predator to benefit from prey reduction presumably because of diminished food

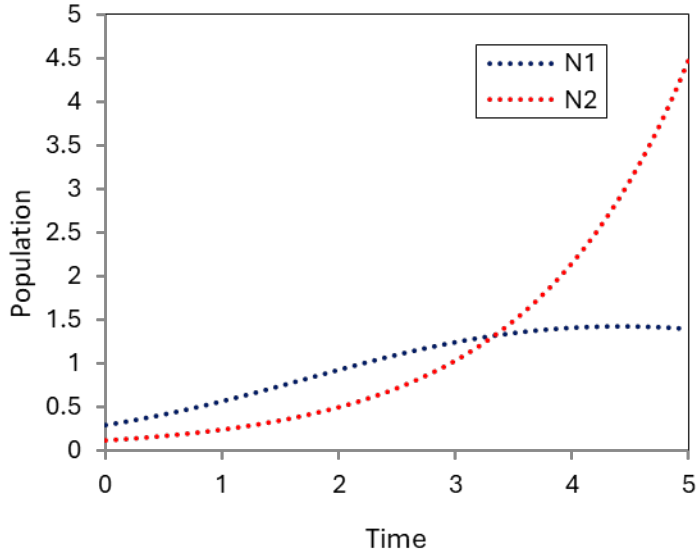

Fig 8. N_1 = 0.289, N_2 = 0.120

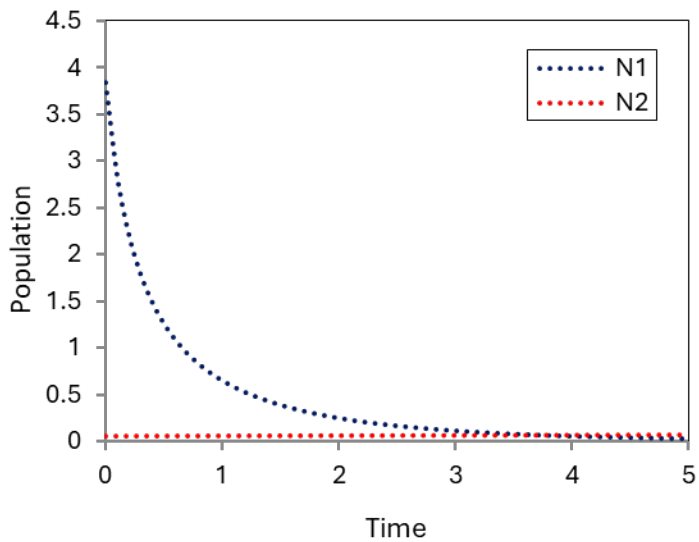

Fig 9. N_1 = 3.840, N_2 = 0.060

availability that leads to an intrinsic regulatory system. This situation demonstrates how delayed predator-prey responses can occur because predator numbers keep increasing following prey decline indicating more than simple mutual influences.

Case (8): In this instance both prey and predator grows steeply up to time t* = 3.48 and after t* = 3.48 prey gets decay whereas the predator grows (Figure 8). The prey begins with the value higher than the predator. Over time, a consistent fluctuation occurs in prey and predator without any growth rate (Figure 9).

The current work examines the stability of a predator-prey eco-system with immigration and migration, where the predator has limitless resources. The model's potential equilibrium states are all identified, and the stability requirements for each are covered. It is found that only the predator washed out state is stable when $M < a_1, a_{11}\beta_2 < a_{21}M$ and neutrally stable if $M < a_1, a_{11}\beta_2 = a_{21}M$; $M = a_1, a_{11}\beta_2 < a_{21}M$; $M = a_1, a_{11}\beta_2 = a_{21}M$ in all equilibrium states. For every equilibrium state, the solution curve trajectories are shown. Further, the system's global stability is achieved with the help of a properly constructed Liapunov's function, and the Runge-Kutta fourth order technique is used to find the numerical solutions for the growth rate equations. Examine some species connections in relation to population dynamics, such as those involving mutualism, agonistic relationships, competition, ammensalism, neutralism, commensalism, and predation. These models can be extended with additional response function types, combination harvesting, selective harvesting, variable rate harvesting, and constant rate harvesting. We are carrying out more study in this area.

Systemic Insights with Keen Observations:

1. Research findings demonstrate how migrations and immigrations affect predator-prey numbers in complex ways. These findings make clear that multiple essential factors determine the results.
2. Natural birth rates of prey along with predator species determine how an ecosystem will evolve throughout its developmental stages.
3. The power dynamics between predator and prey undergo transformation based on the interaction coefficients that describe species interactions like predation and competition.
4. Migration together with Immigration Rates serve as fundamental factors for determining the success of each population species. When more predators immigrate to a population it gives them the advantage to surpass the prey species during periods of prey population decline.
5. Certain situations result in unpredictable population patterns despite showing inconsistent long-term population growth patterns which demonstrate the complicated nature of real-world ecosystems. The predator-prey populations reach an enduring relationship that controls their numbers without achieving sustainable growth.

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