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Phase Transitions and Asymptotic Behaviors in (2+1)-Dimensional Spacetimes with Perfect Fluids and Cosmological Constant

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Abstract

Objectives: The study first examines the (2+1)-dimensional cosmic holographic principle with $p = \omega\rho + \Lambda$, then studies cosmological solutions for this modified equation of state. **Method:** We analyze a (2+1)-dimensional holographic cosmology with the modified Eos $p = \omega\rho + \Lambda$ ($\Lambda = \alpha\rho$), resolving hydrodynamic instabilities in dark/phantom energy. By exploring free parameter space (ω, α), we derive cosmological solutions, identifying three dynamical regimes. Analytical and numerical methods reveal late-time acceleration, bouncing/cyclic universes, and stability criteria. The study connects EoS parameters to observable phenomena, testing holographic cosmology against singularities and observational constraints. **Finding:** This study reveals that the modified equation of state $p = \omega\rho + \alpha$ in (2+1)-dimensional holographic cosmology resolves hydrodynamic instabilities plaguing standard dark energy models $(\omega = const < 0)$. Three distinct dynamical regimes emerge based on ω and α:late-time accelerated expansion, non-singular bouncing universes, and cyclic cosmological models. The scale factor evolution transitions between power-law and de Sitter expansion, offering singularity-free scenarios. Key results include- full characterization of the (ω, α) parameter space, linking Eos parameters to observable cosmic evolution; explicit stability criteria for holographic-inspired cosmologies, ensuring physical viability; demonstration that specific parameter choices reproduce modern cosmological features while avoiding singularities. The work provides theoretical constraints on holographic models and testable predictions to distinguish competing universe evolution scenarios. By connecting Eos modifications to large-scale dynamics, it advances alternative cosmological frameworks within quantum gravity and holography, offering new insights into dark energy, phantom energy, and non-singular cosmologies. These findings enhance the understanding of holographic principles in cosmology and their observational signatures.

Novelty: This work represents the first (2+1)-D holographic cosmological model with a modified EoS ($p = \omega\rho + \alpha\rho$). Unlike (3+1)-D models requiring modified gravity, it offers testable dimensional signatures, new quantum gravity and dark energy insights, and distinct thermodynamics behavior. It predicts novel entropy-area laws, stable bouncing/cyclic solutions without exotic matter, and uniquely resolves singularities, avoiding Big Rip scenarios common in higher dimensions.
Keywords: Holographic cosmology; Modified equation of state; Bouncing universe; Dark energy stability; (2+1)-dimensional gravity

1 Introduction

In (2 + 1)-dimensional Saez-Ballester scalar-tensor theory of gravitation, two fluid cosmological models with matter and radiating source are widely known to have been studied in [1]. In the two-fluid model, the cosmic microwave background (CMB) radiation is described by one fluid, while the matter content of the universe is represented by another. Two-dimensional models are inherently complex and challenging to analyse. These models can only be split into two sub-models when expressed in separable-denominator form, but this requires meeting the minimal rank-decomposition conditions. The study proposes an alternative approach: instead of relying on separable-denominator forms or rank-decomposition criteria, a transformation is introduced that converts the original two-dimensional general model into a decoupled two-dimensional form. This decoupled model can then be separated into two one-dimensional sub-models (effectively cascaded one-dimensional models) [2]. General relativity in three spacetime dimensions (2+1)-D exhibits several distinctive simplifications compared to higher dimensions. For instance, in the absence of a negative cosmological constant, black holes and gravitational waves do not exist. Additionally, the Weyl curvature vanishes identically, and the weak-field limit of the theory does not reproduce Newtonian gravity in (2+1)-dimensions. Due to these unique properties, numerous studies have investigated the structure of general relativistic gravity in (2+1)-dimensional spacetime [3]. Einstein's field equations in (2+1)-dimensional spacetime exhibit unique characteristics, making (2+1)-gravity an especially intriguing subject of study. Lower-dimensional systems, commonly explored in quantum field theory, provide a useful framework for analyzing physical phenomena, and applying this approach to gravity yields valuable insights [4]. It is expected that studying (2+1)-gravity will shed new light on the physically relevant (3+1)-dimensional gravity [5]. Various formulations of the holographic principle have been applied to cosmological models, including a notable one for flat and open universes with an equation of state satisfying $0 \leq p \leq \rho$, but it fails in the case of a closed universe. Cosmological models with perfect fluid obeying a linear equation of state (where pressure is proportional to energy density) have played a fundamental role in modern theoretical cosmology [6], [7]. These models, spanning from the early radiation-dominated era to the present dark energy-dominated epoch, provide crucial understanding of cosmic expansion dynamics. The straightforward nature enables exact solutions that track the universe's development across various physical regimes [8], [9].

Recent cosmological observations reveal that our universe is undergoing accelerated expansion [10]. Within the framework of general relativity, this acceleration implies the existence of dark energy, an enigmatic form of energy characterized by negative pressure ($p = -\frac{\rho}{3}$) that remains poorly understood. Current estimates suggest dark energy constitutes approximately 70% of the universe's total energy density. The fundamental problem facing modern cosmology is how to account for the Universe's observed accelerating expansion. The cosmological Λ-term, also known as vacuum energy, provides the most straightforward theoretical explanation for this occurrence. It is characterized by a constant (Lorentz invariant) pressure $p = -\rho$ and energy density ρ throughout cosmic history. However, this explanation faces a significant fine-tuning problem, as it requires an extraordinarily precise value for the vacuum energy density to match observational data. This challenge has motivated investigations into alternative dynamical models characterized by an effective equation of state parameter $\omega \equiv \frac{p}{\rho} < -\frac{1}{3}$.

Various dynamical mechanisms have been developed to address the constraints of the cosmological constant paradigm. A prominent class of solutions involves quintessence models, featuring slowly evolving scalar field potential [11]. This framework has been generalized through k-essence theories, which incorporate scalar fields with non-canonical kinetic terms in their Lagrangians. These dynamic substitutes provide more adaptable ways to produce the negative pressure needed to propel cosmic acceleration, possibly bypassing the cosmological constant's fine-tuning issues. Our theoretical understanding of cosmic evolution has advanced significantly as a result of the study of such dynamic models. In contrast to the straightforward Λ-term scenario, these methods offer a richer phenomenology that accommodates more intricate evolutionary histories and time-varying equations of state. Modern cosmology is still actively investigating these concepts, which could provide answers to some of the most important queries regarding the underlying causes of the universe's rapid expansion. Modern astronomical observations have placed stringent constraints on the equation-of-state parameter $\omega \equiv \frac{p}{\rho}$ that drives cosmic acceleration. Current analysis confines this parameter to $-1.38 < \omega < -0.82$, with complementary Chandra data reinforcing these bounds.

While the observational evidence most strongly favors models where ω evolves from quintessence-type values ($\omega > -1$) in earlier epochs to phantom-type values ($\omega < -1$) today, the possibility of phantom energy scenarios with $\omega < -1$ remains viable[12].

2 Methodology

We employ analytical and numerical techniques to study a (2+1)-dimensional holographic cosmology with the modified equation of state $p = \omega\rho + \alpha\rho$. By systematically exploring the (ω, α) parameter space, we derive cosmological solutions and identify three distinct dynamical regimes. The approach combines Stability analysis of the modified Eos to resolve hydrodynamic instabilities in dark/phantom energy models, phase-space characterization of cosmological evolution (power-law to de Sitter transitions), and validation against singularity formation and observational constraints. Derivation of stability criteria for holographic-inspired cosmologies, connection of Eos parameters to observable cosmic phenomena through dynamical systems analysis[13]. The methodology bridges theoretical holographic principles with testable cosmological predictions. In this work, we use a modified linear equation of state, $p = \omega\rho + \Lambda$, to study the ideal fluid model, where ω and Λ are constants and exotic cosmological components with negative pressure ($p > 0$) since $\kappa = 0$. The benefit of using the general linear equation of state is that it allows the dark energy to be described with a positive squad sound speed (the dark energy is hydrodynamically unstable for the commonly used equation of state, $p = \omega\rho$, since $\partial p/\partial\rho = \omega < 0$). In the specific situation of $\Lambda = 0$, the examined linear model is reduced to the ideal fluid model with $\omega = constant$[14].

The structure of this model provides analytical cosmological answers for arbitrary values of ω and Λ. Below we demonstrate that this linear model (in contrast to the Chaplygin gas model) represents a wide range of cosmological situations, including the Big Bang, Big Rip, anti-Big Rip, de-Sitter solutions, bouncing solutions, and their diverse combinations. The main portion derives fundamental properties and specific analytical solutions for a perfect fluid cosmology governed by a linear equation of state. The study is set up to present these results in a systematic manner. We first formulate the fundamental equations guiding the scale factor evolution after obtaining and examining the exact solutions for each parameter regime. The asymptotic behaviors around characteristic time points and at early and late times are given particular attention in the oscillatory scenario. Lastly, we talk about our results' physical ramifications and how they relate to recent theoretical advancements and cosmological discoveries[15].

3 Field equation and Solutions

First, we look at the Robertson-Walker line element in (2+1) dimensions

$$ds^2 = dt^2 - a^2(t)\left(\frac{dr^2}{1-kr^2} + r^2 d\theta^2\right) \tag{1}$$

In this case, the scaling factor is denoted by $a(t)$. The comoving coordinates are represented by the coordinates $t,\ r$, and θ while the curvature of the space is indicated by the constant k, which is $k = 0, 1, -1$ for a flat universe, a closed universe, and an open universe, respectively.

In (2+1)-dimension space-time, the Einstein field equation is provided by

$$G_{\mu\vartheta} = R_{\mu\vartheta} - \frac{1}{2}g_{\mu\vartheta} = -2\pi G T_{\mu\vartheta} \tag{2}$$

where $G_{\mu\vartheta}$ is the Einstein tensor, $R_{\mu\vartheta}$ is the Ricci tensor and R is the ricci scalar. In future, we consider $2\pi G = C = 1$

Remark In usual Einstein theory, Einstein field equations can be written as

$$G_{\mu\vartheta} = R_{\mu\vartheta} - \frac{1}{2}g_{\mu\vartheta} = 8\pi G T_{\mu\vartheta}$$

We consider the matter with energy momentum tensor $T\mu\vartheta = (\rho, -p, -p)$. for simplicity's sake. Einstein field **equation (2)** for model **(1)** becomes

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2} = 2\pi G\rho, \tag{3}$$

$$\frac{\ddot{a}}{a} = -2\pi G p, \tag{4}$$

$$\dot{\rho}+2H(\rho+p)=0 \tag{5}$$

where ρ and p stand for the energy density and pressure, respectively and $H=\dot{a}/a$ is the Hubble parameter. Furthermore, a variable's temporal derivative is indicated by a dot over it. In this work, we used a modified equation of state in a more generic form to investigate the cosmic holographic principle.

$$p=\omega\rho+\Lambda \tag{6}$$

As Λ is the cosmological constant. **Equation (6)** is solved for two distinct scenarios, specifically $\Lambda=\alpha\rho$ and $\Lambda=\Lambda_0$

Case (1):$\Lambda=\alpha\rho$, in this case, the solution of the field equation for the flat cosmological model which gives scale factor, comoving distance and the following is the total entropy inside the horizon divided by the area:

$$a(t)=\left[(1+\beta)\sqrt{2\pi G\rho_0}\right]^{\dfrac{1}{1+\beta}}t^{\dfrac{1}{1+\beta}} \tag{7}$$

$$r_H=\frac{1+\beta}{\beta}\frac{1}{\left[(1+\beta)\sqrt{2\pi G\rho_0}\right]^{\dfrac{1}{1+\beta}}}t^{\dfrac{1}{1+\beta}} \tag{8}$$

$$\frac{S}{A}=\sigma\left(\frac{1+\beta}{\beta}\right)\frac{1}{\left[(1+\beta)\sqrt{2\pi GM_0}\right]^{\dfrac{1}{1+\beta}}}t^{\dfrac{\beta-1}{\beta+1}} \tag{9}$$

where $\beta=\omega+\alpha$

for a case (1) all the result is already discuss earlier in (2+1)-dimensional cosmological model for $p=\omega\rho$ and $\omega=0,\omega=\frac{1}{2}$

Case (2):$\Lambda=\Lambda_0$, in this case we put $\omega=\omega_0$ and $\Lambda=\Lambda_0$ in given equation of state (6) which gives

$$\rho=\frac{A_0}{a^{2(\omega_0+1)}}+B_0 \tag{10}$$

where A_0 and B_0 are constant. Using **equations (3) and (10)** we get the relation between the differential da and dt :

$$\frac{da}{a\sqrt{B_1+A_1a^{-2(\omega_0+1)}}}=\pm dt \tag{11}$$

where, $A_1=2\pi GA_0$ and $B_1=2\pi GB_0$ Three distinct outcomes of relation integration can be obtained depending on the signs of B_1 and A_1 (11). For both $A_1>0$ and $B_1>0$, we discover:

$$a(t)=\left\{\sqrt{\beta}\sin h\left[\pm\frac{M_0}{2}\sqrt{B_1t}\right]\right\}^{\dfrac{2}{M_0}} \tag{12}$$

here we denote $M_0=2(\omega_0+1)$ and $\beta=\frac{A_1}{B_1}$ and the selection of the higher or lower sign in **equation (12)** depends on the signs "+" or "-" in **equation (11)**. **Equation (12)** yields the following physical characteristics:

- Sign dependence: the $\pm$ inside the $\sin h$ function determines whether the universe is expanding ($+$) or contracting (–). The $\pm$ in the original **equation (11)** affects the time evolution (forward or backward in time).
- Physical interpretation: the $\sin h$ function describes a smooth transition from a power-law expansion (late times). The solution is valid for $B_1>0$ (positive cosmological constant) and $A_1>0$ (positive fluid energy density).

The asymptotic behavior of (12) for $t \to 0$ is given by

$$a(t) = \left(\pm \frac{M_0}{2}\sqrt{A_1} t\right)^{\dfrac{2}{M_0}} \tag{13}$$

From **equation (13)** following physical properties are obtained:

- Power-law expansion: the early universe is dominated by the fluid component A_1, with dynamics depending on ω_0. for matter-dominated ($\omega_0 = \frac{1}{2}, M_0 = 3$): $a(t) \propto t^{\frac{2}{3}}$, matching standard Friedmann cosmology. For a radiation-dominated ($\omega_0 = 1, M_0 = 4$): $a(t) \propto t^{\frac{1}{2}}$
- Significance: At early times, the cosmological constant B_1 is negligible, and the fluid component A_1 controls the expansion.

The following is a rewrite of **equation (12)** for $t \to \pm\infty$:

$$a(t) = \left(\frac{|\beta|}{4}\right)^{\dfrac{1}{M_0}} \exp\left(\pm\sqrt{B_1} t\right) \tag{14}$$

but with a crucial dependence on the sign of M_0, there are two cases:

case(i):$M_0 > 0\ (i.e.\ \omega_0 > -1)$, the exponential term $\exp\left(\pm\sqrt{B_1} t\right)$ describes accelerated expansion (for +) or contraction (for –). This is the standard dark-energy-dominated phase (de Sitter expansion).

case(ii):$M_0 < 0\ (i.e.\ \omega_0 < -1)$, the sign in the exponent flips, leading to: $a(t) \propto exp\left(\mp\sqrt{B_1} t\right)$.

From **equation (14)** we have the following physical properties:

- For $\omega_0 < -1$ the fluid is phantom-like (equation of state $\omega = \omega_0 < -1$).
- The universe either super-exponentially expands (if + is chosen) or collapses (if – is chosen).

The model naturally describes a universe transitioning from domination (matter/radiation) to dark energy domination. When $M_0 > 0 (\omega_0 > -1)$ the universe begins with a big bang singularity $[a(0) = 0]$, whereas for $M_0 < 0$ ($\omega_0 < -1$), the initial singularity may be avoided depending on the sign choice in the solution.

By adding a modified fluid (with an arbitrary ω_0), this model extended the Λ CDM cosmology and demonstrated how the fluid's equation of state determines the fate of the cosmos.

The integration of (11) in the scenario when $B_1 > 0$ and $A_1 < 0$:

$$a(t) = \left\{\sqrt{-\beta}\cos h\left[\frac{M_0}{2}\sqrt{B_1 t}\right]\right\}^{\dfrac{2}{M_0}} \tag{15}$$

From the above equation, the solution involves a $\cos h$ function, implying a non-singular, bouncing universe. At $t = 0, a(t)$ reaches a minimum finite size $a(0) = (-\beta)^{\frac{1}{M_0}}$, avoiding a big bang singularity. The universe contracts for $t < 0$, reaches a minimum, and then expands for $t > 0$.

For $t \to 0$, the scale factor's asymptotic behavior is provided by:

$$a(t) = (-\beta)^{\dfrac{1}{M_0}} \left(1 + \frac{M_0}{4} B_1 t^2\right) \tag{16}$$

From **equation (16)**, the first term $(-\beta)^{\frac{1}{M_0}}$ is the minimum size of the universe at the bounce. The second term $\propto t^2$ is a smooth, parabolic expansion near the bounce point, driven by the cosmological constant B_1. This is characteristic of a "coasting "phase where the universe transitions from contraction to expansion.

The scale factor is specified by (14), for $t \to \pm\infty$.

The cosmos avoids a singularity and bounces from a finite minimum size when $A_1 < 0$ (with $B_1 > 0$), indicating a non-singular bouncing cosmology. We know the solution (15) for the restricted situation $-1 < \omega_0 < 0$ was achieved in a somewhat different form.

Lastly, the evolution of the universe's scale factor may be found for $B_1 < 0$ and $A_1 > 0$:

$$a(t) = \left\{ \sqrt{-\beta} \sin \left[\frac{M_0}{2} \sqrt{-B_1} t \right] \right\}^{\frac{2}{M_0}} \tag{17}$$

The sine function implies a cyclic/oscillating universe, with periodic expansion and contraction. The universe starts at $a(0) = 0$(Big Bang singularity), expands to a maximum, then collapses. The period of oscillation is $T = \frac{4\pi}{M_0\sqrt{-B_1}}$.

For the cosmology with a negative anti-de-Sitter Λ-term and a scalar fundamental field with a special form a potential in the limit $-1 < \omega_0 < 1$.

The following characterizes the asymptotic behavior of a scale factor close to $t \simeq \frac{\pi}{(M_0 B_1)}$ for a $t \to 0$:

$$a(t) = \left\{ \sqrt{-\beta} \left[1 - \frac{1}{2} \left(\frac{M_0}{2} \sqrt{-B_1} t - \frac{\pi}{2} \right)^2 \right] \right\}^{\frac{2}{M_0}} \tag{18}$$

The parabolic form (quadratic decrease) shows a smooth turnaround from expansion to attraction. The maximum size is $a_{\max} = (-\beta)^{\frac{1}{M_0}}$.

Accordingly, the scale factor's asymptotic behavior at $t \simeq \pm 2 \frac{\pi}{(M_0 B_1)}$ is provided

$$a(t) = \left(\pi \sqrt{-\beta} - \frac{M_0}{2} \sqrt{A_1} t \right)^{\frac{2}{M_0}}$$

The universe collapses to a singularity $(a = 0)\ at$ finite time, ending the cycle. The linear term $\propto t$ dominates near the singularity, resembling the early time power-law (**equation 13**).

4 Conclusion

By considering the cosmology of a perfect fluid with a modified equation of state we have investigated the holographic principle in a (2+1)-dimensional cosmological model. This study examined the dynamical history of the dark energy-filled universe that follows the linear equation of state of the form $p = \omega_0 \rho + \Lambda_0$. It turns out that a wide range of cosmic dynamics may be found in this straightforward linear model for the various values of ω_0 and Λ_0. The integration (11) to three distinct cosmological scenarios depending on the signs of the cosmological constant B_1 and A_1 in the modified linear equation of state: (1) for $B_1 > 0$ and $A_1 > 0$, the scale factor $a(t)$ follows a hyperbolic sin function, transitioning from a power-law expansion at early times ($t \to 0$) to exponential (de-Sitter-like) growth at late times ($t \to \pm\infty$), with $M_0 = 2(\omega_0 + 1)$ determining acceleration ($M_0 > 0$); (2) for $B_1 > 0$ and $A_1 < 0$, $a(t)$ evolves via a hyperbolic cosine, showing quadratic growth near $t \to 0$ and late time exponential expansion, resembling Λ CDM but with distinct early dynamics; (3) for $B_1 < 0$ and $A_1 > 0$, $a(t)$ follows a sin function, leading to a cyclic universe-initially expanding power-law, reaching a maximum at $t \simeq \frac{\pi}{(M_0 B_1)}$, and collapsing symmetrically, characteristic of Ads-like cosmologies.

The asymptotic behaviors highlight the role of the parameter $M_0 = 2(\omega_0 + 1)$ in determining the late time of the universe. For $B_1 > 0$, the late-time behavior is dominated by exponential expansion, resembling a (2+1)-dimensional de Sitter universe. A negative B_1leads to oscillatory solutions, suggesting a possible cyclic cosmology in lower dimensions. The presence of a bounce in ($A_1 < 0$) hints at nonsingular cosmological models in (2+1)-gravity, which may have implications for quantum gravity scenarios. The (3+1)-dimensional Einstein equations have extremely complicated global cosmological solutions that are probably controlled by non-integrability. However, in (2+1)-dimensional spacetime, the structure of the theory allows for significant progress in finding general solutions in various interesting scenarios. A comparable equation of state frequently leads to Big Rip singularities for $\omega < -1$ in (3+1) D-models. In contrast to $\omega < -1$, our (2+1)d-model exhibits super exponential (for $\omega_0 < -1$) without necessarily diverging in finite time , as well as bouncing and cyclic solutions that are less prevalent in (3+1)-dimensions without additional fields like scalar fields, and the entropy area relation $\frac{S}{A}$ lacks a direct (3+1)-dimensional analogue because of dimensional difference. The investigation of Einstein's theory in (2+1)-dimensions has been motivated by this as well as the widely held belief that quantum field theory is better suited to three dimensions than four.

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