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The Stochastic Reliability Modeling of a Centrifugal Pump of a Rectifier Cooling System of DC-Rectiformer with a Standby Unit and Preventive Maintenance

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Abstract

Objectives: To perform a stochastic reliability analysis of a standby Rectifier Cooling System in the aluminum industry, with the aim of evaluating system performance metrics such as Mean Time to System Failure (MTSF), availability, and busy period in the context of an energy-intensive industrial setting. **Methods**: The study employs Weibull distribution to model failure and repair rates of system components, considering two types of repair personnel (regular and expert). The semi-markov process and regenerative point techniques are applied to derive key performance measures. The system, including DC-rectiformers, pumps, and cooling mechanisms, is analyzed under assumed operational data through numerical and graphical illustrations. **Findings**: The analysis yields quantitative insights into system reliability, showing how repair personnel skill level affects system availability and MTSF. The study highlights the role of standby pump operation in maintaining cooling system efficiency, with performance metrics indicating potential improvements in system uptime under optimized conditions. **Novelty**: This work provides a unique application of stochastic modeling techniques, specifically Semi-Markov and Weibull-based reliability analysis, to the Rectifier Cooling System in an aluminum production setting. It also introduces a comparative evaluation of system availability based on repair personnel expertise, which is not commonly addressed in prior reliability studies within this industry.

2020 AMS Classification: 90B25, 60K10

Keywords: DC-Rectiformer; Rectifier Cooling System; Preventive Maintenence; MTSF; Weibull distribution

1 Introduction

The electrolysis process of alumina requires an uninterrupted supply of electricity. Since it takes around three days to resume the electrolysis process when the temperature of the aluminum metal in the pots drops and causes it to freeze, a power outage lasting longer than an hour is never acceptable. In the current period of industrial expansion, it is increasingly challenging to maintain the maximum degree of efficiency and the fewest possible dangers. In order to address these issues, reliability technologies may be essential. The system's capacity to function effectively under predetermined circumstances for a certain period of time is the reliability metric. Preventative maintenance (PM), on the other hand, is typically chosen to avoid failures at times that have significant financial consequences. The main objective of planned PM is to return equipment to as good as new condition. The systems' PM is needed to maintain their effectiveness and maybe reduce the failure rate after a set period of time. Reliability research has made considerable use of the Weibull distribution because of its versatility in replicating the failure times distribution. Researchers have successfully included the Weibull distribution into several reliability models, enhancing the model's ability to predict failure and maintenance intervals. Within this framework, centrifugal pumps used in rectifier cooling systems play a crucial role. These pumps circulate deionized water through a closed loop to maintain optimal operating temperatures of the rectifiers. The failure of a cooling pump not only risks overheating and equipment failure but can trigger cascading effects that compromise the entire power delivery system. Therefore, improving the reliability and availability of such subsystems is essential for uninterrupted operation. The performance degradation, reliability modeling, and maintenance techniques of such systems have been the subject of extensive research over the years. A strong framework for comprehending the failure behavior of centrifugal pumps under dynamic operating environments has been developed using predictive maintenance frameworks, such as those put forth by Pedersen and Vatn [1], and stochastic degradation modeling techniques, as investigated by Chen et al. [2]. Further advancements have been made in the stochastic modeling of industrial systems that operate under varying maintenance and replacement policies. Promila and Bhatia [3] examined a real-world case in a petrochemical complex, emphasizing the role of high-maintenance policies in enhancing pump reliability. Similar studies by Sharma and Tewari [4] and Saw et al. [5] assessed maintenance priorities and profit analysis in repairable systems, incorporating preventive measures and endurance time into standby configurations. Furthermore, studies by Kadyan et al. [6] and Hussien and Sherbeny [7] expanded the analysis to include cold standby and multi-unit systems, taking warranty-based repair methods and concurrent operations into consideration.

Large-scale industrial facilities as in R.Kaur et al. [8] conducted reliability and profit analysis of a metal treatment station, considering the effects of waiting time for standby MTS. Despite these developments, there remains a significant research gap in understanding the effect of waiting time for standby units on the overall reliability and profit of systems. Most existing models either assume instantaneous availability of standby units or overlook the cost and time implications of delayed activation. Moreover, while common cause failures and complex dependencies have been addressed in nuclear and energy systems by Wang et al. [9], such probabilistic interdependencies are rarely examined in the context of industrial environments. Also Omah I et al. [10] have evaluated the economic performance and dependability of systems with standby redundancy and preparation time.

However, the majority of current models either use exponential assumptions or overlook practical industrial issues like Weibull-based failure/repair dynamics, priority-based repair persons task, and patience time before expert intervention. These mistakes make it more difficult for the models to effectively depict system behavior in high-risk settings, such as the manufacture of alumina. Furthermore, cooling pump systems in DC-rectiformer configurations components that are operationally essential but underrepresented in reliability literature have received less attention.

The current study creates a Weibull-based stochastic model of a centrifugal pump-driven rectifier cooling system used in the manufacture of alumina in order to fill up these gaps. Regular and expert personnel are involved in maintenance activities for the system, which consists of one working pump and one standby pump. A patience time system is implemented to reflect realistic delays in escalated repair circumstances, and failure modes are classified into minor and major categories. In order to help with with operational flexibility and preventative maintenance planning in crucial industrial settings, this analysis attempts to evaluate important performance metrics such as system availability, profit, and dependability.

2 Methodology

The research includes system with standby with the purpose of standby systems is to ensure the continuity of essential services during power outages or failures. Pump failures, caused by mechanical or electrical issues, can disrupt operations and impact safety, productivity, and financial performance. To mitigate these effects, the study considers a system influenced by minor and major failures, with repairs handled by regular and expert personnel. Preventive maintenance, performed on a calendar or usage-based schedule, is a critical component of the model. This approach minimizes equipment failures, increases system uptime, and enhances long-term asset management. Following repairs, the system resumes normal operations. Each random variable is assumed to be statistically independent, as system performance variations over time introduce randomness in failure and repair patterns. As a result, their failure is not necessarily constantly distributed. Therefore, the Weibull distribution is adopted due to its versatility in modeling the increasing or decreasing failure rates typically observed in real-world systems.

2.1 Assumptions:

- Regular repair personnel are available for any breakdown.
- Major failures result in complete system failure, while minor failures allow the system to continue operating in a reduced capacity or in normal working condition.
- In cases where regular repair personnel are unable to manage the situation, the concept of patience time is applied, after which expert personnel are called in to address the issue for major failures.
- Although repairs have been made in both partially and completely failed states, the system can fail from a partially failed state.
- Preventive maintenance is conducted at regular intervals to prevent failures and prolong asset life.
- The frequency of preventive maintenance depends on factors such as system usage, manufacturer recommendations, and the operating environment.
- Failure and repair rates follow the Weibull distribution.

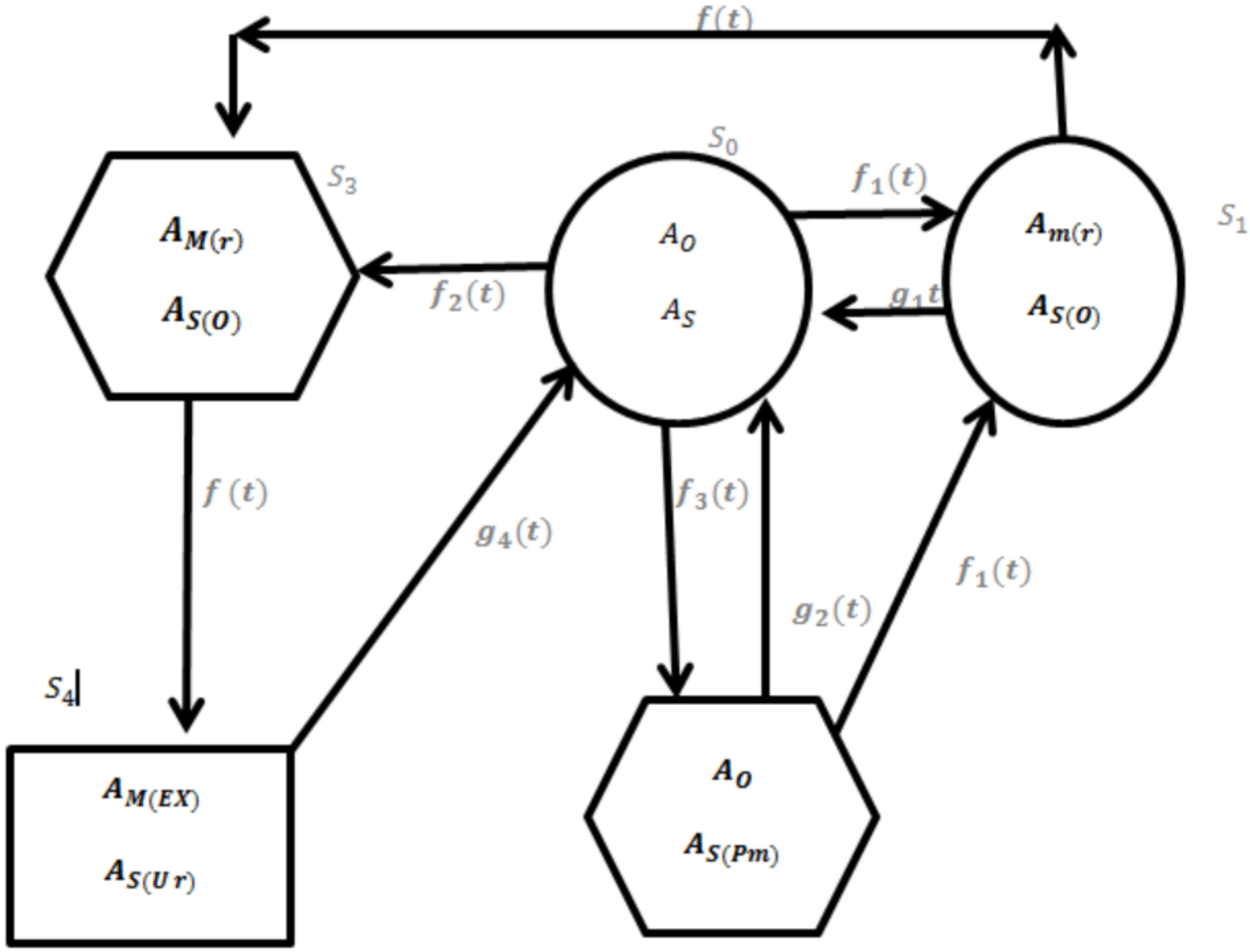

Fig 1. State Transition Diagram

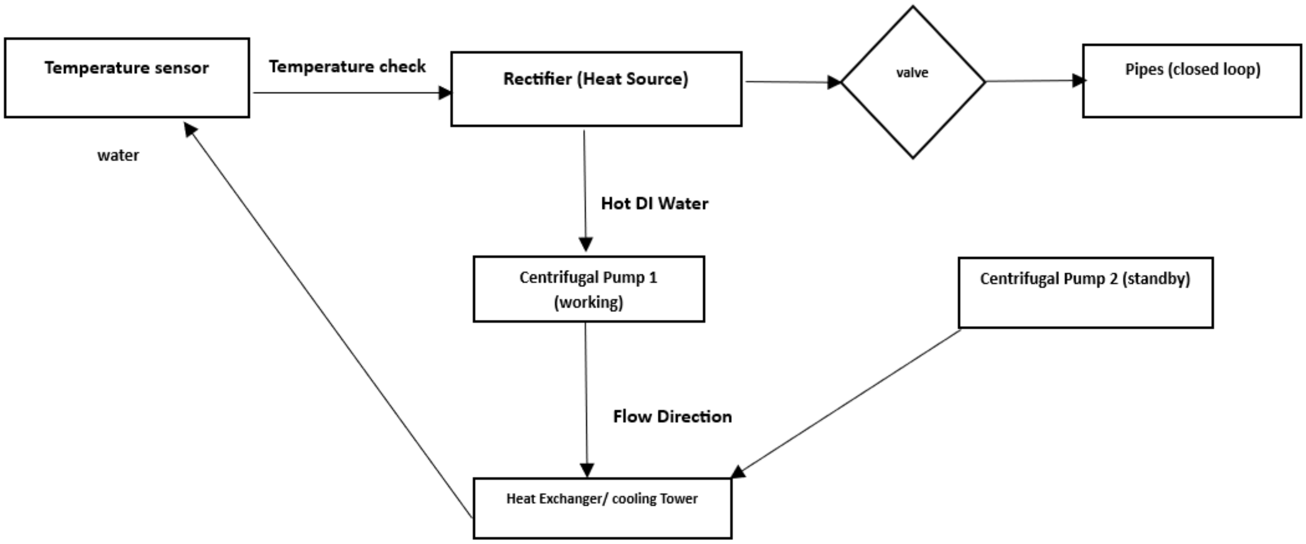

Fig 2. Technical schematic of a rectifier cooling system

2.2 Reliability Analysis of the System:

2.2.1. Transition Probabilities and Mean Sojourn Time

Transition probabilities describe the likelihood of moving between states in a system, while the mean sojourn time represents the average time spent in a specific state before transitioning to another. Using probabilistic principles, the steady-state transition probabilities for the system are derived as follows:

$$P_{ij} = Q_{ij}(\infty) = \int q_{ij}(t)dtas \tag{1}$$

Substituting the relevant failure and repair rates:

$$Q_{01}(t) = \int_0^\infty f_1(t)\overline{F_2(t)F_3(t)}dt = \int_0^\infty \alpha\eta t^{\eta-1}e^{-(\alpha t^\eta)}.e^{-(\beta t^\eta)}e^{-(\gamma t^\eta)}dt \tag{2}$$

This simplifies to:

$$= \alpha\int_0^\infty \eta t^{\eta-1}.e^{-(\alpha+\beta+\gamma).t^\eta}dt \tag{3}$$

$$= \frac{\alpha}{\alpha+\beta+\gamma} \tag{4}$$

$$Q_{02}(t) = \int_0^\infty f_2(t)\overline{F_1(t)F_3(t)}dt = \int_0^\infty \beta\eta t^{n-1}e^{-(\beta t^\eta)}.e^{-(\alpha t^\eta)}e^{-(\gamma t^\eta)}dt \tag{5}$$

$$Q_{03}(t) = \int_0^\infty f_3(t)\overline{F_1(t)F_2(t)}dt = \int_0^\infty \gamma\eta t^{n-1}e^{-(\gamma t^\eta)}.e^{-(\alpha t^\eta)}e^{-(\beta t^\eta)}dt \tag{6}$$

$$P_{01}+P_{02}+P_{03} = \frac{\alpha}{\alpha+\beta+\gamma}+\frac{\beta}{\alpha+\beta+\gamma}+\frac{\gamma}{\alpha+\beta+\gamma} = 1 \tag{7}$$

$$Q_{10}(t) = \int_0^\infty g_1(t)\overline{F(t)}dt = \int_0^\infty \theta\eta t^{n-1}e^{-(\theta t^\eta)}.e^{-(kt^\eta)}dt \tag{8}$$

$$P_{10} = \frac{\theta}{\theta + k} \tag{9}$$

$$Q_{13}(t) = \int_0^\infty f(t).\overline{g_1(t)}dt = \int_0^\infty k\eta t^{n-1} e^{-(kt^\eta)}.e^{-(\theta t^\eta)}dt \tag{10}$$

$$P_{12} = \frac{k}{k + \theta} \tag{11}$$

$$p_{10} + p_{13} = 1 \tag{12}$$

$$Q_{20}(t) = \int_0^\infty g_2(t).\overline{f_1(t)}dt = \int_0^\infty l\eta t^{n-1} e^{-(lt^\eta)} e^{-(\alpha t^\eta)}dt \tag{13}$$

$$P_{20} = \frac{l}{l + \alpha} \tag{14}$$

$$Q_{21}(t) = \int_0^\infty f_1(t).\overline{G_2(t)}dt = \int_0^\infty \alpha\eta t^{n-1} e^{-(\alpha t^\eta)} e^{-(lt^\eta)}dt \tag{15}$$

$$P_{21} = \frac{\alpha}{l + \alpha} \tag{16}$$

$$p_{20} + p_{21} = 1 \tag{17}$$

$$Q_{34}(t) = \int_0^\infty f(t)dt = \int_0^\infty k\eta t^{n-1} e^{-(kt^\eta)}dt \tag{18}$$

$$P_{34} = 1 \tag{19}$$

$$Q_{40}(t) = \int_0^\infty g_4(t)dt = \int_0^\infty h\eta t^{n-1} e^{-(ht^\eta)}dt \tag{20}$$

$$P_{40} = 1 \tag{21}$$

If T denotes the time until system failure, the mean sojourn time (μ_i) or the average duration spent in state S_i , is calculated as:

$$(\mu_i) = E(t) = \int_0^\infty P[T > t]dt \tag{22}$$

Consequently, the average duration of stay (mean sojourn times) (μ_i) at regenerative states S_i is:

$$\mu_0(t) = \int_0^\infty \overline{F_1(t)F_2(t)F_3(t)}dt \tag{23}$$

$$= \frac{\Gamma(1+\frac{1}{\eta})}{(\alpha+\beta+\gamma)^{\frac{1}{\eta}}} \tag{24}$$

$$\mu_1(t) = \int_0^\infty \overline{G_1(t)F(t)}dt = \frac{\Gamma(1+\frac{1}{\eta})}{(\theta+k)^{\frac{1}{\eta}}} \tag{25}$$

$$\mu_2(t) = \int_0^\infty \overline{G_2(t)f_1(t}dt = \frac{\Gamma(1+\frac{1}{\eta})}{(l+\alpha)^{\frac{1}{\eta}}} \tag{26}$$

$$\mu_3(t) = \int_0^\infty \overline{F(t)}dt = \frac{\Gamma(1+\frac{1}{\eta})}{(k)^{\frac{1}{\eta}}} \tag{27}$$

$$\mu_4(t) = \int_0^\infty \overline{G_4(t)}dt = \frac{\Gamma(1+\frac{1}{\eta})}{(h)^{\frac{1}{\eta}}} \tag{28}$$

2.2.2. Mean Time to System Failure

Let $\pi_i(t)$ represent the cumulative distribution function (c.d.f.) of the first passage time from the regenerative state S_i to a failed state. Treating the failed state as an absorbing state, the recursive relations for $\pi_i(t)$ are given as:©

$$\pi_i(t) = \sum_j Q_{ij}(t)\,\pi_j(t) + \sum_k Q_{i,k}(t) \tag{29}$$

$$\pi_0(t) = Q_{01}(t)\,\pi_1(t) + Q_{02}(t) + Q_{03}(t) \tag{30}$$

$$\pi_1(t) = Q_{10}(t)\,\pi_0(t) + Q_{13}(t) \tag{31}$$

For the above equation, we are using the Laplace Steiltjes transform:

$$(\pi_0^{**}, \pi_1^{**}) = Q^{**(-1)}(Q_{02}^{**} + Q_{03}^{**}, Q_{13}^{**}) \tag{32}$$

where,

$$Q^{**(-1)} = \begin{bmatrix} 1 & -Q_{01}^{**} & -Q_{02}^{**} & -Q_{03}^{**} & 0 \\ -Q_{10}^{**} & 1 & 0 & -Q_{13}^{**} & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \pi_0^{**} \\ \pi_1^{**} \\ \pi_2^{**} \\ \pi_3^{**} \\ \pi_4^{**} \end{bmatrix} = \begin{bmatrix} Q_{02}^{**} + Q_{03}^{**} \\ Q_{13}^{**} \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{33}$$

By solving the preceding equation for $\pi_0^{**}(s)$, we derive the following:

$$\pi_0^{**}(s) = \frac{N(s)}{D(s)} \tag{34}$$

where;

$$D(S) = \begin{vmatrix} -1 & Q_{01}^{**} & Q_{02}^{**} & Q_{03}^{**} & 0 \\ Q_{10}^{**} & -1 & 0 & Q_{13}^{**} & 0 \\ 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & -1 \end{vmatrix} \tag{35}$$

and

$$N(S) = \begin{vmatrix} Q_{02}+Q_{03} & Q_{01}^{**} & Q_{02}^{**} & Q_{03}^{**} & 0 \\ Q_{13} & -1 & 0 & Q_{13}^{**} & 0 \\ 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & -1 \end{vmatrix} \tag{36}$$

Therefore,

$$D = 1 - p_{01}p_{10}. \tag{37}$$

and,

$$N = \mu_0 + \mu_1 p_{01}. \tag{38}$$

Taking the inverse Laplace transform of (34) we can obtain the reliability of the system, when the system starts from state S_0. The Mean Time to System Failure (MTSF) is:

$$MTSF = \lim_{s\to 0} \frac{1-\phi_0^{**}(s)}{s} = \frac{D'(0)-N'(0)}{D(0)} = \frac{N}{D} \tag{39}$$

$$E(T) = \frac{\mu_0 + \mu_1 p_{01}}{1 - p_{01}p_{10}} \tag{40}$$

2.2.3. Availability

The availability of a system, denoted as $A_i(t)$, refers to the probability that the system is operational (in an up-state) and capable of providing service at a given time 't', given that it entered the regenerative state S_i at t = 0. The recursive relations for $A_i(t)$ are as follows:

$$A_0(t) = M_0(t) + q_{01}(t)\ A_1(t) + q_{02}(t)\ A_2(t) + q_{03}(t)\ A_3(t) \tag{41}$$

$$A_1(t) = M_1(t) + q_{10}(t)\ A_0(t) + q_{13}(t)\ A_1(t) \tag{42}$$

$$A_2(t) = q_{20}(t)\ A_0(t) + q_{21}(t)\ A_1(t) \tag{43}$$

$$A_3(t) = q_{34}(t)\ A_4(t) \tag{44}$$

$$A_4(t) = q_{40}(t)\ A_0(t) \tag{45}$$

$M_i(t)$ is the probability that the system is up initially in state $S_i \in E$ at time 't' without visiting to any other regenerative state.
Applying the Laplace transformation to the above relations **[41-45]**, we obtain:

$$A_0^*(s) = \frac{N_1(s)}{D_1(s)}. \tag{46}$$

Where

$$D_1(S) = \begin{vmatrix} -1 & Q_{01}^{**} & Q_{02}^{**} & Q_{03}^{**} & 0 \\ Q_{10}^{**} & -1 & 0 & Q_{13}^{**} & 0 \\ Q_{20} & Q_{21} & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 & Q_{34} \\ Q_{40} & 0 & 0 & 0 & -1 \end{vmatrix} \text{ and } N_1(S) = \begin{vmatrix} M_0 & Q_{01}^{**} & Q_{02}^{**} & Q_{03}^{**} & 0 \\ M_1 & -1 & 0 & Q_{13}^{**} & 0 \\ 0 & Q_{21} & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 & Q_{34} \\ 0 & 0 & 0 & 0 & -1 \end{vmatrix}$$

$$\text{therefore, } N_1 = \mu_0 + \mu_1 p_{02} + \mu_1 p_{02} p_{21} \tag{47}$$

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2.2.4. Busy Period of Regular Repair Personnel(due to Minor Failure)

Suppose the system transitions into state S_i at t=0, Let $B_i(t)$ represent the probability that the system is engaged in repairing the unit at a specific time 't'. The steady-state condition, where the regular repair personnel is occupied with repairs, is expressed by the equation

$$B_0 = \lim_{s\to 0} sB_0^*(s) \tag{48}$$

$$= \frac{N_3(s)}{D_1(s)} \tag{49}$$

$$B_0(t) = q_{01}(t)\ B_1(t) + q_{02}(t)\ B_2(t) + q_{03}(t)\ B_3(t) \tag{50}$$

$$B_1(t) = u_1(t) + q_{10}(t)\ B_0(t) + q_{13}(t)\ B_3(t) \tag{51}$$

$$B_2(t) = u_2(t) + q_{20}(t)\ B_0(t) + q_{21}(t)\ B_1(t) \tag{52}$$

$$B_3(t) = u_3(t) + q_{34}(t)\ B_4(t) \tag{53}$$

$$B_4(t) = q_{40}(t)\ B_0(t) \tag{54}$$

The probability that the server is engaged in repair operations at state Si without switching to other regenerative states or returning back to the same state via one or more non-regenerative states is represented by Ui(t). Taking the LT of **equations [50–53]**, we can now solve for $B_0^*(s)$.

Therefore,

$$N_3(S) = \begin{vmatrix} 0 & q_{01}^* & q_{02}^* & q_{03}^* & 0 \\ u_1 & -1 & 0 & q_{13} & 0 \\ u_2 & q_{21} & -1 & 0 & 0 \\ u_3 & 0 & 0 & -1 & q_{34} \\ 0 & 0 & 0 & 0 & -1 \end{vmatrix} \tag{55}$$

$$N_3 = \mu_1 p_{01} + \mu_2 p_{02} + \mu_1 p_{02} p_{21} + \mu_3 p_{03} + \mu_3 p_{01} p_{13} + \mu_3 p_{02} p_{13} p_{21} \tag{56}$$

and D_1 is already mentioned.

2.2.5. Preventive Maintenance by Regular repair personnel

Let $P_i(t)$ denote the probability that the system is undergoing preventive maintenance performed by a regular repair personnel at time 't' By solving the equations derived for $P_0^*(s)$, the busy period of the repair facility due to preventive maintenance can be expressed as:-

$$P_0^* = \lim_{s\to 0} sP_0^*(s) \tag{57}$$

$$= \frac{N_4(s)}{D_1(s)} \tag{58}$$

$$P_0(t) = q_{01}(t)\ P_1(t) + q_{02}(t)\ P_2(t) + q_{03}(t)\ P_3(t) \tag{59}$$

$$P_1(t) = q_{10}(t)\ P_0(t) + q_{13}(t)\ P_3(t) \tag{60}$$

$$P_2(t) = w_2(t) + q_{20}(t)\ P_0(t) + q_{21}(t)\ P_1(t) \tag{61}$$

$$P_3(t) = q_{34}(t)\ P_4(t) \tag{62}$$

$$P_4(t) = q_{40}(t)\ P_0(t) \tag{63}$$

Wi(t) is the probability that the server will remain in the state Si until time 't' without switching to any other regenerative state. Now, find $P_0^*(s)$ by finding the laplace transform of **equations [59–63]**. The following shows the period of time the server is occupied with preventive maintenance:

$$P_0^* = \frac{N_4(s)}{D_1(s)}$$

Where,

$$N_4 = \mu_2 p_{02} \qquad \text{and } D_1 \text{ is already mentioned.} \tag{64}$$

2.2.6. Busy Period of Expert Repair Person (due to Major Failure)

Let $E_i(t)$ represent the probability that the repair facility is occupied with repairing the unit at a specific time 't', given that the system entered state S_i at t=0.

The steady-state condition, where the expert repair facility remains engaged in repairs, is expressed as:

$$E_0 = \lim_{s\to 0} sE_0^*(s) \tag{65}$$

$$= \frac{N_5(s)}{D_1(s)} \tag{66}$$

$$E_0(t) = q_{01}(t)\, E_1(t) + q_{02}(t)\, E_2(t) + q_{03}(t)\, E_3(t) \tag{67}$$

$$E_1(t) = u_1(t) + q_{10}(t)\, E_0(t) + q_{13}(t)\, E_3(t) \tag{68}$$

$$E_2(t) = u_2(t) + q_{20}(t)\, E_0(t) + q_{21}(t)\, E_1(t) \tag{69}$$

$$E_3(t) = u_3(t) + q_{34}(t)\, E_4(t) \tag{70}$$

$$E_4(t) = w_4 + q_{40}(t)\, E_0(t) \tag{71}$$

The probability that the server is engaged in repair operations at state Si without switching to other regenerative states or returning back to the same state via one or more non-regenerative states is given by Ui(t) and Wi(t).

Therefore,

$$N_5(S) = \begin{vmatrix} 0 & q_{01}^* & q_{02}^* & q_{03}^* & 0 \\ 0 & -1 & 0 & q_{13} & 0 \\ 0 & q_{21} & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 & q_{34} \\ w_4 & 0 & 0 & 0 & -1 \end{vmatrix} \tag{72}$$

Taking Laplace transform of (67-71) and computing the relevant elements of the inverse matrix, the Laplace transform of $E_0(t)$ is : $E_0 = \frac{N_5(s)}{D_1(s)}$

$$\text{Where, } N_5 = \mu_4(p_{02}p_{21}p_{13} + p_{01}p_{13} + p_{03}) \tag{73}$$

2.3 Case studies and discussions :

(I) When the shape parameter $\eta = 0.5$, the unit's failure can be attributed to abrupt, wear-out, or intermittent failures. Under these conditions, the preventive maintenance and the repair time distribution handled by regular and expert repair personnel simplify significantly.

$f_3(t) = \gamma\eta t^{\eta-1} e^{-(\gamma t^{\eta})}dt = \frac{\gamma}{2\sqrt{t}} e^{(-\beta\sqrt{t})}$, $f(t) = \frac{K}{2\sqrt{t}} e^{(-K\sqrt{t})}$, $f_1(t) = \frac{\alpha}{2\sqrt{t}} e^{(-\alpha\sqrt{t})}$, $f_2(t) = \frac{\beta}{2\sqrt{t}} e^{(-\beta\sqrt{t})}$, $f_3(t) = \frac{\gamma}{2\sqrt{t}} e^{(-\gamma\sqrt{t})}$, $g_1(t) = \frac{\theta}{2\sqrt{t}} e^{(-\theta\sqrt{t})}$. $g_2(t) = \frac{l}{2\sqrt{t}} e^{(-l\sqrt{t})}$, $g_4(t) = \frac{h}{2\sqrt{t}} e^{(-h\sqrt{t})}$.

(II) When the shape parameter $\eta = 0.1$, the time distributions for repair, unit failure, and preventive maintenance and patience time simplify to an exponential distribution :-

$f(t) = Ke^{-(Kt)}$, $f_1(t) = \alpha e^{-(\alpha t)}$, $f_2(t) = \beta e^{-(\beta t)}$, $f_3(t) = \gamma e^{-(\gamma t)}$, $g_1(t) = \theta e^{-(\theta t)}$, $g_2(t) = le^{-(lt)}$, $g_4(t) = he^{-(ht)}$.

Table 1. List of Abbreviations and terminology used in the stochastic modeling of the rectifier cooling system's reliability and maintenance framework

P M	Preventive Maintenance
MTSF	Mean time to system failure
$/*$	Laplace -Steiltjes transform/ Laplace Transform
	Laplace-Stieltjes convolution /Laplace convolution
A_0	At the initial stage, all of the parts are functional.
S_1	The main unit is being repaired due to a minor failure, and the standby unit is currently operational.
S_2	The standby system is under PM and the first unit is operative.
S_3	System is failed due to Major failure standby unit is working.
S_4	System is under repair by expert repairmen and the standby unit is also under repair.
$\alpha/\beta/\gamma$	Scale Parameter.
k / h / l	Scale Parameter.
$/\lambda\ /\theta\ /\ \chi$	scale parameter.
$\eta > 0$	Common Shape Parameter.
$f(t)$	Probability density function cumulative density function for Patience time.
$g_i(t)/G_i(t)$	pdf /cdf of repair time due to Minor failure and PM.
$f_1(t)$	PDF of the failure rate of operative state to normal state due to minor failure.
$f_2(t)$	PDF of failure rate from normal to Partial failure due to Major failure.
$f_3(t)$	PDF of failure rate from normal to Partial failure due to PM.
$g_4(t)$	CDF of failure rate from Total failure to operative state after repaired by expert repairmen.
$f_1(t)$	$= \alpha = \alpha\eta t^{\eta-1}e^{-(\alpha t^{\eta})}dt$
$f_2(t)$	$= \beta = \beta\eta t^{\eta-1}e^{-(\beta t^{\eta})}dt$
$f_3(t)$	$= \gamma = \gamma\eta t^{\eta-1}e^{-(\gamma t^{\eta})}dt$
$g_4(t)$	$= h\eta t^{\eta-1}e^{-(h t^{\eta})}dt$
$g_1(t)$	$= \theta\eta t^{\eta-1}e^{-(\theta t^{\eta})}dt$
$g_2(t)$	$= l\eta t^{\eta-1}e^{-(l t^{\eta})}dt$
$f(t)$	$= K = K\eta t^{\eta-1}e^{-(K t^{\eta})}dt$

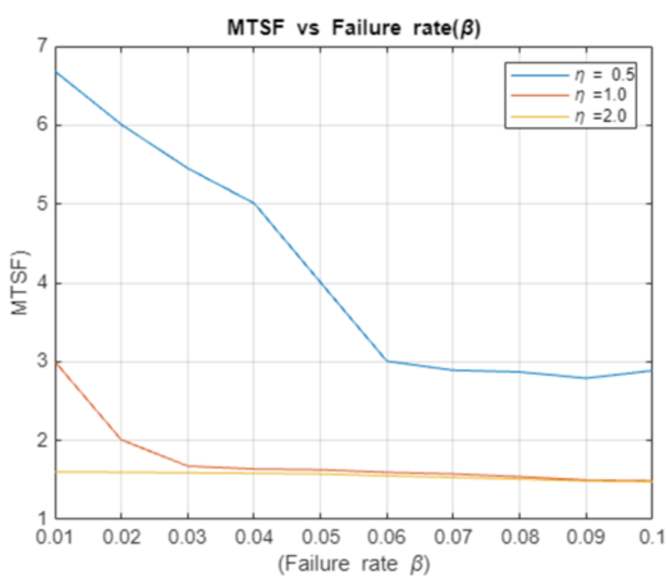

Fig 3. MTSF$_1$ VS Failure Rate

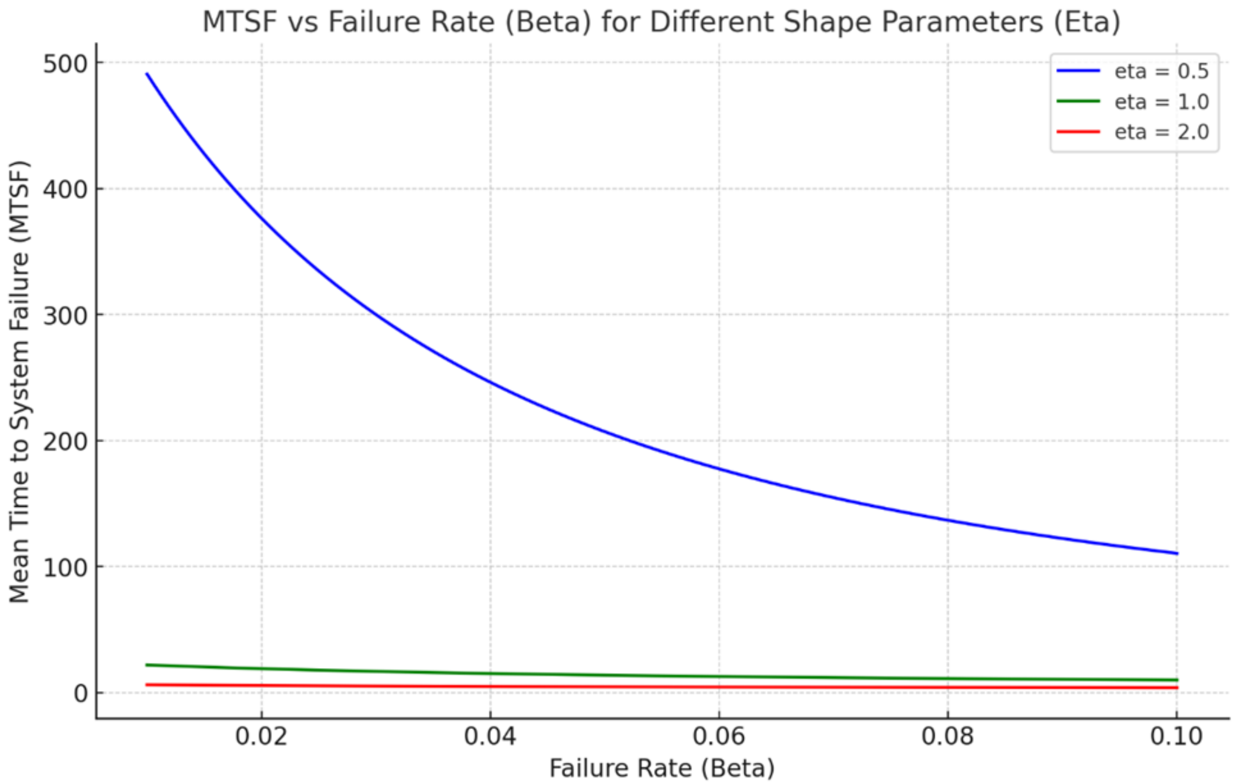

Fig 4. $\mathbf{MTSF_2}$ VS Failure Rate

3 Results and Discussion

The mean time to system failure (MTSF) is a critical metric in reliability modeling, used to estimate the expected time until a system experiences failure. This calculation is based on analyzing the system's failure patterns and degradation processes under varying operational conditions. Centrifugal pumps, when used under normal conditions, are not expected to wear out prematurely. To illustrate, the relationship between MTSF and failure rate is represented in the curve shown in Figure 3. Notably, the graph from the earlier work[7] exhibits a smoother trend, while the current graph presents more irregularities or fluctuations across the same failure range. The smoothness observed in the previous graph can be attributed to either more controlled experimental conditions. In contrast, the present graph, although less smooth, may reflect a higher degree of real-world variability. Incorporating both datasets, allows for a robust understanding of system reliability across both ideal and realistic scenarios.

Shape of the Curves:

For each shape parameter η, the curve 4 begins with an increase in MTSF as β grows from 0.01. This happens because the system transitions into a region where failure interactions are initially less dominant due to the balancing effect of the parameters. After reaching a peak, the MTSF begins to decrease. This indicates that as the failure rate β continues to grow, the system becomes increasingly dominated by failures, reducing the time to failure.

Behavior for Different:

$\eta = 0.5$: The curve shows a steep increase and decrease. This indicates a sharp sensitivity to failure rates, where small changes in β lead to significant changes in MTSF.

$\eta = 1.0$: The curve is smoother and more gradual than $\eta = 0.5$. This reflects a more balanced system where failures accumulate at a more consistent rate over time.

$\eta = 2.0$: The curve is the smoothest and least sensitive. This suggests that the system has a high tolerance for variations in β and the effect on MTSF is less pronounced.

4 Conclusion

A significant gap in the literature, which mostly uses exponential failure models, is filled by this study's thorough stochastic reliability analysis of a centrifugal pump-based rectifier cooling system used in the manufacturing of alumina.

'The novelty of the work lies in the incorporation of Weibull distribution to model failure and repair times, providing greater flexibility and realism in representing system behavior. Additionally, the model distinguishes between minor and major failures, introduces two tiers of repair personnel (regular and expert), and employs a "patience time" mechanism to simulate decision delays in escalated maintenance scenarios , factors often overlooked in conventional approaches. So, with the help of assumed data we have notice that by providing two types of repair person along with the concept of patience time and calender based preventive maintenence system becomes more reliable with decreasing PDF and failure rates which is economically beneficial to the industries and it will increase their working.

Comparative Failure Modeling: Future studies can extend the analysis by comparing Weibull and Exponential distributions to determine the most suitable failure model for different system components, improving predictive accuracy.

Dynamic Failure Rate Analysis: Weibull's shape parameter allows for modeling time-dependent failure rates, making it ideal for ageing components. Further research could explore adaptive maintenance strategies based on dynamically varying failure rates.

5 Author contribution

Hemant Kumar Saw supervised the entire research work, offered valuable feedback on the technique, confirmed theoretical interpretations, and directed the manuscript's finalization and improvement. Rakesh Tiwari examined the model results, offered technical advice in reliability theory and model formulation, and helped to make the manuscript more organized and understandable. Shruti Shukla prepared the initial draft of the manuscript, performed the core research, created the mathematical model, and performed analytical calculations and simulations. All authors completed the final revisions and approval of the manuscripts before submission.

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