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Enhancing Emission Control With Environmental Engineering Services in a Sustainable Fuzzy Inventory Model

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Abstract

Objectives: To present a sustainable fuzzy inventory model that can mitigate emissions and optimize the total cost of inventory processes. **Methods:** To manage uncertainty, this study considers the cost parameters as heptagonal fuzzy numbers and then applies the sub-interval average method to optimize the total inventory cost. **Findings:** From the numerical example solution results, it is clear that the study arrives at a reduced total inventory cost value in the sub-interval average method. **Novelty:** This sustainable fuzzy inventory model adds environmental engineering services costs to mitigate hazardous carbon emissions and pollutants in inventory operations.

2020 Mathematics subject classification: 90B05

Keywords: Emission control; Engineering services; Fuzzy; Inventory; Sustainability

1 Introduction

Sustainable development focuses on economic growth and environmental safety [1].However, greater production often results in emissions from storage and distribution [2,3].Although carbon policies push for pollution control, they are not enough to achieve lasting environmental progress [4,5].Looking at recent research investigations, Kristiyani et al. [6] in 2019, suggest that proper inventory maintenance is vital to optimize available resources and limit environmental pollution. And Chen et al. [7] in 2024, suggest that inventory management together with environmentally friendly procedures can minimize inefficiencies and toxic pollutants. Adoption of sustainable practices in companies can reduce emissions and contaminants, which not only contributes to long-term sustainability [8,9] but ensures the balance of economic and environmental benefits [10,11]. Furthermore, it enhances the company's image and attracts environmentally conscious customers [12,13]. Consequently, these efficient strategies support quality inventory decisions and global environmental targets [14,15]. However, many industries struggle to minimize emissions due to outdated practices. Specialized expertise can improve inventory management and foster sustainable methods to reduce environmental impacts.

1.1. Research Gap

No researchers have previously included environmental engineering services costs in sustainable fuzzy inventory models. This study intends to fill this gap by including environmental engineering services costs to help companies reduce carbon emissions and enhance sustainability practices in inventory management. Environmental engineering services focus on providing expertise to develop pollution reduction plans, which consist of the identification of high-emission activities, examining inefficiencies in inventory operations, and implementing targeted approaches such as process optimization, cleaner technologies, and energy-saving strategies. They also provide detection systems to monitor emission levels, create compliance reports, educate employees on sustainable practices, and incorporate renewable energy sources for long-term environmental advantages.

The article is presented as follows: Section 2 covers the methodology. Section 3 provides results and discussion. Finally, Section 4 makes a conclusion of the study.

2 Methodology

This article expands on Kristiyani et al.[6] inventory model by including environmental engineering services costs[10] to mitigate hazardous industrial emissions. To manage uncertainty, we consider the cost parameters as heptagonal fuzzy numbers, and then we apply the sub-interval average method[11] to optimize the total inventory cost. This method ensures a simple formula and solution process, making it easy to understand without the need for complex computational processes.

2.1 Definitions

2.1.1 Heptagonal Fuzzy number

A Fuzzy number $\widetilde{K}_H = \left(k_{h_1}, k_{h_2}, k_{h_3}, k_{h_4}, k_{h_5}, k_{h_6}, k_{h_7}\right)$ is a heptagonal fuzzy number, and the definition of its membership function is,

$$\mu_{K\ H}(m) = \begin{cases} \frac{1}{2}\frac{(m-k_{h1})}{(k_{h2}-k_{h3})}, for\ k_{h1} \le m \le k_{h2} \\ 0.5,\ for\ k_{h2} \le m \le k_{h3} \\ \frac{1}{2}+\frac{1}{2}\frac{(m-kh_3)}{(k_{h4}-k_{h3})},\ for\ k_{h3} \le m \le k_{h4} \\ \frac{1}{2}+\frac{1}{2}\frac{(kh_5-m)}{(k_{h5}-k_{h4})},\ for\ k_{h4} \le m \le k_{h5} \\ 0.5,\ for\ k_{h5} \le m \le k_{h6} \\ \frac{1}{2}\frac{(kh_7-m)}{(k_{h7}-k_{h6})},\ for\ k_{h6} \le m \le k_{h7} \\ 0,\ otherwise \end{cases}$$

Example: Suppose the fuzzy number is said to be $\widetilde{K}_H = (1,2,3,4,5,6,7)$, then we get membership values as follows, $\mu_{\widetilde{K}_H}(1) = 0$, $\mu_{\widetilde{K}_H}(2) = 0.5$, $\mu_{\widetilde{K}_H}(3) = 0.5$, $\mu_{\widetilde{K}_H}(4) = 1$, $\mu_{\widetilde{K}_H}(5) = 0.5$, $\mu_{\widetilde{K}_H}(6) = 0.5$, $\mu_{\widetilde{K}_H}(7) = 0$. The graphical representation of heptagonal fuzzy number is given below.

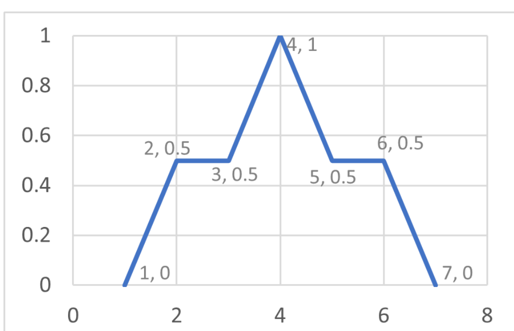

Fig 1. Graphical representation of heptagonal fuzzy number

2.1.2 Arithmetic Operations of Heptagonal Fuzzy Number

Assume that $\widetilde{K}_{HD} = \left(k_{hd_1}, k_{hd_2}, k_{hd_3}, k_{hd_4}, k_{hd_5}, k_{hd_6}, k_{hd_7}\right)$ and $\widetilde{K}_{HF} = \left(k_{hf_1}, k_{hf_2}, k_{hf_3}, k_{hf_4}, k_{hf_5}, k_{hf_6}, k_{hf_7}\right)$ be two heptagonal fuzzy numbers. The arithmetic operations for two heptagonal fuzzy numbers are provided below,

$$\widetilde{K}_{HD} + \widetilde{K}_{HF} = \left(k_{hd_1} + k_{hf_1}, k_{hd_2} + k_{hf_2}, k_{hd_3} + k_{hf_3}, k_{hd_4} + k_{hf_4}, k_{hd_5} + k_{hf_5}, k_{hd_6} + k_{hf_6}, k_{hd_7} + k_{hf_7}\right)$$

$$\widetilde{K}_{HD} - \widetilde{K}_{HF} = \left(k_{hd_1} - k_{hf_1}, k_{hd_2} - k_{hf_2}, k_{hd_3} - k_{hf_3}, k_{hd_4} - k_{hf_4}, k_{hd_5} - k_{hf_5}, k_{hd_6} - k_{hf_6}, k_{hd_7} - k_{hf_7}\right)$$

$$\widetilde{K}_{HD} \times \widetilde{K}_{HF} = \left(k_{hd_1}k_{hf_1}, k_{hd_2}k_{hf_2}, k_{hd_3}k_{hf_3}, k_{hd_4}k_{hf_4}, k_{hd_5}k_{hf_5}, k_{hd_6}k_{hf_6}, k_{hd_7}k_{hf_7}\right)$$

$$\frac{\widetilde{K}_{HD}}{\widetilde{K}_{HF}} = \left(\frac{k_{hd_1}}{k_{hf_1}}, \frac{k_{hd_2}}{k_{hf_2}}, \frac{k_{hd_3}}{k_{hf_3}}, \frac{k_{hd_4}}{k_{hf_4}}, \frac{k_{hd_5}}{k_{hf_5}}, \frac{k_{hd_6}}{k_{hf_6}}, \frac{k_{hd_7}}{k_{hf_7}}\right)$$

Example: if $\widetilde{K}_{HD} = (4,6,10,8,12,7,11)$and $\widetilde{K}_{HF} = (2,2,5,4,3,7,1)$ be two heptagonal fuzzy numbers, then

$$\widetilde{K}_{HD} + \widetilde{K}_{HF} = (4+2,\ 6+2,\ 10+5,\ 8+4,\ 12+3,\ 7+7,\ 11+1) = (6,\ 8,\ 15,12,15,\ 14,\ 12)$$

$$\widetilde{K}_{HD} - \widetilde{K}_{HF} = (4-2, 6-2, 10-5, 8-4, 12-3, 7-7, 11-1) = (2,\ 4,\ 5,\ 4,\ 9,\ 0,\ 10)$$

$$\widetilde{K}_{HD} \times \widetilde{K}_{HF} = (4 \times 2,\ 6 \times 2,\ 10 \times 5,\ 8 \times 4,\ 12 \times 3,\ 7 \times 7,\ 11 \times 1) = (8,\ 12,\ 50, 32,\ 36,\ 49,\ 11)$$

$$\frac{\widetilde{K}_{HD}}{\widetilde{K}_{HF}} = \left(\frac{4}{2}, \frac{6}{2}, \frac{10}{5}, \frac{8}{4}, \frac{12}{3}, \frac{7}{7}, \frac{11}{1}\right) = (2,3,2,2,4,1,11)$$

2.1.3 Sub-interval Average method for heptagonal fuzzy number[(11)]

If $\widetilde{K}_H = \left(k_{h_1}, k_{h_2}, k_{h_3}, k_{h_4}, k_{h_5}, k_{h_6}, k_{h_7}\right)$ is supposed to be heptagonal fuzzy number, then the sub-interval average method formula is given as,

$$T_{V_{HEP}} = \frac{8\left(k_{h_1} + k_{h_2} + k_{h_3} + k_{h_4} + k_{h_5} + k_{h_6} + k_{h_7}\right)}{56}$$

2.2 Notations

$U_{VT}-$ Number of goods requests for each period
$S_{TB}-$ Cost for each item
$A_{BF}-$ Cost of ordering for each cycle
$C_{PG}-$ Cost of Storage for each period
$K_{TE}-$ Weight of the item
$K_{LM}-$ Basic costs for shipping items
$M_{ND}-$Variable costs for shipping items
$L_{GY}-$ Mileage from the supplier
$E_{FM}-$ Fuel consumption when the vehicle is empty
$S_{KU}-$ Additional fuel consumption per unit load of transportation
$G_{HC}-$ Standard emissions from the power plant

$B_{CX}-$ Carbon emission tax
$P_{QS}-$ Average carbon emissions per unit in storage
$G_{FK}-$ Cost of carbon emissions from vehicles
$S_{LB}-$ Additional costs of carbon emissions from transportation of one item
$H_{MS}-$ Environmental engineering services costs
${Q_q}^*-$ Optimal order quantity
$TC_{jk}-$ Total inventory cost
Q_q- Kristiyani et.al [6] optimal order quantity
TC_b- Kristiyani et.al [6] total inventory cost

2.2.1 Fuzzy parameters

$\widetilde{U}_{VT}$—Fuzzy Number of goods requests for each period
$\widetilde{S}_{TB}-$ Fuzzy cost for each item
$\widetilde{A}_{BF}$—Fuzzy cost of ordering for each cycle
$\widetilde{C}_{PG}$—Fuzzy cost of Storage for each period
$\widetilde{K}_{TE}$—Fuzzy weight of the item
$\widetilde{K}_{LM}$—Fuzzy Basic costs for shipping items
$\widetilde{M}_{ND}$—Fuzzy Variable costs for shipping items
$\widetilde{L}_{GY}-$ Fuzzy Mileage from the supplier
$\widetilde{G}_{HC}-$ Fuzzy Standard emissions from the power plant
$\widetilde{B}_{CX}-$ Fuzzy Carbon emission tax
$\widetilde{P}_{QS}-$ Fuzzy Average carbon emissions per unit in storage
$\widetilde{S}_{KU}-$ Fuzzy Additional fuel consumption per unit load of transportation
$\widetilde{S}_{LB}-$ Fuzzy Additional costs of carbon emissions from transportation of one item
$\widetilde{E}_{FM}-$ Fuzzy Fuel consumption when the vehicle is empty
$\widetilde{G}_{FK}-$ Fuzzy Cost of carbon emissions from vehicles
$\widetilde{H}_{MS}-$ Fuzzy Environmental Engineering services Costs
$T\widetilde{C}_{jk}$—Fuzzy total inventory cost
$\widetilde{Q}_q^*-$ Fuzzy optimal order quantity using the sub-interval average method
$S\left(T\widetilde{C}_{jk}\right)$—Fuzzy total inventory cost using the sub-interval average method

2.3 Assumptions

1. **Ordering and Purchasing expenses:** Cost of ordering, cost for each item, and number of goods requested per period are taken as uncertain due to the possibility of changes in market demand and customer needs.
2. **Storage expenses:** Storage cost per period is assumed to be uncertain due to the possibility of variations in warehouse capacity and external cost factors.
3. **Transportation expenses:** Base costs for shipping items, variable costs for shipping items, mileage from the supplier, weight of the item, additional fuel consumption per unit load of transportation, and fuel consumption for empty vehicles are taken as uncertain because there may be changes in product packaging, vehicle efficiency, and fuel rates.
4. **Emissions expenses**: Average carbon emissions per unit in storage, the cost of carbon emissions from vehicles, additional costs of carbon emissions from the transportation of one item, standard emissions from the power plant, and carbon emission tax are considered uncertain because there may be fluctuations in energy usage, transportation conditions, and regulatory policy changes.
5. **Engineering services expenses**: Environmental engineering services costs are considered uncertain due to the possibility of fluctuations in service rates, project demands, and regulatory requirements.
6. **Real life Application:** In the pharmaceutical sector, the cost of storage can vary, especially when specific temperature control is required for certain medications and vaccines. Similarly, transportation expenses can change based on the need for refrigerated shipping, such as for insulin or blood plasma. Emissions may vary based on the energy required for refrigerated storage units and the fuel consumed by refrigerated trucks during transport. Costs for environmental

engineering services can also change, particularly when specialized storage is required for expired drugs and the management of hazardous emission control.

2.4 Mathematical Model

In this section, we provide an overview of Kristiyani et.al[6] inventory model, followed by an explanation of our proposed sustainable fuzzy inventory model. The model developed by Kristiyani et.al[6] serves as the base reference, and we expand it by including environmental engineering services cost and a fuzzy solution approach.

2.4.1 Kristiyani et al[6] inventory model

In the Kristiyani et.al[6] model**,** the total inventory cost is calculated with the sum of purchasing expenses, ordering expenses, storage expenses, and transportation expenses. Carbon emission is considered in storage and transportation areas.

Purchasing expenses: This is determined by the cost for each item and the number of goods requested per period, $(S_{TB} \times U_{VT})$

Ordering expenses: These costs are related to placing an order and the number of goods requested per period, $\frac{(A_{BF} \times U_{VT})}{Q_q}$

Storage expenses: These are the expenses associated with maintaining storage facilities, including the average carbon emissions generated within the storage environment,

$$\frac{(Q_q \times C_{PG} \times S_{TB})}{2} + \frac{(Q_q \times P_{QS} \times G_{HC} \times B_{CX})}{2}$$

Transportation expenses: This is the total expenses for shipping goods, which include fuel consumption, item weight, mileage from the supplier, and carbon emissions from transport vehicles, given as,

$$\frac{U_{VT}}{Q_q}\left\{K_{LM} + (2 \times L_{GY} \times E_{FM} \times M_{ND}) + (L_{GY} \times S_{KU} \times K_{TE} \times Q_q \times M_{ND}) + (2 \times L_{GY} \times G_{FK}) + (L_{GY} \times S_{LB} \times Q_q)\right\}$$

Kristiyani et al.[6] model, total inventory cost is given below,

$$TC_b = \left[\begin{array}{l} (S_{TB} \times U_{VT}) + \dfrac{(A_{BF} \times U_{VT})}{Q_q} + \dfrac{(Q_q \times C_{PG} \times S_{TB})}{2} + \dfrac{(Q_q \times P_{QS} \times G_{HC} \times B_{CX})}{2} \\ + \dfrac{U_{VT}}{Q_q}\left\{\begin{array}{l} K_{LM} + (2 \times L_{GY} \times E_{FM} \times M_{ND}) + (L_{GY} \times S_{KU} \times K_{TE} \times Q_q \times M_{ND}) \\ + (2 \times L_{GY} \times G_{FK}) + (L_{GY} \times S_{LB} \times Q_q) \end{array}\right\} \end{array}\right] \tag{4.1}$$

By differentiating the Equation (4.1) with respect to Kristiyani et al. model, optimal order quantity,

$$Q_q = \sqrt{\frac{2 \times U_{VT}\left[(A_{BF} + K_{LM}) + (2 \times L_{GY} \times E_{FM} \times M_{ND}) + (2 \times L_{GY} \times G_{FK})\right]}{\left[(C_{PG} \times S_{TB}) + (P_{QS} \times G_{HC} \times B_{CX})\right]}} \tag{4.2}$$

2.4.2 Our model: Sustainable fuzzy inventory model

In Kristiyani et al.[6] model, emissions from storage and transportation were included in the overall inventory processes. In our model, we expanded it by adding the cost of environmental engineering services to mitigate carbon emissions. Moreover, we apply the sub-interval average method to handle the uncertainties in the cost parameters. In this model, total inventory cost can be calculated with the sum of purchasing expenses, ordering expenses, storage expenses, transportation expenses, and environmental engineering services expenses.

Environmental engineering services expenses: This is related to the costs for professional services for designing and executing sustainable practices, pollution prevention measures, and eco-friendly infrastructure solutions, expressed as, $\frac{(H_{MS} \times U_{VT})}{Q_q}$

The total inventory cost is given below,

$$TC_{jk} = \left[\begin{array}{l} (S_{TB} \times U_{VT}) + \dfrac{(A_{BF} \times U_{VT})}{Q_q} + \dfrac{(Q_q \times C_{PG} \times S_{TB})}{2} + \dfrac{(Q_q \times P_{QS} \times G_{HC} \times B_{CX})}{2} \\ + \dfrac{U_{VT}}{Q_q}\left\{\begin{array}{l} K_{LM} + (2 \times L_{GY} \times E_{FM} \times M_{ND}) + (L_{GY} \times S_{KU} \times K_{TE} \times Q_q \times M_{ND}) \\ + (2 \times L_{GY} \times G_{FK}) + (L_{GY} \times S_{LB} \times Q_q) \end{array}\right. \\ + \dfrac{(H_{MS} \times U_{VT})}{Q_q} \end{array}\right] \tag{4.3}$$

By differentiating the Equation (4.3) with respect to equating to zero, we can get theoptimal order quantity,

$$Q_q* = \sqrt{\frac{2 \times U_{VT}\left[(A_{BF} + K_{LM} + H_{MS}) + (2 \times L_{GY} \times E_{FM} \times M_{ND}) + (2 \times L_{GY} \times G_{FK})\right]}{\left[(C_{PG} \times S_{TB}) + (P_{QS} \times G_{HC} \times B_{CX})\right]}} \tag{4.4}$$

Fuzzy solution approach

Now, we consider the cost parameters as fuzzy, Then the fuzzy total inventory cost is

$$T\tilde{C}_{jk} = \left[\begin{array}{l} \left(\tilde{S}_{TB} \times \tilde{U}_{VT}\right) + \dfrac{\left(\tilde{A}_{BF} \times \tilde{U}_{VT}\right)}{Q_q} + \dfrac{\left(Q_q \times \tilde{C}_{PG} \times \tilde{S}_{TB}\right)}{2} + \dfrac{\left(Q_q \times \tilde{P}_{QS} \times \tilde{G}_{HC} \times \tilde{B}_{CX}\right)}{2} \\ + \dfrac{\tilde{U}_{VT}}{Q_q}\left\{\begin{array}{l} \tilde{K}_{LM} + \left(2 \times \tilde{L}_{GY} \times \tilde{E}_{FM} \times \tilde{M}_{ND}\right) + \left(\tilde{L}_{GY} \times \tilde{S}_{KU} \times \tilde{K}_{TE} \times Q_q \times \tilde{M}_{ND}\right) \\ + \left(2 \times \tilde{L}_{GY} \times \tilde{G}_{FK}\right) + \left(\tilde{L}_{GY} \times \tilde{S}_{LB} \times Q_q\right) \end{array}\right. \\ + \dfrac{\left(\tilde{H}_{MS} \times \tilde{U}_{VT}\right)}{Q_q} \end{array}\right] \tag{4.5}$$

Now, the cost parameters are expressed as heptagonal fuzzy numbers,

$$\widetilde{S}_{TB} = \left\{s_{tb_1}, s_{tb_2}, s_{tb_3}, s_{tb_4}, s_{tb_5}, s_{tb_6}, s_{tb_7}\right\}$$

$$\widetilde{A}_{BF} = \left\{a_{bf_1}, a_{bf_2}, a_{bf_3}, a_{bf_4}, a_{bf_5}, a_{bf_6}, a_{bf_7}\right\}$$

$$\widetilde{U}_{VT} = \left\{u_{vt_1}, u_{vt_2}, u_{vt_3}, u_{vt_4}, u_{vt_5}, u_{vt_6}, u_{vt_7}\right\}$$

$$\widetilde{K}_{LM} = \left\{k_{lm_1}, k_{lm_2}, k_{lm_3}, k_{lm_4}, k_{lm_5}, k_{lm_6}, k_{lm_7}\right\}$$

$$\widetilde{L}_{GY} = \left\{l_{gy_1}, l_{gy_2}, l_{gy_3}, l_{gy_4}, l_{gy_5}, l_{gy_6}, l_{gy_7}\right\}$$

$$\widetilde{M}_{ND} = \left\{m_{nd_1}, m_{nd_2}, m_{nd_3}, m_{nd_4}, m_{nd_5}, m_{nd_6}, m_{nd_7}\right\}$$

$$\widetilde{P}_{QS} = \left\{p_{qs_1}, p_{qs_2}, p_{qs_3}, p_{qs_4}, p_{qs_5}, p_{qs_6}, p_{qs_7}\right\}$$

$$\widetilde{G}_{HC} = \left\{g_{hc_1}, g_{hc_2}, g_{hc_3}, g_{hc_4}, g_{hc_5}, g_{hc_6}, g_{hc_7}\right\}$$

$$\widetilde{K}_{TE} = \left\{k_{te_1}, k_{te_2}, k_{te_3}, k_{te_4}, k_{te_5}, k_{te_6}, k_{te_7}\right\}$$

$$\widetilde{B}_{CX} = \left\{b_{cx_1}, b_{cx_2}, b_{cx_3}, b_{cx_4}, b_{cx_5}, b_{cx_6}, b_{cx_7}\right\}$$

$$\widetilde{C}_{PG} = \left\{c_{pg_1}, c_{pg_2}, c_{pg_3}, c_{pg_4}, c_{pg_5}, c_{pg_6}, c_{pg_7}\right\}$$

$$\widetilde{S}_{KU} = \left\{ s_{ku_1}, s_{ku_2}, s_{ku_3}, s_{ku_4}, s_{ku_5}, s_{ku_6}, s_{ku_7} \right\}$$

$$\widetilde{S}_{LB} = \left\{ s_{lb_1}, s_{lb_2}, s_{lb_3}, s_{lb_4}, s_{lb_5}, s_{lb_6}, s_{lb_7} \right\}$$

$$\widetilde{E}_{FM} = \left\{ e_{fm_1}, e_{fm_2}, e_{fm_3}, e_{fm_4}, e_{fm_5}, e_{fm_6}, e_{fm_7} \right\}$$

$$\widetilde{G}_{FK} = \left\{ g_{fk_1}, g_{fk_2}, g_{fk_3}, g_{fk_4}, g_{fk_5}, g_{fk_6}, g_{fk_7} \right\}$$

$$\widetilde{H}_{MS} = \left\{ h_{ms_1}, h_{ms_2}, h_{ms_3}, h_{ms_4}, h_{ms_5}, h_{ms_6}, h_{ms_7} \right\}$$

By substituting these heptagonal fuzzy cost parameters in equation Equation (4.5), we get Equation 4.6 which is given below,

$$T\tilde{C}_{jk} =$$
$$\left[\begin{array}{l}
\left(\begin{array}{l} \left(s_{tb_1} \times u_{vt_1}\right) + \frac{\left(a_{bf_1} \times u_{vt_1}\right)}{Q_q} + \frac{\left(Q_q \times c_{pg_1} \times s_{tb_1}\right)}{2} + \frac{\left(Q_q \times p_{qs_1} \times g_{hc_1} \times b_{cx_1}\right)}{2} + \frac{u_{vt_1}}{Q_q} \\ \left\{k_{lm_1} + \left(2 \times l_{gy_1} \times e_{fm_1} \times m_{nd_1}\right) + \left(l_{gy_1} \times s_{ku_1} \times k_{te_1} \times Q_q \times m_{nd_1}\right) + \left(2 \times l_{gy_1} \times g_{fk_1}\right) + \left(l_{gy_1} \times s_{lb_1} \times Q_q\right)\right\} + \frac{\left(h_{ms_1} \times u_{vt_1}\right)}{Q_q} \end{array}\right), \\
\left(\begin{array}{l} \left(s_{tb_2} \times u_{vt_2}\right) + \frac{\left(a_{bf_2} \times u_{vt_2}\right)}{Q_q} + \frac{\left(Q_q \times c_{pg_2} \times s_{tb_2}\right)}{2} + \frac{\left(Q_q \times p_{qs_2} \times g_{hc_2} \times b_{cx_2}\right)}{2} + \frac{u_{vt_2}}{Q_q} \\ \left\{k_{lm_2} + \left(2 \times l_{gy_2} \times e_{fm_2} \times m_{nd_2}\right) + \left(l_{gy_2} \times s_{ku_2} \times k_{te_2} \times Q_q \times m_{nd_1}\right) + \left(2 \times l_{gy_2} \times g_{fk_2}\right) + \left(l_{gy_2} \times s_{lb_2} \times Q_q\right)\right\} + \frac{\left(h_{ms_2} \times u_{vt_2}\right)}{Q_q} \end{array}\right), \\
\left(\begin{array}{l} \left(s_{tb_3} \times u_{vt_3}\right) + \frac{\left(a_{bf_3} \times u_{vt_3}\right)}{Q_q} + \frac{\left(Q_q \times c_{pg_3} \times s_{tb_3}\right)}{2} + \frac{\left(Q_q \times p_{qs_3} \times g_{hc_3} \times b_{cx_3}\right)}{2} + \frac{u_{vt_3}}{Q_q} \\ \left\{k_{lm_3} + \left(2 \times l_{gy_3} \times e_{fm_3} \times m_{nd_3}\right) + \left(l_{gy_3} \times s_{ku_3} \times k_{te_3} \times Q_q \times m_{nd_3}\right) + \left(2 \times l_{gy_3} \times g_{fk_3}\right) + \left(l_{gy_3} \times s_{lb_3} \times Q_q\right)\right\} + \frac{\left(h_{ms_3} \times u_{vt_3}\right)}{Q_q} \end{array}\right), \\
\left(\begin{array}{l} \left(s_{tb_4} \times u_{vt_4}\right) + \frac{\left(a_{bf_4} \times u_{vt_4}\right)}{Q_q} + \frac{\left(Q_q \times c_{pg_4} \times s_{tb_4}\right)}{2} + \frac{\left(Q_q \times p_{qs_4} \times g_{hc_4} \times b_{cx_4}\right)}{2} + \frac{u_{vt_4}}{Q_q} \\ \left\{k_{lm_4} + \left(2 \times l_{gy_4} \times e_{fm_4} \times m_{nd_4}\right) + \left(l_{gy_4} \times s_{ku_4} \times k_{te_4} \times Q_q \times m_{nd_4}\right) + \left(2 \times l_{gy_4} \times g_{fk_4}\right) + \left(l_{gy_4} \times s_{lb_4} \times Q_q\right)\right\} + \frac{\left(h_{ms_4} \times u_{vt_4}\right)}{Q_q} \end{array}\right), \\
\left(\begin{array}{l} \left(s_{tb_5} \times u_{vt_5}\right) + \frac{\left(a_{bf_5} \times u_{vt_5}\right)}{Q_q} + \frac{\left(Q_q \times c_{pg_5} \times s_{tb_5}\right)}{2} + \frac{\left(Q_q \times p_{qs_5} \times g_{hc_5} \times b_{cx_5}\right)}{2} + \frac{u_{vt_5}}{Q_q} \\ \left\{k_{lm_5} + \left(2 \times l_{gy_5} \times e_{fm_4} \times m_{nd_5}\right) + \left(l_{gy_5} \times s_{ku_5} \times k_{te_5} \times Q_q \times m_{nd_5}\right) + \left(2 \times l_{gy_5} \times g_{fk_5}\right) + \left(l_{gy_5} \times s_{lb_5} \times Q_q\right)\right\} + \frac{\left(h_{ms_5} \times u_{vt_5}\right)}{Q_q} \end{array}\right), \\
\left(\begin{array}{l} \left(s_{tb_6} \times u_{vt_6}\right) + \frac{\left(a_{bf_6} \times u_{vt_6}\right)}{Q_q} + \frac{\left(Q_q \times c_{pg_6} \times s_{tb_6}\right)}{2} + \frac{\left(Q_q \times p_{qs_6} \times g_{hc_6} \times b_{cx_6}\right)}{2} + \frac{u_{vt_6}}{Q_q} \\ \left\{k_{lm_6} + \left(2 \times l_{gy_6} \times e_{fm_6} \times m_{nd_6}\right) + \left(l_{gy_6} \times s_{ku_6} \times k_{te_6} \times Q_q \times m_{nd_6}\right) + \left(2 \times l_{gy_6} \times g_{fk_6}\right) + \left(l_{gy_6} \times s_{lb_6} \times Q_q\right)\right\} + \frac{\left(h_{ms_6} \times u_{vt_6}\right)}{Q_q} \end{array}\right), \\
\left(\begin{array}{l} \left(s_{tb_7} \times u_{vt_7}\right) + \frac{\left(a_{bf_7} \times u_{vt_7}\right)}{Q_q} + \frac{\left(Q_q \times c_{pg_7} \times s_{tb_7}\right)}{2} + \frac{\left(Q_q \times p_{qs_7} \times g_{hc_7} \times b_{cx_7}\right)}{2} + \frac{u_{vt_7}}{Q_q} \\ \left\{k_{lm_7} + \left(2 \times l_{gy_7} \times e_{fm_7} \times m_{nd_7}\right) + \left(l_{gy_7} \times s_{ku_7} \times k_{te_6} \times Q_q \times m_{nd_7}\right) + \left(2 \times l_{gy_7} \times g_{fk_7}\right) + \left(l_{gy_7} \times s_{lb_7} \times Q_q\right)\right\} + \frac{\left(h_{ms_7} \times u_{vt_7}\right)}{Q_q} \end{array}\right)
\end{array}\right]$$

—————– (4.6)

By applying the sub-interval average method formula [2.1.3], $T_{V_{HEP}} = \frac{8\left(k_{h_1}+k_{h_2}+k_{h_3}+k_{h_4}+k_{h_5}+k_{h_6}+k_{h_7}\right)}{56}$ in Equation 4.6, We get the fuzzy total inventory cost using the sub-interval average method, which is given below

$$S\left(T\tilde{C}_{jk}\right)=\frac{8}{56}\left[\begin{array}{l}
\left(\begin{array}{l}\left(s_{tb_1}\times u_{vt_1}\right)+\frac{\left(a_{bf_1}\times u_{vt_1}\right)}{Q_q}+\frac{\left(Q_q\times c_{ps_1}\times s_{tb_1}\right)}{2}+\frac{\left(Q_q\times p_{qs_1}\times g_{hc_1}\times b_{cx_1}\right)}{2}+\frac{u_{vt_1}}{Q_q}\\ \left\{k_{lm_1}+\left(2\times l_{gy_1}\times e_{fm_1}\times m_{nd_1}\right)+\left(l_{gy_1}\times s_{ku_1}\times k_{te_1}\times Q_q\times m_{nd_1}\right)+\left(2\times l_{gy_1}\times g_{fk_1}\right)+\left(l_{gy_1}\times s_{lb_1}\times Q_q\right)\right\}+\frac{\left(h_{ms_1}\times u_{vt_1}\right)}{Q_q}\end{array}\right)\\
+\left(\begin{array}{l}\left(s_{tb_2}\times u_{vt_2}\right)+\frac{\left(a_{bf_2}\times u_{vt_2}\right)}{Q_q}+\frac{\left(Q_q\times c_{ps_2}\times s_{tb_2}\right)}{2}+\frac{\left(Q_q\times p_{qs_2}\times g_{hc_2}\times b_{cx_2}\right)}{2}+\frac{u_{vt_2}}{Q_q}\\ \left\{k_{lm_2}+\left(2\times l_{gy_2}\times e_{fm_2}\times m_{nd_2}\right)+\left(l_{gy_2}\times s_{ku_2}\times k_{te_2}\times Q_q\times m_{nd_2}\right)+\left(2\times l_{gy_2}\times g_{fk_2}\right)+\left(l_{gy_2}\times s_{lb_2}\times Q_q\right)\right\}+\frac{\left(h_{ms_2}\times u_{vt_2}\right)}{Q_q}\end{array}\right)\\
+\left(\begin{array}{l}\left(s_{tb_3}\times u_{vt_3}\right)+\frac{\left(a_{bf_3}\times u_{vt_3}\right)}{Q_q}+\frac{\left(Q_q\times c_{ps_3}\times s_{tb_3}\right)}{2}+\frac{\left(Q_q\times p_{qs_3}\times g_{hc_3}\times b_{cx_3}\right)}{2}+\frac{u_{vt_3}}{Q_q}\\ \left\{k_{lm_3}+\left(2\times l_{gy_3}\times e_{fm_3}\times m_{nd_3}\right)+\left(l_{gy_3}\times s_{ku_3}\times k_{te_3}\times Q_q\times m_{nd_3}\right)+\left(2\times l_{gy_3}\times g_{fk_3}\right)+\left(l_{gy_3}\times s_{lb_3}\times Q_q\right)\right\}+\frac{\left(h_{ms_3}\times u_{vt_3}\right)}{Q_q}\end{array}\right)\\
+\left(\begin{array}{l}\left(s_{tb_4}\times u_{vt_4}\right)+\frac{\left(a_{bf_4}\times u_{vt_4}\right)}{Q_q}+\frac{\left(Q_q\times c_{ps_4}\times s_{tb_4}\right)}{2}+\frac{\left(Q_q\times p_{qs_4}\times g_{hc_4}\times b_{cx_4}\right)}{2}+\frac{u_{vt_4}}{Q_q}\\ \left\{k_{lm_4}+\left(2\times l_{gy_4}\times e_{fm_4}\times m_{nd_4}\right)+\left(l_{gy_4}\times s_{ku_4}\times k_{te_4}\times Q_q\times m_{nd_4}\right)+\left(2\times l_{gy_4}\times g_{fk_4}\right)+\left(l_{gy_4}\times s_{lb_4}\times Q_q\right)\right\}+\frac{\left(h_{ms_4}\times u_{vt_4}\right)}{Q_q}\end{array}\right)\\
+\left(\begin{array}{l}\left(s_{tb_5}\times u_{vt_5}\right)+\frac{\left(a_{bf_5}\times u_{vt_5}\right)}{Q_q}+\frac{\left(Q_q\times c_{ps_5}\times s_{tb_5}\right)}{2}+\frac{\left(Q_q\times p_{qs_5}\times g_{hc_5}\times b_{cx_5}\right)}{2}+\frac{u_{vt_5}}{Q_q}\\ \left\{k_{lm_5}+\left(2\times l_{gy_5}\times e_{fm_5}\times m_{nd_5}\right)+\left(l_{gy_5}\times s_{ku_5}\times k_{te_5}\times Q_q\times m_{nd_5}\right)+\left(2\times l_{gy_5}\times g_{fk_5}\right)+\left(l_{gy_5}\times s_{lb_5}\times Q_q\right)\right\}+\frac{\left(h_{ms_5}\times u_{vt_5}\right)}{Q_q}\end{array}\right)\\
+\left(\begin{array}{l}\left(s_{tb_6}\times u_{vt_6}\right)+\frac{\left(a_{bf_6}\times u_{vt_6}\right)}{Q_q}+\frac{\left(Q_q\times c_{ps_6}\times s_{tb_6}\right)}{2}+\frac{\left(Q_q\times p_{qs_6}\times g_{hc_6}\times b_{cx_6}\right)}{2}+\frac{u_{vt_6}}{Q_q}\\ \left\{k_{lm_6}+\left(2\times l_{gy_6}\times e_{fm_6}\times m_{nd_6}\right)+\left(l_{gy_6}\times s_{ku_6}\times k_{te_6}\times Q_q\times m_{nd_6}\right)+\left(2\times l_{gy_6}\times g_{fk_6}\right)+\left(l_{gy_6}\times s_{lb_6}\times Q_q\right)\right\}+\frac{\left(h_{ms_6}\times u_{vt_6}\right)}{Q_q}\end{array}\right)\\
+\left(\begin{array}{l}\left(s_{tb_7}\times u_{vt_7}\right)+\frac{\left(a_{bf_7}\times u_{vt_7}\right)}{Q_q}+\frac{\left(Q_q\times c_{ps_7}\times s_{tb_7}\right)}{2}+\frac{\left(Q_q\times p_{qs_7}\times g_{hc_7}\times b_{cx_7}\right)}{2}+\frac{u_{vt_7}}{Q_q}\\ \left\{k_{lm_7}+\left(2\times l_{gy_7}\times e_{fm_7}\times m_{nd_7}\right)+\left(l_{gy_7}\times s_{ku_7}\times k_{te_7}\times Q_q\times m_{nd_7}\right)+\left(2\times l_{gy_7}\times g_{fk_7}\right)+\left(l_{gy_7}\times s_{lb_7}\times Q_q\right)\right\}+\frac{\left(h_{ms_7}\times u_{vt_7}\right)}{Q_q}\end{array}\right)
\end{array}\right]$$

———— (4.7)

Now, we differentiate the Equation 4.7 with respect to Q_q and equating to zero,

$$\frac{\left[\begin{array}{l}
\left\{\begin{array}{l}\left(\begin{array}{l}\left(a_{bf_1}\times u_{vt_1}\right)+\left(a_{bf_2}\times u_{vt_2}\right)+\left(a_{bf_3}\times u_{vt_3}\right)+\\ \left(a_{bf_4}\times u_{vt_4}\right)+\left(a_{bf_5}\times u_{vt_5}\right)+\left(a_{bf_6}\times u_{vt_6}\right)+\left(a_{bf_7}\times u_{vt_7}\right)\end{array}\right)\\
+\left(\begin{array}{l}\left(u_{vt_1}\times k_{lm_1}\right)+\left(u_{vt_2}\times k_{lm_2}\right)+\left(u_{vt_3}\times k_{lm_3}\right)+\left(u_{vt_4}\times k_{lm_4}\right)\\ +\left(u_{vt_5}\times k_{lm_5}\right)+\left(u_{vt_6}\times k_{lm_6}\right)+\left(u_{vt_7}\times k_{lm_7}\right)\end{array}\right)\\
+\left(\begin{array}{l}\left(h_{ms_1}\times u_{vt_1}\right)+\left(h_{ms_2}\times u_{vt_2}\right)+\left(h_{ms_3}\times u_{vt_3}\right)+\\ \left(h_{ms_4}\times u_{vt_4}\right)+\left(h_{ms_5}\times u_{vt_5}\right)+\left(h_{ms_6}\times u_{vt_6}\right)+\left(h_{ms_7}\times u_{vt_7}\right)\end{array}\right)\end{array}\right\}\\
+2\left(\begin{array}{l}\left(u_{vt_1}\times l_{gy_1}\times m_{nd_1}\times e_{fm_1}\right)+\left(u_{vt_2}\times l_{gy_2}\times m_{nd_2}\times e_{fm_2}\right)+\\ \left(u_{vt_3}\times l_{gy_3}\times m_{nd_3}\times e_{fm_3}\right)+\left(u_{vt_4}\times l_{gy_4}\times m_{nd_4}\times e_{fm_4}\right)\\ +\left(u_{vt_5}\times l_{gy_5}\times m_{nd_5}\times e_{fm_5}\right)+\left(u_{vt_6}\times l_{gy_6}\times m_{nd_6}\times e_{fm_6}\right)+\\ \left(u_{vt_7}\times l_{gy_7}\times m_{nd_7}\times e_{fm_7}\right)\end{array}\right)\\
+2\left(\begin{array}{l}\left(u_{vt_1}\times l_{gy_1}\times g_{fk_1}\right)+\left(u_{vt_2}\times l_{gy_2}\times g_{fk_2}\right)+\\ \left(u_{vt_3}\times l_{gy_3}\times g_{fk_3}\right)+\left(u_{vt_4}\times l_{gy_4}\times g_{fk_4}\right)+\left(u_{vt_5}\times l_{gy_5}\times g_{fk_5}\right)\\ +\left(u_{vt_6}\times l_{gy_6}\times g_{fk_6}\right)+\left(u_{vt_7}\times l_{gy_7}\times g_{fk_7}\right)\end{array}\right)
\end{array}\right]}{Q_q^{\,2}}=\frac{\left[\begin{array}{l}\left(\begin{array}{l}\left(c_{ps_1}\times s_{tb_1}\right)\\ +\left(c_{ps_2}\times s_{tb_2}\right)\\ +\left(c_{ps_3}\times s_{tb_3}\right)\\ +\left(c_{ps_4}\times s_{tb_4}\right)\\ +\left(c_{ps_5}\times s_{tb_5}\right)\\ +\left(c_{ps_6}\times s_{tb_6}\right)\\ +\left(c_{ps_7}\times s_{tb_7}\right)\end{array}\right)\\
+\left(\begin{array}{l}\left(p_{qs_1}\times g_{hc_1}\times b_{cx_1}\right)\\ +\left(p_{qs_2}\times g_{hc_2}\times b_{cx_2}\right)\\ +\left(p_{qs_3}\times g_{hc_3}\times b_{cx_3}\right)\\ +\left(p_{qs_4}\times g_{hc_4}\times b_{cx_4}\right)\\ +\left(p_{qs_5}\times g_{hc_5}\times b_{cx_5}\right)\\ +\left(p_{qs_6}\times g_{hc_6}\times b_{cx_6}\right)\\ +\left(p_{qs_7}\times g_{hc_7}\times b_{cx_7}\right)\end{array}\right)\end{array}\right]}{2}$$

(4.8)

$$\tilde{Q}_q{}^{*}=\sqrt{\frac{2\left[\begin{array}{l}\left\{\begin{array}{l}\left(\left(a_{bf_1}\times u_{vt_1}\right)+\left(a_{bf_2}\times u_{vt_2}\right)+\left(a_{bf_3}\times u_{vt_3}\right)+\left(a_{bf_4}\times u_{vt_4}\right)+\left(a_{bf_5}\times u_{vt_5}\right)+\left(a_{bf_6}\times u_{vt_6}\right)+\left(a_{bf_7}\times u_{vt_7}\right)\right)\\+\left(\left(u_{vt_1}\times k_{lm_1}\right)+\left(u_{vt_2}\times k_{lm_2}\right)+\left(u_{vt_3}\times k_{lm_3}\right)+\left(u_{vt_4}\times k_{lm_4}\right)+\left(u_{vt_5}\times k_{lm_5}\right)+\left(u_{vt_6}\times k_{lm_6}\right)+\left(u_{vt_7}\times k_{lm_7}\right)\right)\\+\left(\left(h_{ms_1}\times u_{vt_1}\right)+\left(h_{ms_2}\times u_{vt_2}\right)+\left(h_{ms_3}\times u_{vt_3}\right)+\left(h_{ms_4}\times u_{vt_4}\right)+\left(h_{ms_5}\times u_{vt_5}\right)+\left(h_{ms_6}\times u_{vt_6}\right)+\left(h_{ms_7}\times u_{vt_7}\right)\right)\end{array}\right\}\\+2\left(\begin{array}{l}\left(u_{vt_1}\times l_{gy_1}\times m_{nd_1}\times e_{fm_1}\right)+\left(u_{vt_2}\times l_{gy_2}\times m_{nd_2}\times e_{fm_2}\right)+\left(u_{vt_3}\times l_{gy_3}\times m_{nd_3}\times e_{fm_3}\right)+\left(u_{vt_4}\times l_{gy_4}\times m_{nd_4}\times e_{fm_4}\right)\\+\left(u_{vt_5}\times l_{gy_5}\times m_{nd_5}\times e_{fm_5}\right)+\left(u_{vt_6}\times l_{gy_6}\times m_{nd_6}\times e_{fm_6}\right)+\left(u_{vt_7}\times l_{gy_7}\times m_{nd_7}\times e_{fm_7}\right)\end{array}\right)\\+2\left(\begin{array}{l}\left(u_{vt_1}\times l_{gy_1}\times g_{fk_1}\right)+\left(u_{vt_2}\times l_{gy_2}\times g_{fk_2}\right)+\left(u_{vt_3}\times l_{gy_3}\times g_{fk_3}\right)+\left(u_{vt_4}\times l_{gy_4}\times g_{fk_4}\right)+\left(u_{vt_5}\times l_{gy_5}\times g_{fk_5}\right)\\+\left(u_{vt_6}\times l_{gy_6}\times g_{fk_6}\right)+\left(u_{vt_7}\times l_{gy_7}\times g_{fk_7}\right)\end{array}\right)\end{array}\right]}{\left[\begin{array}{l}\left(\left(c_{ps_1}\times s_{tb_1}\right)+\left(c_{ps_2}\times s_{tb_2}\right)+\left(c_{ps_3}\times s_{tb_3}\right)+\left(c_{ps_4}\times s_{tb_4}\right)+\left(c_{ps_5}\times s_{tb_5}\right)+\left(c_{ps_6}\times s_{tb_6}\right)+\left(c_{ps_7}\times s_{tb_7}\right)\right)\\+\left(\begin{array}{l}\left(p_{qs_1}\times g_{hc_1}\times b_{cx_1}\right)+\left(p_{qs_2}\times g_{hc_2}\times b_{cx_2}\right)+\left(p_{qs_3}\times g_{hc_3}\times b_{cx_3}\right)+\left(p_{qs_4}\times g_{hc_4}\times b_{cx_4}\right)+\\\left(p_{qs_5}\times g_{hc_5}\times b_{cx_5}\right)+\left(p_{qs_6}\times g_{hc_6}\times b_{cx_6}\right)+\left(p_{qs_7}\times g_{hc_7}\times b_{cx_7}\right)\end{array}\right)\end{array}\right]}} \tag{4.9}$$

Equation (4.9) is the fuzzy optimal order quantity using the sub-interval average method.

3 Results and Discussion

3.1 Numerical example

A numerical example is given to illustrate the application of this proposed research work. The parameter data values are mainly adopted and modified from the studies of Katariya et al.[2] and Kristiyani et al.[6] inventory models. However, this methodology has not yet been tested with data from any specific industry or sector. Future empirical validation could involve applying the methodology to data from industries such as the electronics, pharmaceuticals, or textiles sectors.

3.1.1 Data values

$A_{BF}=$ \$ 500 / run, $U_{VT}=$ 10,000 unit/period, $S_{TB}=$ \$ 30/unit, $C_{PG}=$ \$ 0.8/period, $L_{GY}=$ 200 km $K_{TE}=$ 0.03 ton/ unit, $K_{LM}=$ \$ 90 / delivery, $M_{ND}=$ \$ 0.78 / L, $G_{HC}=0.0006$ ton CO_2 / kWh$S_{KU}=$ 0.75 L/ Truck load, $P_{QS}=$ 2.56 kWh / unit / period, $B_{CX}=$ \$ 150 / ton CO_2, $E_{FM}=$ 45 L/ km$G_{FK}=$ \$ 0.06265 / km, $S_{LB}=$ \$ 1.2115×10^{-5} / unit / km, $H_{MS}=$ \$ 600 / unit

3.1.2 Heptagonal fuzzy data values

Now, we consider all the parameter data values as uncertain, and they are represented as heptagonal fuzzy numbers.

$\widetilde{A}_{BF}=(493,495,498,500,501,502,503)$, $\widetilde{U}_{VT}=(10,000,$ 10,000, 10,000, 10,000, 10,000, 10,000, 10,000$)$

$\widetilde{G}_{HC}=(0.0003,$ 0.0004, 0.0005, 0.0006, 0.0007, 0.0008, 0.0009$)$, $\widetilde{S}_{KU}=(0.69,$ 0.71, 0.73, 0.75, 0.76, 0.77, 0.78$)$$\widetilde{P}_{QS}=$ (2.53, 2.54, 2.55, 2.56, 2.57, 2.58, 2.59)

$\widetilde{B}_{CX}=(147,$ 148, 149, 150, 151, 152, 153$)$, $\widetilde{C}_{PG}=(0.5,$ 0.6, 0.7, 0.8, 0.9, 0.95, 1.0$)$

$\widetilde{K}_{LM}=(84,$ 86, 88, 90, 91, 92, 93$)$, $\widetilde{M}_{ND}=(0.73,$ 0.75, 0.77, 0.78, 0.79, 0.80, 0.81$)$

$\widetilde{H}_{MS}=(594,$ 596, 598, 600, 601, 602, 603$)$, $\widetilde{E}_{FM}=(40,$ 41, 43, 45, 46, 47, 48$)$

$\widetilde{L}_{GY}=(194,$ 196, 198, 200, 201, 202, 203$)$, $\widetilde{K}_{TE}=(0.01,$ 0.015, 0.02, 0.03, 0.04, 0.05, 0.06$)$

$\widetilde{G}_{FK}=(0.06255,$ 0.06257, 0.06262, 0.06265, 0.06266, 0.06267, 0.06268$)$

$\tilde{S}_{LB}=\left(\begin{smallmatrix}1.2111\times 10^{-5},1.2112\times 10^{-5},1.2113\times 10^{-5},1.2114\times 10^{-5},\\1.2115\times 10^{-5},1.2116\times 10^{-5},1.2117\times 10^{-5}\end{smallmatrix}\right)$

3.2 Solution

Solution results from Kristiyani et al.[6] model and our proposed inventory model are provided below,

3.2.1 Kristiyani et al[6] inventory model solution

By substituting the data values of (3.1.1) in Equations (4.1) and (4.2) , we get

Kristiyani et al.[6] optimal order quantity $Q_q=$ 3477.99 units and Kristiyani et al.[6] total inventory cost $TC_b=$ \$ 419,259.57

3.2.2 Our inventory model solution

1. By substituting the data values of (3.1.1) in equations (4.3) and (4.4), we get

Optimal order quantity $Q_q{}^* =$ 3548.47 units and Total inventory cost $TC_{jk} =$ \$ 424173.60

2. By substituting the heptagonal fuzzy data values of (3.1.2) in Equations (4.7) and (4.9), we get,

Fuzzy total inventory cost using the sub-interval average method, $S\left(T\widetilde{C}_{jk}\right) =$ \$ 415648.71 and

Fuzzy optimal order quantity using the sub-interval average method, $\widetilde{Q}_q^* =$ 3533.81 units

3.3 Comparative Analysis

Based on the above numerical example solution results, we observe that in our inventory model, the total inventory cost value is $TC_{jk} =$ \$ 424173.60 and fuzzy total inventory cost using the sub-interval average method is $S\left(T\widetilde{C}_{jk}\right) =$ \$ 415648.71. So, we can clearly say that the total inventory cost value is reduced in the fuzzy solution approach.

Furthermore, the results of our proposed inventory model are compared with Kristiyani et al.[6] inventory model. In that, we can say that Kristiyani et al.[6] total inventory cost value is $TC_b =$ \$ 419,259.57and our model fuzzy total inventory cost using the sub-interval average method is $S\left(T\widetilde{C}_{jk}\right) =$ \$ 415648.71. From this, we can say that our model achieves 0.86% cost savings compared to the Kristiyani et al.[6] model.

Therefore, by using the sub-interval average method, we can optimize the total inventory cost, which is beneficial for enhancing financial profitability in inventory management. The table of comparative analysis is displayed below.

Table 1. Comparative Analysis Table

Models	Order quantity value	Total cost Value	Cost savings	Percentage of cost savings
Kristiyani et al.[6] Model	3477.99	419,259.57	—	—
Our model (Fuzzy)	3533.81	415648.71	\$ 3610.86	0.86

3.4 Sensitivity Analysis

In this section, sensitivity analyses for parameters such as fuzzy ordering costs $\widetilde{A}_{BF}$, fuzzy average carbon emissions per unit in storage $\widetilde{P}_{QS}$, fuzzy cost of carbon emissions from vehicles $\widetilde{G}_{FK}$, fuzzy environmental engineering services costs $\widetilde{H}_{MS}$, and the fuzzy optimal order quantity $\widetilde{Q}_q$ are calculated with -40%, -20%, and +20%, +40% changes in fuzzy optimal order quantity and fuzzy total inventory cost. Table 2 shows the sensitivity analysis results.

Table 2. Sensitivity Analysis table

Parameters	Changes	Fuzzy optimal order quantity	Fuzzy total inventory cost
(Fuzzy Ordering cost)	+40%	3557.38	416682.41
	+20%	3545.61	416166.77
	0	3533.81	415648.71
	-20%	3521.96	415127.79
	-40%	3510.08	414536.37
(Fuzzy Average carbon emission in storage)	+40%	3527.08	415676.39
	+20%	3530.44	415661.92
	0	3533.81	415648.71
	-20%	3537.29	415635.48
	-40%	3540.68	415351.41
(Fuzzy cost of carbon emission from vehicle)	+40%	3534.99	415700.27
	+20%	3534.40	415674.49
	0	3533.81	415648.71
	-20%	3533.22	415622.89
	-40%	3532.63	415597.12
(Fuzzy environmental engineering services cost)	+40%	3562.07	416885.42
	+20%	3547.97	416521.55
	0	3533.81	415648.71
	-20%	3519.59	415023.41
	-40%	3505.31	414394.31

Continued on next page

Table 2 continued

(Fuzzy order quantity)	+40%	4947.33	420599.63
	+20%	4240.57	416216.23
	0	3533.81	415648.71
	-20%	2827.05	417662.99
	-40%	2120.29	427335.60

Some of the key observations and managerial insights from the sensitivity analysis results are listed below.

1. An increase in fuzzy ordering costs increases total cost, therefore specific focus on the reduction of ordering expenses can result in substantial financial savings.
2. As fuzzy average carbon emissions per unit in storage increases and fuzzy total inventory cost reduces.Thissuggests that lowering emissionsfrom storage can result in costsavingsorder quantity optimization.
3. As the fuzzy cost of carbon emissions from vehicles decreases, total cost decrease. So, concentration on making small modifications to vehicle operations and fuel efficiency to optimize costs.
4. A decrease in fuzzy environmental engineering services costs reduces total cost. Therefore, thorough cost-benefit evaluation is important when including environmental services.
5. As the fuzzy optimal order quantity costs experience fluctuations, initially decreasing and then rising beyond a certain point. small changes in order quantity may significantly affect financial management. Soconcentrate onoptimize inventory. The relationship between cost, which is shown below.

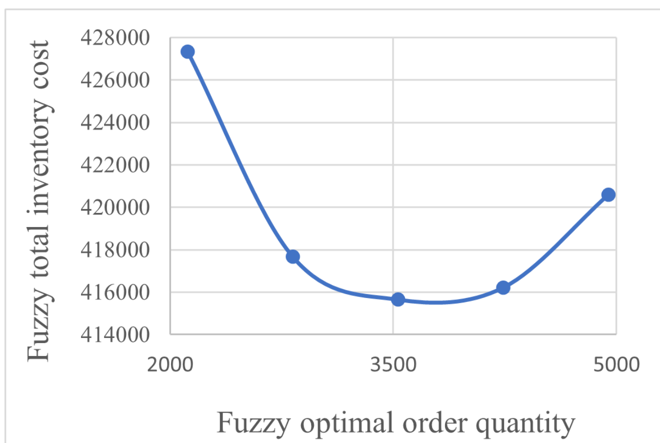

Fig 2. Convexity of the fuzzy optimum total inventory cost

4 Conclusion

Industries are facing enormous challenges in lowering carbon emissions due to their energy-intensive inventory processes. For this reason, this study included environmental engineering services costs to achieve emission reduction in inventory operations. Additionally, it applied the sub-interval average method to optimize the total cost of the inventory processes. Thus, this study contributes to economic efficiency and environmental responsibility in industrial inventory management. Moreover, this article only addresses direct emissions from inventory operations and does not consider the implications of indirect emissions from both upstream and downstream supply chain activities. Future research studies can be expanded with this model by including the indirect environmental impacts across the entire supply chain processes from sourcing to distribution networks with emission intensity analysis to provide a more comprehensive sustainability framework.

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