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ORIGINAL ARTICLE

A Closed-Loop Sustainable Production Inventory Model for Profit Optimization and Carbon Emission Control

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Abstract

Objectives: To analyze and justify the significance of Rework and Screening technology combined with green technology investments in production inventory model for maximizing profit, minimizing emissions and enhancing manufacturing sustainability. **Methods:** A Closed loop production inventory model is constructed under two methods namely (i) Model with Screening and Rework technology investment and (ii) Model based on Screening and Rework technology combined with Green technology investment. The proposed methods were implemented to evaluate their effectiveness in maximizing total profit, reducing carbon emissions and enhancing sustainability in production processes by using secondary data from Umakanta Mishra, Jei-Zheng Wu and Biswajit Sarkar's research. **Findings:** This study formulates a closed-loop sustainable production inventory model and comparative analysis has been made to study the effectiveness of the models under two categories namely (i) a model incorporating screening and rework technology investment and (ii) a model that integrates both screening and rework technology with green technology investment. A computational example is presented to show the efficiency of the models, and the results shows that the total profit and carbon emission of the system incorporating screening and rework technology is $222.812 and 46.5956 respectively whereas the total profit and carbon emission of the system integrates both screening and rework technology with green technology is $237.291 and 2.7946 respectively. The findings suggest that integrating screening and rework with green technology into a production-inventory model leads to significant improvements in economic performance through cost reduction, while also minimizing environmental impact, thereby enhancing overall sustainability. **Novelty:** Screening and rework combined with green technology investment are integrated into a production inventory model to optimize profit and emissions, enhancing sustainability and efficiency, yet unexplored in existing literature.

2020 Mathematics Subject Classification: 90B05.

Keywords: Closed-loop inventory model; Sustainable production; Profit optimization; Carbon emission control; Screening and Rework technology; Green Technology; Environmental sustainability

1 Introduction

Sustainable inventory management is a holistic approach to inventory management that takes into account the environmental, social and economic impacts of all inventory related decisions. It involves optimizing inventory levels to reduce waste, conserving resources, and supporting sustainable suppliers. Sustainable inventory management can be significantly strengthened through the incorporation of screening, rework procedures and environmentally friendly methods. By incorporating these methods with the right technology, companies can strike a balance between operational objectives and the preservation of the environment. Suchitra Pattnaik and Mitali Madhusmita Nayak reviewed a recent trend in sustainable inventory models.[1] Wakhid Ahmad Jauhari considered a sustainable inventory management for a closed-loop supply chain with energy usage, imperfect production, and green investment.[2] Seung Ho Yoo, Taesu Cheong developed an inventory model for sustainable operations of a closed-loop supply chain.[3] Technology enhances accuracy and efficiency in inventory operations by automating processes such as screening, reworking, and tracking which reduces human error, minimizes waste and supports faster, more sustainable decision making. These innovations collectively contribute to reduce waste, maximize profit, preserve resources and improve the inventory system's overall efficacy. Zsuzsanna Hauck, Boualem Rabta & Gerald Reiner analysed the screening decisions in inventory models with imperfect quality items.[4] Ata Allah Taleizadeh and Mahshid Yadegari constructed a production model of multiple products using a single machine under quality screening and reworking policies.[5] Screening technology plays a vital role in modern inventory and quality control systems by enabling the early identification and separate subpar, damaged, or flawed goods before they are put into storage or move on to the next phase of the supply chain. This guarantees that only quality assured items move through the supply chain while lowering needless processing, transportation and energy consumption. This proactive strategy improves entire product dependability and consumer satisfaction by ensuring that only quality-assured items were reach the customers. Additionally, by eliminating defective products early on, companies may drastically cut down on needless processing, packaging, shipping, and energy use, which helps them achieve their sustainability and cost cutting objectives. Simultaneously, rework technology is equally crucial for resource optimization. It enables the methodical examination, repair and restoration of rejected or flawed goods by using automated or semi-automatic methods. Rework systems assist in lowering production expenses and the ecological effects of producing goods through reducing the need for fresh raw materials and minimizing the number of further production cycles. The circular economy's emphasis on recycling, reuse, and repair is further supported by the incorporation of intelligent rework systems. These technologies contribute to the reduction of waste accumulation, the maintenance of constant quality standards, and the extension of the functional lifespan of items. Furthermore, the implementation of green technologies across the inventory lifecycle strengthens sustainability initiatives by increasing energy efficiency, lowering greenhouse gas emissions and bringing operations into compliance with environmental laws and corporate social responsibility pledges. Green technology offers substantial advantages by facilitating cleaner and more effective manufacturing methods through investments in cutting-edge machinery, unconventional energy sources, and the usage of green products. It promotes sustainable development by lowering carbon emissions per unit. The emission decrease can be scaled to meet industry demands with this investment. Umakanta Mishra, Jei-Zheng Wu and Biswajit Sarkar proposed a sustainable production inventory model for a controllable carbon emissions rate under shortages.[6] Abu Hashan Md Mashud developed a sustainable inventory model with controllable carbon emissions in green warehouse farms.[7] M. Palanivel constructed the comprehensive model incorporating price-sensitive stochastic demand, promotional strategies, rework, preservation technology, and carbon emission reduction.[8] Rahmi Yuniarti Model of multiperiod production distribution for closed-loop supply chain considering carbon emission and traceability for agri-food products.[9] Santsoh Shekhawat developed a profit optimization for a manufacturing supply chain under carbon emission and with inventory price based demand.[10] Hence, these technologies support ecological objectives and environmental requirements while enabling continuous surveillance, improved inventory control, and reduced surplus stock.

To the best of our knowledge, there is no existing literature that focuses on maximizing total profit and minimizing carbon emissions by incorporating screening and rework technology alongside green technology into a production inventory model. In contemporary manufacturing processes, striking a balance between economic goals and environmental sustainability is essential, particularly as businesses come under growing pressure to lower their carbon footprint. Existing research primarily focuses on environmental or profitability issues separately, or on particular aspects like production efficacy, emissions management, or quality assurance, without combining these variables into a single framework for decision-making. This highlights a research gap where cutting edge technologies like screening and rework, when combined with green activities, can be crucial to accomplishing the dual objectives of environmental and economic efficiency. In the context of global sustainability goals and changing regulatory environments constructing models which promote both economical and ecologically friendly production processes is crucial. Future studies can bridge this gap and build comprehensive solutions that support sustainable growth while simultaneously improving operational effectiveness. Therefore, research on production inventory models that integrate green technology, screening and rework technologies is crucial in order to optimize profit and reduce emissions at the same time. In this study, a closed loop sustainable production inventory model is developed through the integration of these technologies to fill the noted research gap and offer a unified framework for effective and ecologically conscious manufacturing.

This study aims to respond to the following research issues in light of the discussion above and to fill the identified research gap:

1. How can the integration of screening and rework technology with green technology investment in a production-inventory system maximize total profit while lowering carbon emissions?
2. What is the comparative impact of implementing only screening and rework technology versus combining it with green technology on the economic and environmental performance of a closed-loop sustainable production-inventory model?
3. How can the proposed model guide manufacturers in balancing economic objectives with environmental sustainability under carbon cap-and-trade policies?

Study	**Focus Area**	**Technology Considered**	**Environmental Aspect**	**Profit Optimization**	**Research Gap**
Mishra et al. (2020)	Sustainable production-inventory	Green investment only	Yes (carbon emissions)	Yes	Does not consider quality improvement (screening/rework)
Jauhari (2022)	Closed-loop supply chain	Energy usage & green investment	Yes	Limited	No focus on rework/screening
Hauck et al. (2021)	Imperfect quality inventory	Screening decisions	No	Partial	No green technology or sustainability analysis
Taleizadeh & Yadegari (2019)	Multi-product rework & screening	Screening & Rework	No	Yes	No environmental integration
Palanivel et al. (2024)	Stochastic demand & preservation	Rework & promotion	Partial	Yes	No integration of green technology
This Study (2025)	Closed-loop sustainable production	Screening + Rework + Green Tech	Yes (full integration)	Yes (maximized)	First integrated approach for dual optimization

2 Methodology

A closed loop sustainable production inventory model was developed with the integration of screening and rework technology combined with green technology. This study compared with two strategies namely (i) Model with Screening and Rework technology investment and (ii) Model based on Screening and Rework combined with Green technology investment. To demonstrate the efficiency of the proposed model, secondary data from the research of Umakanta Mishra, Jei-Zheng Wu and Biswajit Sarkar were utilized[6], and the proposed methods were implemented and evaluated to assess their effectiveness in minimizing carbon emissions and maximizing total profits. The notations, assumptions and mathematical formulations used to formulate the proposed models are detailed in Sections 2.1, 2.2 and 2.3.

2.1 Notations

The notation in this paper is as follows:

Decision Variables:

S_R: amount of capital investment on screening and rework technology

T: discrete cycle time (year), i.e.,$T = 1, 2, 3, 4$

Parameters:

r_c: reordering cost ($/unit)

g_l: goodwill loss resulting from unmet demand ($/unit)

h_c: holding cost ($/unit/year)

p: optimum production rate (unit/year)

p_c: production cost ($/unit)
α: percentage of time spent with backordered stock outs
S_c: setup cost ($/setup)
n: the product's sales expenses ($/unit)
s_p: scrap cost ($/unit)
d: total demand rate (unit/year)
G: investment on green technology
Emission parameters:
X: annual cap on carbon emissions (kg)
c_1: inventory related carbon emissions (kg/year)
c_2: carbon emissions due to obsolescence of inventory (kg/year)
U: cost of setup for a carbon emissions unit (kg/year)
β: carbon tax ($/kg)
y: emissions related to manufacturing (kg/year)
i: environmental impact of carbon emissions from inventory operations
m: effectiveness of greener technology in lowering emissions
ξ: proportion of carbon emissions after investments in green technology
m_1: effectiveness of screening and rework technology in lowering emissions
ξ_1: proportion of carbon emissions after investments in screening and rework technology
technology
γ: rate of obsolescence of inventory (%)
ρ: weight of the warehouse's obsolete inventory (kg/unit)
σ: space needed for every product unit (meters/unit)
Expressions:
CE: carbon emissions (kg/year)
M: optimum quantity of inventory (unit/year)
Q: production quantity (unit/year)
TP: total profit ($/year)
Other functions:
GTI: annual amount of capital invested in green technology (per year)
SP: selling price per year ($/year)
CA: annual cost of carbon permit ($/kg)
SCE: total setup cost related emissions (kg/setup/year)
EIC: total environmental effect of emissions for the inventory (kg/year)
HCE: inventory wide total emissions (kg/year)
OBSE: inventory related total obsolescence emissions (kg/year)
CPE: total manufacturing emissions (kg/year)
GLC: the entire cost of goodwill loss ($/year)
HC: total cost of holding ($/year)
OBS: total inventory obsolescence cost ($/year)
CP: total cost of production ($/year)
SC: total annual setup cost (per year)

2.2 Assumptions

1. This model takes into account a single type of item and all of the goods are delivered to customers via a single mode of delivery with a manufacturing cycle time (discrete cycle time T in year, i.e., $T = 1, 2, 3, 4$). Additionally, inventory durations are similar.
2. This method builds a deterministic model that allows a single type of things in order to estimate stock reserves based on identifying economic facts. As a result, this study has taken into account the constant rate of production and predictable demand.
3. This model is developed with no shortages.

4. Not every product made is of perfect quality; A specific quantity of the products produced are defective and need to be discovered and segregated before they are delivered to customers or the inventory.
5. The manufacturer intends to make investments in energy efficient machinery, unconventional energy sources, and other things in order to transition to a greener production system. Fraction of average emission decrease $F=\xi\left(1-e^{-mG}\right)$ where ξ, m, G are specified in the notation sections.
6. The firm makes investments in cutting-edge technologies for both the screening and rework processes to increase product quality and minimize the waste. Defective items are identified through screening and those items are reprocessed through rework technologies. Fraction of average emission decrease after investing in these technologies is given by $F=\xi_1\left(1-e^{-m_1 S_R}\right)$ where ξ_1, m_1, S_R are specified in the notation sections. The manufacturer can invest in advanced production technology to make it cleaner, more sustainable, and environmentally friendly. The incorporation of these technologies guarantees increased output and a sustainable manufacturing system by reducing waste and optimizing the recovery of defective products.

2.3 Mathematical Formulation

In this section, the Sustainable production quantity model without shortages has divided into two parts: (i) Model with Screening and Rework technology investment and (ii) Model based on Screening and Rework combined with Green technology investment. Companies have billed a dollar for each ton of greenhouse gas emissions they create since the carbon tax clearly sets a price on them, while a cap-and-trade system issues a certain number of emissions “allowances” annually. In secondary markets, these allowances can be sold to the highest bidder, which would raise the price. There are two components namely the cap-and-trader or carbon tax. These are employed to lower carbon emissions. In wealthy nations, either of these two policies can be implemented to lower carbon emissions. The producer allocating an initial cap for emissions and is allowed to sell or purchase the rights to emit in the cap. If the producer’s carbon emissions surpass the carbon quota, it must obtain additional carbon permits on the consent carbon trading marketplace to address the gap between its emissions and the cap. On the other hand, if the carbon emissions are lower than carbon cap, it might be permitted to trade its excess carbon allowance to other companies and sectors on the obligingness carbon trading market. Hence, the producer’s annual revenue for a single year is provided below under the carbon cap-tax function.

Annual total profit = Annual selling price – Annual total cost

The overall selling price of the inventory is obtained by selling the order quantities sold at the time (dfT) where f is the fraction period length in positive inventory level, i.e., fraction period length in positive inventory level, i.e., $0<f<1$, which is yearly selling price is $SP=\frac{dnT}{T}=dn$.

The entire annual cost is divided into two parts.

The first part is:

1. The production cost is computed using the order quantity made in time (dfT), which equals $CP=\frac{dp_cT}{T}=dp_c$.
2. The yearly fixed setup costs have been computed as $SC=\frac{S_c}{T}$.
3. The total holding cost is determined by multiplying the average inventories level in stock $\frac{dT(p-d)}{2p}$ by h_c. Consequently, the total holding cost is $HC=\frac{h_c dT(p-d)}{2p}$.
4. The cost includes getting rid of and gathering old or useless items from storage is $\left(n-s_p\right)$. Thus,$OBS=\frac{\gamma\left(n-s_p\right)dT(p-d)}{2p}$ is the aggregate obsolescence cost of inventory.

The Second part is:

1. The total expense of investing in green technology is determined by multiplying the number of cycles each year by the green technology investment cost (G), which can be expressed as $GTM=\frac{GT}{T}=G$.
2. The total expense of investing in screening & rework technology is determined by multiplying the number of cycles each year by the screening & rework technology investment cost (S_R), which can be expressed as $S_RTM=\frac{S_RT}{T}=S_R$.
3. The total carbon allowance cost is computed by multiplying the aggregate carbon tax (β) by the total carbon emissions cap (X), which can be expressed as $CA=\beta X$.
4. The carbon emission related setup cost is computed by multiplying the entire carbon tax (β). Thus, the overall emission related to setup costs is $SCEC=\frac{\beta U}{T}$.

5. The overall emission of holding costs is determined by multiplying the space needed for each product unit by σ, c_1, the average inventories level in stock $\frac{dT(p-d)}{2p}$, and the overall carbon tax (β). Thus, the entire emission of holding cost is $HCEC = \frac{\beta c_1 \sigma dT(p-d)}{2p}$.
6. Net emission of obsolescence cost is computed by averaged inventories level in stock $\frac{dT(p-d)}{2p}$ with multiplied by c_2, stock obsolescence rate (γ), obsolete inventory weight in the storage area (ρ) and overall carbon tax (β). Therefore,$OBSEC = \frac{\beta\gamma\rho c_2 dT(p-d)}{2p}$ is the entire emission of holding costs.
7. The overall carbon tax (β) multiplied by the manufacturing emissions y and total ordering quantity $\left(\frac{dT}{T}\right)$ yields the aggregate emission of production costs. Consequently, the total cost of production is $CPEC = \frac{\beta y dT}{T} = \beta yd$.
8. The net emission of production cost is determined by multiplying by the sum of the carbon tax (β), environmental effect of carbon emissions for stock i, and net ordering quantity $\left(\frac{dT}{T}\right)$. Hence, the overall emissions of environmental impact expense of inventory are $EICS = \frac{\beta i dT}{T} = \beta id$.

Thus, the overall cost of carbon emissions is $\beta\left(\frac{U}{T} + c_1 M + \gamma\rho c_2 M + (y+i)d\right)$, where $CE = \left(\frac{U}{T} + c_1 M + \gamma\rho c_2 M + (y+i)d\right)$, $M = \frac{dT(p-d)}{2p}$.

a) Model with Screening and Rework technology investment:

Therefore, the producer's annual profit under a carbon cap-and-tax policy with screening & rework technology investment is given as follows:

$$\begin{aligned} TP &= SP - CP - SC - HC\left(1-\xi_1\left(1-e^{-m_1 S_R}\right)\right) - OBS\left(1-\xi_1\left(1-e^{-m_1 S_R}\right)\right) - S_R TM \\ &+ \beta\left[X - \left(SCE + HCE\left(1-\xi_1\left(1-e^{-m_1 S_R}\right)\right) + OBSE\left(1-\xi_1\left(1-e^{-m_1 S_R}\right)\right)\right.\right. \\ &+ \left.\left.(CPE + EIC)\left(1-\xi_1\left(1-e^{-m_1 S_R}\right)\right)\right)\right] \end{aligned} \tag{1}$$

where all the terms in the above equations which are multiplied with $\left(1-\xi_1\left(1-e^{-m_1 S_R}\right)\right)$ is the reduction of emissions and costs after investment of screening & rework technology S_R. The price of the emissions is $\beta\left[X - \left(SCE + HCE\left(1-\xi_1\left(1-e^{-m_1 S_R}\right)\right) + OBSE\left(1-\xi_1\left(1-e^{-m_1 S_R}\right)\right) + (CPE+EIC)\left(1-\xi_1\left(1-e^{-m_1 S_R}\right)\right)\right)\right]$

The producer's carbon emissions are just below the emission cap X when $X - \left(SCE + HCE\left(1-\xi_1\left(1-e^{-m_1 S_R}\right)\right) + OBSE\left(1-\xi_1\left(1-\right.\right.\right.$
0

Thus, the producer can make money by selling the permit. The manufacturer's emissions are higher than the emission cap X when $X - \left(SCE + HCE\left(1-\xi_1\left(1-e^{-m_1 S_R}\right)\right) + OBSE\left(1-\xi_1\left(1-e^{-m_1 S_R}\right)\right) + (CPE+EIC)\left(1-\xi_1\left(1-e^{-m_1 S_R}\right)\right)\right) <$
0

Hence, the manufacturer must purchase a permit, which comes at a price. Therefore, the total profit can be expressed as follows:

$$\begin{aligned} TP(T,S_R) &= d(n-p_c) - \frac{S_c}{T} - h_c M\left(1-\xi_1\left(1-e^{-m_1 S_R}\right)\right) - \gamma\left(n-s_p\right)M\left(1-\xi_1\left(1-e^{-m_1 S_R}\right)\right) \\ &- S_R + \beta\left(X - \left(\frac{U}{T} + c_1 M\left(1-\xi_1\left(1-e^{-m_1 S_R}\right)\right) + \gamma\rho c_2 M\left(1-\xi_1\left(1-e^{-m_1 S_R}\right)\right)\right.\right. \\ &+ \left.\left.(y+i)\left(1-\xi_1\left(1-e^{-m_1 S_R}\right)\right)\right)\right) \end{aligned} \tag{2}$$

Substituting $M = \frac{dT(p-d)}{2p}$ in **eq. (2)**, the total profit is

$$\begin{aligned} TP(T,S_R) &= d(n-p_c) - \frac{S_c}{T} - \frac{h_c dT(p-d)}{2p}\left(1-\xi_1\left(1-e^{-m_1 S_R}\right)\right) \\ &- \frac{\gamma\left(n-s_p\right)dT(p-d)}{2p}\left(1-\xi_1\left(1-e^{-m_1 S_R}\right)\right) \\ &- S_R + \beta\left(X - \left(\frac{U}{T} + \frac{c_1\sigma dT(p-d)}{2p}\left(1-\xi_1\left(1-e^{-m_1 S_R}\right)\right) + \frac{\gamma\rho c_2 dT(p-d)}{2p}\right.\right. \\ &\left.\left.\left(1-\xi_1\left(1-e^{-m_1 S_R}\right)\right) + (y+i)\left(1-\xi_1\left(1-e^{-m_1 S_R}\right)\right)\right)\right) \end{aligned} \tag{3}$$

Now, to determine the solution of T and S_R by maximizing **eq. (3)** in the next step. To find T and S_R first to show the concavity of **eq.(3)** with respect to T and S_R.

Theorem 1. For any positive value of S_R, the profit function $TP(T,S_R)$ expressed in **eq.(3)** is concave in T.

Theorem 2. Considering any discrete value of T, the profit function $TP(T,S_R)$ expressed in **eq.3** is concave in S_R, then the optimum solution of S_R can be found from $\frac{\partial TP(T,S_R)}{\partial S_R}=0$.

The carbon emission CE can be calculated as follows,

$$CE=\left(\frac{U}{T}+\frac{c_1\sigma dT(p-d)}{2p}\left(e^{-m_1S_R}\right)+\frac{\gamma\rho c_2 dT(p-d)}{2p}\left(e^{-m_1S_R}\right)+(y+i)\left(e^{-m_1S_R}\right)\right) \tag{4}$$

The optimum quantity of inventory M can be computed by

$$M=\frac{dT(p-d)}{p} \tag{5}$$

$$Q=dT \tag{6}$$

b) Model based on Screening and Rework technology combined with Green technology investment:

The producer's annual profit under a carbon cap-and-tax policy with screening & rework technology combined with green technology investment is given as follows:

$$\begin{aligned} TP \;=\;& SP-CP-SC-HC\left(1-\xi_1\left(1-e^{-m_1S_R}\right)\right)-OBS\left(1-\xi_1\left(1-e^{-m_1S_R}\right)\right)\\ -\;& S_RTM-GTM+\beta\left[X-\left(SCE+HCE\left(1-\xi_1\left(1-e^{-m_1S_R}\right)\right)+OBSE\right.\right.\\ & \left.\left.\left(1-\xi_1\left(1-e^{-m_1S_R}\right)\right)+(CPE+EIC)\left(1-\xi_1\left(1-e^{-m_1S_R}\right)\right)\right)\times\left(1-\xi\left(1-e^{-mG}\right)\right)\right]\end{aligned} \tag{7}$$

where all the terms in the above equations which are multiplied with $\left(1-\xi_1\left(1-e^{-m_1S_R}\right)\right)$ is the reduction of emissions and costs after investment of screening & rework technology S_R and the terms which are multiplied with $\left(1-\xi\left(1-e^{-mG}\right)\right)$ is the reduction of emissions after investment of green technology G. Therefore, the total profit can be expressed as follows:

$$\begin{aligned} TP(T,S_R) \;=\;& d(n-p_c)-\frac{S_c}{T}-h_cM\left(1-\xi_1\left(1-e^{-m_1S_R}\right)\right)-\gamma\left(n-s_p\right)M\left(1-\xi_1\left(1-e^{-m_1S_R}\right)\right)\\ -\;& S_R-G+\beta\left[X-\left(\frac{U}{T}+c_1M\left(1-\xi_1\left(1-e^{-m_1S_R}\right)\right)+\gamma\rho c_2M\left(1-\xi_1\left(1-e^{-m_1S_R}\right)\right)\right.\right.\\ +\;& \left.\left.(y+i)\left(1-\xi_1\left(1-e^{-m_1S_R}\right)\right)\right)\times\left(1-\xi\left(1-e^{-mG}\right)\right)\right]\end{aligned} \tag{8}$$

Substituting $M=\frac{dT(p-d)}{2p}$ in **eq. (8)**, the total profit is computed by

$$\begin{aligned} TP(T,S_R) \;=\;& d(n-p_c)-\frac{S_c}{T}-\frac{h_cdT(p-d)}{2p}\left(1-\xi_1\left(1-e^{-m_1S_R}\right)\right)\\ -\;& \frac{\gamma\left(n-s_p\right)dT(p-d)}{2p}\left(1-\xi_1\left(1-e^{-m_1S_R}\right)\right)-S_R-G+\beta\\ & \left[X-\left(\frac{U}{T}+\frac{c_1\sigma dT(p-d)}{2p}\left(1-\xi_1\left(1-e^{-m_1S_R}\right)\right)+\frac{\gamma\rho c_2dT(p-d)}{2p}\right.\right.\\ & \left.\left.\left(1-\xi_1\left(1-e^{-m_1S_R}\right)\right)+(y+i)\left(1-\xi_1\left(1-e^{-m_1S_R}\right)\right)\right)\times\left(1-\xi\left(1-e^{-mG}\right)\right)\right]\end{aligned} \tag{9}$$

Now, to determine the solution of T and S_R by maximizing **eq. (9)** in the next step. To find T and S_R first to show the concavity of **eq.(9)** with respect to T and S_R.

The Carbon emission CE for this proposed model can be calculated by

$$\begin{aligned} CE \;=\;& \left(\frac{U}{T}\left(e^{-mG}\right)+\frac{c_1\sigma dT(p-d)}{2p}\left(e^{-mG}\right)\left(e^{-m_1S_R}\right)\right.\\ +\;& \left.\frac{\gamma\rho c_2dT(p-d)}{2p}\left(e^{-mG}\right)\left(e^{-m_1S_R}\right)+(y+i)\left(e^{-mG}\right)\left(e^{-m_1S_R}\right)\right)\end{aligned} \tag{10}$$

The optimum quantity of inventory M and production quantity Q can be determined using **equations (5) and (6)**, respectively.

3 Result and discussion

There is a lack of literature focused on optimizing total profit, carbon emissions by integrating both screening and rework technology with green technology investment in a production inventory model. Most existing studies tend to examine these components independently, either on environmentally friendly practices or on quality control techniques without examining their combined impact on both economic and environmental outcomes through appropriate technological implementation. This highlights a substantial research gap as the relation between quality improvement through screening & rework technology and carbon reduction via green technology remains underexplored. This study aims to fill that gap by developing a comprehensive production inventory model that incorporates screening and rework technology in conjunction with green technology investment, with the goal of optimizing profit while simultaneously reducing carbon emissions.

3.1 Numerical Example:

Numerical examples were performed to demonstrate the model's effectiveness, using parameters from Umakanta Mishra, Jei-Zheng Wu and Biswajit Sarkar's research [6] are given below:

$$\begin{aligned} n &= \$40/unit, s_p = \$2/unit, p_c = \$6/unit, X = 900kg/year, y = 40kg/year, i = 60kg/year, \beta = \$0.33/kg, \\ p &= 6unit/year, d = 3unit/year, S_c = \$80/setup/year, \gamma = 0.6, U = 60kg/year, \rho = 0.2kg/unit, \\ \sigma &= 0.4meters/units, c_1 = 4kg/year, c_2 = 3kg/year, \xi = 0.2unit, m = 0.8unit, \xi_1 = 0.15unit, m_1 = 0.85unit \end{aligned}$$

3.2 Optimal solution and comparative study:

Example 1:

The obtained optimum solutions for the model with screening & rework technology investment are presented in [Table 1].

Table 1. Optimum solutions for the model with Screening & Rework technology investment

With Screening & Rework technology investment					
T	$\mathbf{S_R}^*$	**TP***	**CE***	**M***	**Q***
1	3.21983	191.881	79.5273	1.5	3
2	**3.41694**	**222.812**	**46.5956**	**3**	**6**
3	3.58571	220.504	34.4475	4.5	9
4	3.73328	209.901	27.8059	6	12

Example 2:

The obtained optimum solutions for the model with screening & rework technology combined with green technology investment are presented in [Table 2].

Table 2. Optimum solutions for the model with Screening & Rework technology combined with Green technology investment

With Screening & Rework technology combined with Green technology investment					
T	$\mathbf{S_R}^*$	**TP***	**CE***	**M***	**Q***
1	3.02069	208.173	4.3556	1.5	3
2	**3.24994**	**237.291**	**2.7946**	**3**	**6**
3	3.44171	234.419	2.1209	4.5	9
4	3.60656	223.568	1.7272	6	12

Example 3:

The existing optimum solution of the model, incorporating only green technology investment and assuming no shortages, as reported in [6], is presented below.

$$T = 2, G^* = 3.58340, TP^* = 223.07, CE^* = 18.9394, I^* = 3, Q^* = 6$$

The existing optimum solution of the model without any investment and with the consideration of no shortages, as reported in [6], is presented below.

$$T = 2, TP^{*} = 205.93, CE^{*} = 332.94, I^{*} = 3, Q^{*} = 6$$

The above numerical examples show the efficiency of the suggested paradigm, and the results highlight the novelty of the work, demonstrating that the integration of green technology with screening and rework technologies significantly improves system performance yielding a higher total profit and drastically reducing carbon emissions, a combination not addressed together in existing literature.

3.3 Managerial Insights:

The findings of this research have a number of applications for managers and decision-makers in supply chain and manufacturing settings who want to attain sustainability and profitability.

1. **Integrated Technology Investments Drive Dual Benefits:** The study shows that investing in screening, rework, and green technologies at the same time produces better results in terms of reducing carbon emissions and maximizing profits. Managers are urged to see these investments as complementing tactics that increase system efficiency as a whole rather than as distinct projects.
2. **Optimal Investment Levels Are Critical:** Sensitivity analysis shows that whereas technological investments greatly enhance economic and environmental performance, excessive investment may result in decreasing benefits. The best amount of money to spend on screening, rework, and green technology should be carefully considered by managers in order to maintain cost-effectiveness without sacrificing sustainability objectives.
3. **Compliance with Environmental Regulations:** With the growing global emphasis on carbon taxation and emission caps, the model gives managers with a quantitative tool to evaluate the financial impact of carbon-related regulations. It enables proactive decision-making to optimize regulatory costs while maintaining competitiveness.
4. **Quality Improvement as a Sustainability Lever:** The study emphasizes how crucial it is to incorporate quality improvement procedures like screening and rework into a larger sustainability plan. These solutions not only improve customer happiness but also make a substantial contribution to environmental conservation by lowering the number of defective products and the waste that goes along with them.
5. **Strategic Flexibility through Scenario Analysis:** Managers can evaluate a variety of scenarios using the closed-loop model, such as alternative carbon tax rates, demand levels, and investment capacity. This flexibility supports better preparedness for market fluctuations, policy changes, and sustainability requirements.

By adopting the proposed integrated production-inventory approach, firms can align their operational strategies with long-term sustainability objectives, enhance resource efficiency, and strengthen their market position in an increasingly eco-conscious business landscape.

4 Conclusion

This study highlights the significance of integrating screening and rework technologies with green technology investments in order to maximize overall profit and carbon emissions in an environmentally friendly production inventory system while existing literature has examined them separately, concentrating on environmental sustainability, quality assurance, or profit maximization. This research fills that gap by developing an integrated model that makes use of technological interventions to improve product quality, eliminate waste and lessen environmental impact, all while enhancing economic outcomes. The suggested model takes into account a closed-loop production system that includes reworking defective things, using green technologies to decrease carbon footprints. Hence a closed loop production inventory model is constructed under two methods namely (i) Model with Screening and Rework technology investment and (ii) Model based on Screening and Rework technology combined with Green technology investment. The model's efficacy is demonstrated by a computational example, which shows that combining these technologies results in lower emissions and higher overall profit when compared to traditional systems. Results shows that the total profit and carbon emission of the system incorporating screening and rework technology is \$222.812 and 46.5956 respectively whereas the total profit and carbon emission of the system integrates both screening and rework technology with green technology is \$237.291 and 2.7946 respectively, underscoring the dual benefits of economic efficiency and environmental responsibility. However, this study has certain limitations that offer opportunities for future research. First, the model is developed under the assumption of no shortages, which may not fully reflect real world market dynamics where demand fluctuations and supply disruptions are common. Second, the model assumes deterministic demand and production rates, without considering variability or uncertainty that could arise in practice. Third, the model focuses on a single product type, extending this to multi-item inventory systems could increase its practical relevance. Additionally, the carbon emission factors are treated as fixed and may vary in real-life applications depending on energy sources, production processes, and regulatory changes. Future research could explore the adaptability of this model across different industry types and scale levels. The prospectus of this study includes incorporating shortages into the production-inventory model, as it is vital for representing realistic market conditions where demand may exceed supply. This extension would enhance the model's applicability by enabling more effective decision-making in scenarios involving backorders, lost sales, and customer satisfaction. Therefore, future research could be directed towards extending the model by incorporating the possibility of shortages. The novelty of this study lies in its dual optimization strategy, which strikes a balance between environmental responsibility and economic performance, and it provides a practical pathway for industries aiming to align with global sustainability norms, while maintaining competitiveness.

References

1) Pattnaik S, Nayak MM, Abbate S, Centobelli P. Recent trends in sustainable inventory models: A literature review. *Sustainability*. 2021 Oct;13(21):11756. https://doi.org/10.3390/su132111756.
2) Jauhari WA. Sustainable inventory management for a closed-loop supply chain with energy usage, imperfect production, and green investment. *Cleaner Logistics and Supply Chain*. 2022 Jul;4:100055. https://doi.org/10.1016/j.clscn.2022.100055.
3) Yoo SH, Cheong T. Inventory model for sustainable operations of a closed-loop supply chain: Role of a third-party refurbisher. *Journal of Cleaner Production*. 2021 Sep;315:127810. https://doi.org/10.1016/j.jclepro.2021.127810.
4) Hauck Z, Rabta B, Reiner G. Analysis of screening decisions in inventory models with imperfect quality items. *International Journal of Production Research*. 2021 Nov;59(21):6528–6543. https://doi.org/10.1080/00207543.2020.1818862.
5) Taleizadeh AA, Yadegari M, Sana SS. Production models of multiple products using a single machine under quality screening and reworking policies. *Journal of Modelling in Management*. 2019 Feb;14(1):232–259. https://doi.org/10.1108/JM2-06-2018-0086.
6) Mishra U, Wu JZ, Sarkar B. A sustainable production-inventory model for a controllable carbon emissions rate under shortages. *Journal of Cleaner Production*. 2020 May;256:120268. https://doi.org/10.1016/j.jclepro.2020.120268.
7) Mashud AH, Pervin M, Mishra U, Daryanto Y, Tseng ML, Lim MK. A sustainable inventory model with controllable carbon emissions in green-warehouse farms. *Journal of Cleaner Production*. 2021 May;298:126777. https://doi.org/10.1016/j.jclepro.2021.126777.
8) Palanivel M, Venkadesh M, Vetriselvi S, Sundararajan R. Dynamic Optimization of Inventory: A Comprehensive Model Incorporating Price-Sensitive Stochastic Demand, Promotional Strategies, Rework, Preservation Technology, and Carbon Emission Reduction. *Discrete Dynamics in Nature and Society*. 2024;2024(1):5444880. https://doi.org/10.1155/2024/5444880.
9) Yuniarti R, Masudin I, Rusdiansyah A, Handayani DI. Model of multiperiod production-distribution for closed-loop supply chain considering carbon emission and traceability for agri-food products. *International Journal of Industrial Engineering and Operations Management*. 2023 Aug;5(3):240–263. https://doi.org/10.1108/IJIEOM-10-2022-0045.
10) Shekhawat S, Rathore H, Sharma K. Profit Optimization for a Manufacturing Supply Chain Under Carbon Emission and with Inventory-Price Based Demand. *Brazilian Journal of Operations & Production Management*. 2023 Apr;20(2):1471. https://doi.org/10.14488/BJOPM.1471.2023.