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RESEARCH ARTICLE

# Supply Chain Inventory Model for Defective Items with Sustainable Repair and Recovery Operations in Pythagorean Fuzzy Environment

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## Abstract

**Objectives:** To propose and justify a supply chain inventory model with sustainable repair and recovery operations under a triangular Pythagorean fuzzy environment. **Methods:** An economic order quantity model for minimizing the waste generated from repair and recovery operations of the supply chain with two storage units is formulated by introducing associated costs. Pythagorean fuzzy parameters were employed in this model to better represent the uncertainty present in everyday life. By solving the model, the ideal order quantity is determined. A numerical example is illustrated for the model, and its profit is compared with the existing model. **Findings:** A comparative study has been made for the proposed sustainable supply chain inventory model with double storage yields higher order quantities and maximizes the profit, which is quite beneficial to the usual production practices. The damaged items are taken as triangular Pythagorean fuzzy numbers. Sustainability costs such as rework, reuse, recycle, defect liability, and incentive costs are included in formulating the model. A numerical example is solved and shows an increase in ordering a quantity of 1173.551 and $62041.5 in profit. **Novelty:** Sustainability practices are integrated into the supply chain management of double storage units for defective items in triangular Pythagorean fuzzy environment which is not done in previous inventory models.

**Mathematics Subject Classification:** 90B05

**Keywords:** Pythagorean fuzzy number; Sustainability; Dual storage unit; Supply chain; Repair and Recovery operations

## 1 Introduction

Inventory management has evolved significantly since Ford W. Harris introduced the Economic Order Quantity (EOQ) model in 1913, revolutionizing supply chain optimization. Traditional inventory models relied on precise values for variables like lead time, inventory costs, and demand. However, real-world scenarios often involve ambiguous, imprecise, or fuzzy values. To address these uncertainties, fuzzy logic was incorporated into inventory models, starting with Kaufmann and Gupta's work in 1985. Recently, researchers have explored innovative approaches, such as the Pythagorean fuzzy TODIM methodology, to enhance inventory management decisions [(1)]. Moreover, sustainable supply chain inventory models have emerged as decision-support tools, integrating sustainability performance measures to analyze financial, social, and ecological implications [(2)]. Notably, Sarkar B, Sarkar M, Ganguly B, Cárdenas-Barrón LE. identified the most effective strategy for reducing carbon footprint in supply chains, highlighting the importance of environmental considerations in inventory management [(3)].

As the manufacturing industry navigates the complexities of integrating sustainable practices, innovative approaches to sustainable supply chain inventory models offer valuable insights into achieving conservation [(4)]. A significant challenge in this context is the presence of substandard products, which can arise due to imperfect production under uncertain environments [(5)]. To address this issue, researchers have proposed various models, including Economic Order Quantity (EOQ) models with defective items that can be remanufactured under fuzzy environments [(6)]. Other studies have developed sustainable inventory models for defective items under fuzzy environments and sustainable supply chain models for substandard growing items with prepayment and learning effects using fuzzy logic [(7,8)]. Furthermore, Alsaedi, B. S. O., Alamri, O. A., Jayaswal, M. K., Mittal, M. have developed green supply chain models with carbon emissions for defective items under learning effects in fuzzy environments [(9)]. In addition, Alsaedi has constructed sustainable supply chain models with variable production rates and remanufacturing for imperfect production inventory systems under learning in fuzzy environments [(10)]. Recently, Haripriya Barman, Magfura Pervin, Sankar Kumar Roy have developed back-ordered inventory models that account for inflation in fuzzy-cloud environments to meet customer demand [(11)].

Despite the growing importance of sustainable supply chain management, a significant research gap exists in the development of inventory models that integrate sustainable repair and recovery operations for defective items in a Pythagorean fuzzy environment. This study aims to address this research gap by developing a novel sustainable supply chain inventory model that integrates sustainable repair and recovery operations with Pythagorean fuzzy numbers. In this paper, to fulfill the demand, the retailers apply an inspection period to separate the defective items and send them to the refurbishment hub. In that hub, while reducing waste sustainability practices are employed. The defective items turn into good ones by the rework, reuse, and recycling process. Also, a dual storage unit is considered to avoid the shortage of storage capacity in each cycle with a lockdown situation. Once the product is delivered by the supplier, if there is any damage, there is a defect liability cost to compensate for the loss, advance payment, and incentive cost for the worker during the screening period. Compared to the existing model, this strategy improves operational efficiency, minimizes overall distribution costs, and preserves solid relationships with clients. The proposed model exists in a fuzzy Pythagorean environment. Yager 2013 introduced the Pythagorean fuzzy numbers. It is an extension of intuitionistic fuzzy numbers by integrating a relationship between membership and non-membership degrees satisfying the condition that the sum of the squares of the membership and non-membership functions is constrained to be less than or equal to unity. To date, no studies have examined the effectiveness of supply chain inventory models that incorporate sustainable practices, such as refurbishment, rework, reuse, and recycling, under uncertain conditions. Furthermore, existing models fail to account for the complexities of Pythagorean fuzzy environments, which are characterized by vagueness and uncertainty. In this paper, a parameter for the fraction of damaged items is taken as a Pythagorean fuzzy number that can appropriately demonstrate vagueness and uncertainty. A numerical example is illustrated to show the effectiveness of the proposed model.

The summarized literature review is mentioned in Table 1.

**Table 1. Summarized literature review**

| References | Sustainable supply chain | Defective Items and Repair/Recovery Operations | Fuzzy and Pythagorean Fuzzy Environment | Environmental Considerations | Mathematical Modeling and Optimization |
|---|---|---|---|---|---|
| (12) | ✓ | X | X | ✓ | X |
| (13) | X | ✓ | X | X | ✓ |
| (7) | X | ✓ | ✓ | ✓ | ✓ |
| (14) | X | X | ✓ | X | ✓ |
| (15) | ✓ | ✓ | X | ✓ | ✓ |
| (5) | X | ✓ | ✓ | ✓ | ✓ |
| (6) | X | X | ✓ | ✓ | ✓ |
| (11) | X | X | ✓ | X | ✓ |

*Continued on next page*

*Table 1 continued*

| | | | | | |
|---|---|---|---|---|---|
| This Paper | ✓ | ✓ | ✓ | ✓ | ✓ |

The following subsections comprise the remaining portion of this paper. Basic definitions, Notations, and assumptions are given in section 2, mathematical modeling is done in section 3, a numerical example is demonstrated in section 4 and a conclusion is provided in section 5.

# 2 Methodology

## 2.1 Basic definitions

To deal with ambiguity and uncertainty in decision-making scenarios, Zadeh created fuzzy sets in 1965. Intuitionistic fuzzy sets were first introduced by Atanassov in 1986, and Yager in 2013 expanded the fuzzy set theory by introducing Pythagorean fuzzy sets. Here, we covered the concepts and basic operations on fuzzy sets, including Intuitionistic fuzzy sets, and Pythagorean fuzzy sets.

### *Definition 2.1.1*[(16)]

"An intuitionistic fuzzy number (IFN) $\bar{A} = \{(x, \mu_{\bar{A}}(x), \gamma_{\bar{A}}(x) : x \in X)\}$ in a universal set $X$. where $\mu_{\bar{A}}(x), \gamma_{\bar{A}}(x) : X \to [0,1]$ are the degrees of membership function and of non-membership function that are satisfying $0 \le \mu_{\bar{A}}(x) + \gamma_{\bar{A}}(x) \le 1$ for all $x \in X$. The degree of uncertainty of an element $x$ in the set $\overline{A}$is defined as a function $\pi_{\bar{A}}(x) = 1 - \mu_{\bar{A}}(x) - \gamma_{\bar{A}}(x)$. Note that when $\pi_{\bar{A}}(x) = 0, x \in X$, then an IFS reduces to an FS".

### *Definition 2.1.2*[(16)]

"A Triangular Intuitionistic Fuzzy number (TIFN) $\bar{A} = (a_1, a_2, a_3; \bar{a}_1, a_2, \bar{a}_3)$ where $\bar{a}_1 \le a_1 \le a_2 \le a_3 \le \bar{a}_3$ , membership function and non-membership function are denoted as

$$\mu_{\bar{A}(x)} = \begin{cases} \frac{x-a_1}{a_2-a_1}, & \text{if } a_1 \le x \le a_2 \\ \frac{a_3-x}{a_3-a_2}, & \text{if } a_2 \le x \le a_3 \\ 0 & \text{otherwise.} \end{cases} \quad \text{and } \mu_{\bar{A}(x)} = \begin{cases} \frac{a_2-x}{a_2-\bar{a}_1}, & \text{if } \bar{a}_1 \le x \le a_2 \\ \frac{x-a_2}{\bar{a}_3-a_2}, & \text{if } a_2 \le x \le \bar{a}_3 \\ 1 & \text{otherwise} \end{cases}$$

### *Definition 2.1.3*[(17)]

A Pythagorean fuzzy Set $\overline{A}_P$ in $X$ is defined as $\bar{A}_P = \left\{\left(x, \mu_{\bar{A}(x)}, \gamma_{\bar{A}(x)}\right) : x \in X\right\}$,, where $\mu_{\bar{A}(x)}, \gamma_{\bar{A}(x) : X \to [0,1]}$ are the membership function and non-membership functions, which satisfy $0 \le \mu^2_{\overline{A}(x) + \gamma^2_{\overline{A}(x) \le 1}}$ for all $x \in X$. The function establishes an element in the set's range of indeterminacy $\pi_{\overline{A}_P}(x) = \sqrt{1 - \mu^2_{\overline{A}(x) - \gamma^2_{\overline{A}(x)}}}$. We remark that both $\mu\overline{A}(x) + \gamma\overline{A}(x) \ge 1 or$ $\mu\overline{A}(x) + \gamma\overline{A}(x) \le 1$ can occur, which is the main difference between IFS and PFS, for this reason, every IFS is a PFS; but the converse does not hold.

### *Definition 2.1.4*[(17)]

Considering two Pythagorean fuzzy numbers$\overline{A}_P$and $\overline{B}_P$ in $X$, where $\overline{A}_P$ and $\overline{B}_P$ are described as $\overline{A}_P = \left\{\left(x, \mu_{\overline{A}(x)}, \gamma_{\overline{A}(x)}\right) : x \in X\right\}$ and $\overline{B}_P = \left\{\left(x, \mu_{\overline{B}(x)}, \gamma_{\overline{B}(x)}\right) : x \in X\right\}$ respectively, let $r \ge 0$ then the following arithmetic operations be given:

1. **Sum:** $\overline{A}_P \oplus \overline{B}_P = \left\{\left(x, \sqrt{\mu^2_{\overline{A}_P}(x) + \mu^2_{\overline{B}_P}(x) - \mu^2_{\overline{A}_P}(x)\mu^2_{\overline{B}_P}(x)}, \gamma_{\overline{A}_P}(x)\gamma_{\overline{B}_P}(x)\right) : x \in X\right\}$
2. **Inner product:**$\overline{A}_P \odot \overline{B}_P = \left\{\left(x, \mu_{\overline{A}_P}(x)\mu_{\overline{B}_P}(x)\sqrt{\gamma^2_{\overline{A}_P}(x) + \gamma^2_{\overline{B}_P}(x) - \gamma^2_{\overline{A}_P}(x)\gamma^2_{\overline{B}_P}(x)}\right) : x \in X\right\}$
3. **Scalar product:** $r\overline{A}_P = \left\{\left(x, \sqrt{1 - \left(1 - \mu^2_{\overline{A}_P}(x)\right)^r}, \left(\gamma_{\overline{A}_P}(x)\right)^r\right) : x \in X\right\}$
4. **Power:** $r\overline{A}_P = \left\{\left(x, \sqrt{1 - \left(1 - \mu^2_{\overline{A}_P}(x)\right)^r}, \left(\gamma_{\overline{A}_P}(x)\right)^r\right) : x \in X\right\}$
5. **Complement:** $\left(\overline{A}_P\right)^c = \left\{\left(x, \gamma_{\overline{A}_P}(x), \mu_{\overline{A}_P}(x)\right) : x \in X\right\}$

### *Definition 2.1.5[17]*

The triangular Pythagorean fuzzy number on the real axis is described as $\overline{A}_P = \left\{\left((a,b,c);\mu_{\overline{A}_P}(x),\gamma_{\overline{A}_P}(x)\right):x\in X\right\}$ where $\mu_{\overline{A}_P}(x)\geq 0,\gamma_{\overline{A}_P}(x)\geq 0,\mu^2{}_{\overline{A}_P}(x)+\gamma^2{}_{\overline{A}_P}(x)\leq 1, 0\leq a\leq b\leq c$ and $(a,b,c)$ is a triangular fuzzy number. The membership and non-membership functions are

$$\mu_{\overline{A}_P}(x)=\begin{cases}\frac{x-a}{b-a}, if\, a\leq x\leq b\\ \frac{c-x}{c-b} if\, b\leq x\leq c\\ 0\; Otherwise\end{cases}\quad \text{and } \gamma_{\overline{A}_P}(x)=\begin{cases}\sqrt{1-\left(\frac{x-a}{b-a}\right)^2}, if\, a\leq x\leq b\\ \sqrt{1-\left(\frac{c-x}{c-b}\right)^2} if\, b\leq x\leq c\\ 1\; Otherwise\end{cases}$$

### *Definition 2.1.6[17]*

Considering $\overline{A}_P$ and $\overline{B}_P$ are Triangular Pythagorean fuzzy numbers, and $X$ be the universal set where $\overline{A}_P$ and $\overline{B}_P$ are given and $\overline{B}_P=\left\{\left((a_2,b_2,c_2);\mu_{\overline{A}_P}(x),\gamma_{\overline{A}_P}(x)\right):x\in X\right\}$. Let $r\geq 0$ for TPFN, the following arithmetic operations are defined:

Sum: $\overline{A}_P\oplus\overline{B}_P=\left\{\left((a_1+a_2,b_1+b_2,c_1+c_2);\sqrt{\mu^2_{\overline{A}_P}(x)+\mu_{\overline{B}_P}(x)-\mu^2_{\overline{A}_P}(x)\mu_{\overline{B}_P}(x)},\gamma_{\overline{A}_P}(x)\gamma_{\overline{B}_P}(x)\right):x\in X\right\}$

1. Inner product: $\overline{A}_P\odot\overline{B}_P=\left\{\left((a_1a_2,b_1b_2,c_1c_2);\mu_{\overline{A}_P}(x)\mu_{\overline{B}_P}(x),\sqrt{\gamma^2{}_{\overline{A}_P}(x)+\gamma^2{}_{\overline{B}_P}(x)-\gamma^2{}_{\overline{A}_P}(x)\gamma^2{}_{\overline{B}_P}(x)}\right):x\in X\right\}$
2. Scalar product: $r\overline{A}_P=\left\{\left((ra,rb,rc);\sqrt{1-\left(1-\mu^2{}_{\overline{A}_P}(x)\right)^r},\left(\gamma_{\overline{A}_P}(x)\right)^r\right):x\in X\right\}$
3. Power: $(A_P)^r=\{(a^r,b^r,c^r);\quad (\mu_{\overline{A}_P}(x))^r\sqrt{1-(1-\gamma^2{}_{\overline{A}_P}(x))^r}):x\in X\}$

### *Definition 2.1.7*

Let $\overline{A}_P=\left\{\left((a,b,c);\mu_{\overline{A}_P}(x),\gamma_{\overline{A}_P}(x)\right):x\in X\right\}$ be a TPFN. Let $\lambda\in(0,1)$ be a fixed parameter. Then the ranking index is defined as:

$$R(\overline{A}_P)=\frac{1}{4}\frac{\lambda(a+2b+c)+(1-\lambda)(5a-2b+5c)}{(2-\lambda)}$$

## 2.2 Notations

$o_S$—Cycle order quantity
$z$— In-house storage limit
$D_r$—Yearly demand volume
$S_r$—Inspection throughput
$a$—Purchase cost per unit
$C_O$—Order processing fee
$b$—Quality control cost
$G_S$—Revenue per unit
$F_{O_S}$—Defective item rate
$\overline{q}$— Uncertain defect rate
$S_P$—In-house inspection time
$T_{sd}$—Pre-demand period
$T_d$—Sales fulfillment duration
$A_r$—Prepayment installments
$I_{h_r}$—Commercial storage expense
$I_{h_o}$—In-house storage expense
$R_C$—Maintenance initiation
$R_r$—Maintenance efficiency
$T_f$—Shipping setup fee
$L_S$—Repair operational expense
$T_u$—Shipping cost per unit
$I_r$—Rented storage expense
$\gamma$—Down payment percentage
$M$—Total purchase expense

$P_d$—Discount percentage
$T_p$—Advance payment period
$L$—Replenishment cycle duration
$I_p$—Commercial warehouse inspection time
$T_c$—Replenishment lead time
$L_p$—Period of lockdown
$T_{L_P}$—Replenishment lead time during the lockdown period
$M_r$—Repair and maintenance expense
$\lambda$—Rate of imperfect goods
$BI$—Business development investment
$i$—Product reacquisition
$U_1$—Revised product value
$U_2$—Secondary market value
$U_3$—Recycled material value
$g_{r_1}$—Scrap rate
$g_{r_2}$—Reusability rate
$g_{r_3}$—Material recovery rate

## 2.3 Assumptions

1. Demand is consistent with forecasts.
2. Investigation resources exceed demand needs.
3. Every product is thoroughly checked, with demand met instantly.
4. The storage capacity of an In-house storage unit is finite, totalling $z$units, while commercial storage unit capacity is indefinitely expandable.
5. Holding expenses in commercial storage units are higher than In-house storage unit inventory costs.
6. Supplier offers flexible payment terms, including advance payment.
7. No stockouts permitted.
8. Lockdown restrictions affect operations for a limited time t.

## 2.4 Model formulation

During each cycle, $z$ goods from Lot $O_S$ are restocked in the In-house storage unit while the remaining items are held in the Commercial storage unit. A portion of the goods received are faulty $q$ due to manufacturing/maintenance/transportation concerns. Both storage units screen at a rate of $S_r$ during $[0, I_P]$ in the Commercial storage units and $[0, S_P]$ In-house storage unit. Defective products $q$ $O_S$are pulled and fixed locally, which requires time $T_C$. A lockdown is expected during time $L_P$ (constant $t$). At the conclusion $T_{L_P}$Commercial storage units stock becomes zero, whereas In-house storage unit items deteriorate at the same rate of demand $D_r$. Suppliers provide advance payment: $\gamma M$ is paid in advance and divided into $A_r$payments at previous cycle dates. The remaining amount $(1-\gamma)M$ is paid upon replenishment, with a price discount of $\gamma M$at rate $P_d$ for prepaid amounts. The prepayment period corresponds to the preceding cycle ($T_P \in [0, L]$).

Non-Defective Item Revenue

$$ND_r = G_S O_S (1-q) \tag{1}$$

Defective Item Revenue

$$D_r = G_S q O_S \tag{2}$$

Purchase Discount

$$PD = \gamma a T_p P_d \left(\frac{A_r + 1}{2A_r}\right) \tag{3}$$

Gross sales revenue

$$G_{sr} = G_S O_S + \gamma a T_p P_d \left(\frac{A_r + 1}{2A_r}\right) \tag{4}$$

Acquisition cost

$$A_C = C_O + aO_S \tag{5}$$

Warehouse Inspection cost:

Commercial Storage Unit

$$O_S W_C = b(1-z) \tag{6}$$

In-house Storage Unit

$$O_S W_h = bz \tag{7}$$

The maintenance costs charged by the service firm include:

1. Maintenance setup cost
2. Fixed shipping charge
3. Unit freightage expense for shipping between the distributor's warehouse and the repair business (both directions)
4. Warehousing costs at the maintenance unit
5. Material and labor expenses for fixing faulty items"

Thus, the maintenance cost is shown as

$$M_C = R_C + 2T_f + F_{O_S} O_S \left( L_S + 2T_u + I_r \left( \frac{(O_S - z)\left(1 - F_{O_S} - \frac{D_r}{S_r}\right)}{D_r} \right) \right) \tag{8}$$

The holding costs for commercial storage units and In-house storage units is

$$H_C = \frac{I_{h_r}}{2}(O_S - z)^2 \left( \frac{2}{S_r} - \frac{D_r}{S_r^2} + \frac{\left(1 - F_{O_S} - \frac{D_r}{S_r}\right)}{D_r} \right) + I_{h_r} F_{O_S} t_1 O_S + \frac{I_{h_r} F_{O_S}^2 O_S^2}{2D_r} \tag{9}$$

$$H_H = \frac{I_{h_0}}{2} z^2 \left( \frac{1}{S_r} - \frac{2\left(1 - F_{O_S}\right)}{S_r} - \frac{\left(1 - F_{O_S}\right)\left(1 - F_{O_S} - \frac{D_r}{S_r}\right)}{D_r} + \frac{\left(1 - 2F_{O_S} + {F_{O_S}}^2\right)}{2D_r} \right)$$

$$+I_{h0} O_s z = \left( \frac{1 - F_{Os}}{S_r} + \frac{(1 - F_{Os})\left(1 - F_{Os} - \frac{D_r}{S_r}\right)}{D_r} + \frac{(1 - F_{Os}) F_{Os}}{D_r} \right) + I_{h0} z (1 - F_{Os}) t_1 \tag{10}$$

The expense of lockdown penalties in both storage units is

$$L_C = l F_{O_S} t_1 O_S \tag{11}$$

$$L_H = lz\left(1 - F_{O_S}\right) t_1 \tag{12}$$

Defect liability cost

$$D_l = M_r \lambda O_S \tag{13}$$

Incentive compensation cost

$$I_{CC} = BIiO_S \tag{14}$$

Rework, reuse, and recycling are the three R's we've employed here.

$$R_w = U_1 g_{r_1} O_S \tag{15}$$

$$R_u = U_2 g_{r_2} O_S \tag{16}$$

$$R_c = U_3 g_{r_3} O_S \tag{17}$$

All the costs in every process are shown as

$$Total\ cost\,(O_S) = A_C + O_S W_C + O_S W_h + M_C + H_C + H_H + L_C + L_H + D_l + I_{C_C}$$

Thus,

$$T(O_S) = C_O + aO_S + bO_S + R_c + 2T_f + M_r \lambda O_S + BIiO_S + U_1 g_{r_1} O_S + U_2 g_{r_2} O_S + U_3 g_{r_3} O_S$$

$$+F_{O_S} O_S \left( L_s + 2T_u - \frac{I_c\left(1 - F_{O_S} - \frac{D_r}{S_r}\right)}{D_r} \right) + \frac{F_{O_S} O_S^2 I_c \left(1 - F_{O_S} - \frac{D_1}{S_r}\right)}{D_r} + \frac{F_{O_S} O_S^2}{2} \left( \frac{2}{S_r} - \frac{D_r}{S_r^2} + \frac{\left(1 - F_{O_S} - \frac{D_r}{S_r}\right)^2}{D_r} \right)$$

$$-I_{h_r} O_S z \left( \frac{2}{S_r} - \frac{D_r}{S_r^2} + \frac{\left(1 - F_{O_S} - \frac{D_r}{S_r}\right)^2}{D_r} \right) + \frac{I_{h_r} z^2}{2} \left( \frac{2}{S_r} - \frac{D_r}{S_r^2} + \frac{\left(1 - F_{O_S} - \frac{D_r}{S_r}\right)^2}{D_r} \right) + I_{h_r} F_{O_S} t_1 O_S$$

$$+\frac{I_h F_{O_S}^2 O_S^2}{2D_r} + \frac{I_{h_0} z^2}{2} \left( \frac{1}{S_r} - \frac{2(1 - F_{O_S})}{S_r} - \frac{(1 - F_{O_S})\left(1 - F_{O_S} - \frac{D_r}{S_r}\right)}{D_r} + \frac{(1 - 2F_{O_S} + F_{O_S}^2)}{2D_r} \right)$$

$$+I_h O_S z \left\{ \frac{(1 - F_{O_S})}{S_r} + \frac{(1 - F_{O_S})\left(1 - F_{O_S} - \dfrac{D_r}{S_r}\right)}{D_r} + \frac{(1 - F_{O_S}) F_{O_S}}{D_r} \right\} + I_{h_0} z (1 - F_{O_S}) t_1 + l F_{O_S} t_1 O_S + l z (1 - F_{O_S}) t_1 \tag{18}$$

Profit annually per unit of time is the variation between the gross sales revenue and the total cost per cycle $P(O_S) = G_{S_r}(O_S) - T(O_S)$

The profit annually is shown as $T_P = \frac{P(O_S)}{L}, then\ L = \frac{O_S}{D_r}$

We have to consider that faulty items in $O_S$are uncertain. To deal with uncertainty we choose fuzzy sets. Here I have taken the Triangular Pythagorean fuzzy number for faulty items $F_{O_S}$ as $q$

$.Soweusedtherankingmethodtofindthetotalprofit.$

The ranking method for profit annually is given as

$RT_P(O_S) = \frac{R(P(O_S))}{R(L)}$

Hence,

$$RT_P(O_S) = O_S \left[ -D_r \left( \frac{I_{h_r}}{2} \left( \frac{2}{S_r} - \frac{D_r}{{S_r}^2} + \frac{\left(1 - R[\tilde{q}] - \frac{D_r}{S_r}\right)^2}{D_r} \right) + \frac{I_{h_r} R[\tilde{q}^2]}{2D_r} + \frac{R[\tilde{q}] I_C \left(1 - R[\tilde{q}] - \frac{D_r}{S_r}\right)}{D_r} \right) \right]$$

$$+\frac{1}{O_S} \left[ -D_r \left( \begin{array}{l} C_O + R_c + 2T_f + \frac{I_{h_r} z^2}{2} \left( \frac{2}{S_r} - \frac{D_r}{S_r^2} + \frac{\left(1 - R[\overline{q}] - \frac{D_r}{S_r}\right)^2}{D_r} \right) \\ + \frac{I_{h_0} z^2}{S_r} - I_{h_0} z^2 (1 - R[\overline{q}]) \left( \frac{\left(1 - R[\overline{q}] - \frac{D_r}{S_r}\right)}{D_r} + \frac{2}{S_r} - t_1 \right) + \left( \frac{I_{h_o} z^2 (1 - 2R[\overline{q}] + R[\overline{q}^2])}{2D_r} \right) + l z (1 - R[\overline{q}]) t t_1 \end{array} \right) \right]$$

$$+D_r\left[\begin{array}{l} G_S+\gamma a T_p P_d\left(\frac{A_r+1}{2A_r}\right)+BI\,i+U_1 g_{r_1}+U_2 g_{r_2}+U_3 g_{r_3}+M_r\lambda-a-b \\ -R[\bar{q}]\left(L_S+2T_u-\frac{I_c\left(1-R[\bar{q}]-\frac{D_r}{S_r}\right)}{D_r}\right)+I_{h_r}z\left(\frac{2}{S_r}-\frac{D_r}{S_r{}^2}+\frac{\left(1-R[\bar{q}]-\frac{D_r}{S_r}\right)^2}{D_r}\right)-I_{h_r}R[\bar{q}]t_1\end{array}\right]$$

$$-I_{h_0}z(1-R[\bar{q}])\left(\frac{1}{S_r}+\frac{R[\bar{q}]}{D_r}+\frac{\left(1-R[\bar{q}]-\frac{D_r}{S_r}\right)}{D_r}\right)-lR[\bar{q}]t_1 \tag{19}$$

$\mathrm{R}\overline{\mathrm{T_p}\left(\mathrm{O_S}\right)}=\sum \mathrm{O_S}+\frac{\gamma}{\mathrm{O_S}}+\pi$

Thus, $\Sigma=-D_r\left(\frac{I_{hr}}{2}\left(\frac{2}{S_r}-\frac{D_r}{S_r^2}+\frac{\left(1-R[\bar{q}]-\frac{D_r}{S_r}\right)^2}{D_r}\right)+\frac{I_{hr}R[\bar{q}]^2}{2D_r}+\frac{R[\bar{q}]\left[I_c\left(1-R[\bar{q}]-\frac{D_r}{S_r}\right)\right]}{D_r}\right)\prec 0$

$$\Upsilon=-D_r\left(\begin{array}{c} C_O+R_c+2T_f+\frac{I_{h_r}z^2}{2}\left(\frac{2}{S_r}-\frac{D_r}{S_r{}^2}+\frac{\left(1-R[\bar{q}]-\frac{D_r}{S_r}\right)^2}{D_r}\right)+\frac{I_{h_0}z^2}{S_r} \\ -I_{h_0}z^2(1-R[\bar{q}])\left(\frac{\left(1-R[\bar{q}]-\frac{D_r}{S_r}\right)}{D_r}+\frac{2}{S_r}-t_1\right)+\left(\frac{I_{h_o}z^2(1-2R[\bar{q}]+R[\bar{q}^2])}{2D_r}\right)+lz(1-R[\bar{q}])t_1\end{array}\right)\prec 0$$

$$\pi=D_r\left[\begin{array}{c} G_s+\gamma a T_p P_d\left[\frac{A_r+1}{2A_r}\right]+BI_i+U_1 g_{r1}+U_2 g_{r2}+U_3 g_{r3}+M_r\lambda-a-b \\ \left[-R[\bar{q}]\left[L_S+2T_u-\frac{I_c\left(1-R[\bar{q}]-\frac{D_r}{S_r}\right)}{D_r}\right]+I_{h_r}z\left[\frac{2}{S_r}-\frac{D_r}{S_r^2}+\frac{\left(1-R[\bar{q}]-\frac{D_r}{S_r}\right)^2}{D_r}\right]-I_{h_r}R[\bar{q}]t\right]\end{array}\right]$$
$$-I_{h0}z(1-R[\bar{q}])\left(\frac{1}{S_r}+\frac{R[\bar{q}]}{D_r}+\frac{\left(1-R[\bar{q}]-\frac{D_r}{S_r}\right)}{D_r}\right)-lR[\bar{q}]t_1$$

Differentiating partially $RT_P\left(O_S\right)$ with respect to $O_S$ and equating to zero, we get the optimum quantity for orders.

$$O_S=\sqrt{\frac{\gamma}{\sum}} \tag{20}$$

## 3 Results And Discussion

Despite the numerous studies in the literature that have explored the evolution of inventory models in supply chain management, sustainability, and imprecise or vague situations, they have not adequately addressed sustainability in supply chain management[5,7]. While some research focuses on supply chain management in fuzzy environments[6,11], it often overlooks the concept of sustainability. This paper aims to bridge this gap by developing a sustainable supply chain inventory model under a Pythagorean fuzzy environment. The parameter for the fraction of damaged items is represented as a Pythagorean fuzzy number, effectively capturing vagueness and uncertainty. Additionally, this model incorporates defect liability costs to compensate for losses, advance payments, and incentive costs for workers during the screening period. Compared to existing models, it demonstrates a \$62041.5 profit increase and an increase in ordering a quantity of 1173.551. This strategy enhances operational efficiency, minimises overall distribution costs, and fosters strong relationships.

### 3.1 Numerical example

To show the efficiency of model performance. It includes defective and non-defective item revenue, purchase discount, gross sales revenue, acquisition cost, warehouse inspection cost, commercial storage unit, In-house storage unit, maintenance cost, holding cost for both commercial and In-house storage units, lockdown cost, defect liability cost, incentive compensation cost, and sustainability cost are formulated. The following parameters are taken from Rajeswari, S., Sugapriya, C. & Nagarajan, D.[6].

$S_r=1,75,200(unit/yr)$, $G_S=\$50/$ unit, $a=\$25/unit$, $I_{hr}=\$6/$ unit, $L_S=\$5/$ unit, $T_u=\$2/$ unit, $C_0=\$100$ $R_C=\$100, T_f=\$200, A_r=5, T_p=0.20 years, t_1=0.05 years, \gamma=0.4, P_d=0.1$

The fraction of the defective items is taken as Pythagorean fuzzy numbers $q$ $=(0.0150, 0.0200, 0.0300)$ from the above parameters. We have obtained the optimum quantity order is $O_s*=4290.151$ and the profit annually is $RT_P(Q_s)=\$1066741.5$

**Table 2. Optimum Solution**

| | Existing model | Proposed model |
|---|---|---|
| Order quantity | 3116.6 | $4290.151$ |
| Total Profit | $1004700 | $\$1066741.5$ |

### *3.1.1 Optimum solution and comparative study*

We compared the optimal solution for order quantity and the total profit with the existing model given in Table 2.

It indicates a rise in ordering number of 1173.551 and a profit of $62041.5.

## 4 Conclusion

The Supply chain inventory model without Sustainability is one of the significant disadvantages of the traditional inventory model, as it affects the environment during the distribution process. This study addresses a critical gap in sustainable supply chain inventory models by focusing on sustainability in repair and recovery operations. Unlike existing models, this research incorporates Pythagorean fuzzy numbers to account for uncertainty in defective products. Additionally, it considers defect liability costs and incentive compensation costs to minimise defects and promote effective recovery operations. These results demonstrates that the integrated approach shows the effectiveness of the proposed model in optimising order quantity $O_s* = 4290.151$and increasing profit $RT_P(O_s) = \$1066741.5$. The key advantages of the model include improved sustainability, uncertainty management, cost minimization, and profit maximization. However, the model has limitations, such as a single-product focus and static pricing. This research contributes significantly to inventory management literature, offering valuable insights for retailers seeking to optimise supply chain operations and maximise profits. Future research directions include exploring multi-product inventory optimisation, fuzzy stochastic models, and dynamic pricing strategies to further enhance the model's capabilities.

### Data availability statement

The data used in this paper were obtained from [6] Rajeswari S, Sugapriya C, Nagarajan D. An analysis of uncertain situation and advance payment system on a double-storage fuzzy inventory model. Opsearch. 2022 Mar;59(1):20-40. https://doi.org/10.1007/s12597-021-00530-8

### Details of Ethical clearance

No ethical clearance was necessary for this research as it was based on existing data.

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