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ORIGINAL ARTICLE

Study of Bianchi Type-III Dark Energy Model in the Presence of $f(R,T)$ Gravity

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Abstract

Objective: To get a solution for the dark energy model using a Bianchi type III model with an EoS (Equation of State) parameter filled with a perfect fluid in the scalar tensor theory of gravity. **Methods**: To acquire exact solutions of field equations, we considered: (i) the shear scalar is proportional to scalar expansion. (ii) a special form of scale factor, and (iii) the EoS (Skewness) parameter is proportional to the skewness parameter. **Findings**: We have seen that the EoS and the Skewness are functions of time. We also observed some physical and kinematic aspects of the resulting model. It is found that the resultant model is consistent with the current observations of SNeIa and CMBR. **Novelty**: We obtained a Bounce Bianchi type III dark energy model with an EoS parameter filled with a perfect fluid in the scalar tensor gravitational theory.

Keywords: Bianchi Type III space time metric; Perfect Fluid; $f(R,T)$ Gravity; Dark Energy

1 Introduction

Our universe is dominated by a component with large negative pressure known as dark energy (DE), according to recent cosmological observational data from Type Ia Supernovae, the Cosmic Microwave Background, Large Scale Structure, the Sloan Digital Sky Survey, the Wilkinson Microwave Anisotropy Probe, and the Chandra X-ray observatory. It is a fictitious type of energy that permeates space and tends to accelerate the expansion of the universe. According to the results of WMAP, the universe is made up of 23% dark matter, 73% dark energy, and 4%

Ordinary matter.

As a result, a number of authors find it intriguing to study dark energy cosmological theories.

Dark energy is defined by the equation of state (EoS) parameter, given by $\omega(t) = p/\rho$, which is not always constant, where p is the fluid pressure and ρ is energy density.

The existence of dark energy, dark matter, and the process underlying the universe's late-time acceleration have all been explained by several modified theories of gravity like $f(R), f(T), f(G)$, and $f(R,T)$ gravity. Harko et al.[1] have developed a new modified theory of gravity known as $f(R,T)$ gravity. This theory attracted many researchers since it provides a natural gravitational alternative to dark energy.

Several researchers [2–10] have investigated cosmological models that incorporate the *f(R,T)* theory of gravitation.

Recently Kumari et al. studied the Bianchi Type- III Perfect Fluid Dark Energy Cosmological Model with Dynamical Equation of State Parameter in $f(R,T)$ Theory of Gravity

Motivated by these recent investigations, we have explored the Bounce Bianchi type III dark energy cosmological model in the presence of $f(R, T)$ gravity. We present Einstein field equations with the conditions. Some physical and kinematical aspects are also discussed.

2 Methodology

2.1 Metric and Field Equations

We consider the Bianchi type-III space time in the form

$$ds^2 = dt^2 - \chi_1^2 dx^2 - e^{-2m_0 x}\chi_2^2 dy^2 - \chi_3^2 dz^2 \tag{1}$$

where m_0 is a constant and $\chi_1, \chi_2 \chi_3$ are function of cosmic time *t*.

The energy momentum tensor for anisotropic dark energy is given by

$$T_i^j = diag[\rho, -p_x, -p_y, -p_z] = diag[1, -\omega_x, -\omega_y, -\omega_z]\rho = diag[1, -\omega, -(\omega+\delta), -(\omega+\gamma)]\rho \tag{2}$$

where ρ is the energy density of the fluid, p_x, p_y and p_z are the pressure and ω_x, ω_y and $, \omega_z$ are the directional EoS parameters along the *x, y* and *z* axis respectively. ω is the deviation-free EoS parameter of the dark energy, and one can parameterize the deviation from isotropy by setting $\omega_x = \omega$ and then introducing skewness parameters δ and γ that are deviation from ω along *y* and *z* axes, respectively.

The field equation in the $f(R,T)$ theory of gravity for the function $f(R,T) = R + f(T)$ when the matter source is a perfect fluid is given by (Harko *et al.,* 2011).

$$R_{ij} - \frac{1}{2}Rg_{ij} = 8\pi T_{ij} + 2f'(T)T_{ij} + [2pf'(T) + f(T)]g_{ij} \tag{3}$$

$$\text{where } T_{ij} = (\rho + p)u_i u_j - pg_{ij} \tag{4}$$

We choose the function $f(T)$ of the trace of the energy tensor of the matter so that

$$f(T) = \eta T \tag{5}$$

where η is a constant.

Now with the help of **Equations (2), (4) and (5)**, the field **equations (3)** for the metric **(1)** can be written as

$$\frac{\ddot{\chi_2}}{\chi_2} + \frac{\ddot{\chi_3}}{\chi_3} + \frac{\dot{\chi_2}}{\chi_2}\frac{\dot{\chi_3}}{\chi_3} = \rho\left[(8\pi + 2\eta)\omega - \eta(1 - 3\omega - \delta - \gamma)\right] - 2p\eta \tag{6}$$

$$\frac{\ddot{\chi_1}}{\chi_1} + \frac{\ddot{\chi_3}}{\chi_3} + \frac{\dot{\chi_1}}{\chi_1}\frac{\dot{\chi_3}}{\chi_3} = \rho\left[(8\pi + 2\eta)(\omega + \delta) - \eta(1 - 3\omega - \delta - \gamma)\right] - 2p\eta \tag{7}$$

$$\frac{\ddot{\chi_1}}{\chi_1} + \frac{\ddot{\chi_2}}{\chi_2} + \frac{\dot{\chi_1}}{\chi_1}\frac{\dot{\chi_2}}{\chi_2} - \frac{m_0^2}{\chi_1^2} = \rho\left[(8\pi + 2\eta)(\omega + \gamma) - \eta(1 - 3\omega - \delta - \gamma)\right] - 2p\eta \tag{8}$$

$$\frac{\dot{\chi_1}}{\chi_1}\frac{\dot{\chi_2}}{\chi_2}+\frac{\dot{\chi_2}}{\chi_2}\frac{\dot{\chi_3}}{\chi_3}+\frac{\dot{\chi_1}}{\chi_1}\frac{\dot{\chi_3}}{\chi_3}-\frac{m_0^2}{\chi_1^2}=-\rho\left[(8\pi+2\eta)+\eta(1-3\omega-\delta-\gamma)\right]-2p\eta \tag{9}$$

$$m_0\left(\frac{\dot{\chi_1}}{\chi_1}-\frac{\dot{\chi_2}}{\chi_2}\right)=0 \tag{10}$$

where an overhead dot $(\cdot)$ denotes differentiation with respect to time.

Integrating **Equation (10)**, we obtain

$$\chi_1=D_1\chi_2 \tag{11}$$

where D_1 is a positive constant of integration and without loss of generality we take $D_1=1$.

Using **Equation (11)** in **Equation (7)** and subtract the result from **Equation (6)**, we obtain that the skewness parameter on y-axis is null i.e.

$$\delta=0 \tag{12}$$

Thus, system of **Equations (6)** - **(9)** reduces to

$$\frac{\ddot{\chi_2}}{\chi_2}+\frac{\ddot{\chi_3}}{\chi_3}+\frac{\dot{\chi_2}}{\chi_2}\frac{\dot{\chi_3}}{\chi_3}=\rho\left[(8\pi+2\eta)\omega-\eta(1-3\omega-\gamma)\right]-2p\eta \tag{13}$$

$$2\frac{\ddot{\chi_2}}{\chi_2}+\frac{\dot{\chi_2^2}}{\chi_2^2}-\frac{m_0^2}{\chi_2^2}=\rho\left[(8\pi+2\eta)(\omega+\gamma)-\eta(1-3\omega-\gamma)\right]-2p\eta \tag{14}$$

$$2\frac{\dot{\chi_2}}{\chi_2}\frac{\dot{\chi_3}}{\chi_3}+\frac{\dot{\chi_2^2}}{\chi_2^2}-\frac{m_0^2}{\chi_2^2}=-\rho\left[(8\pi+2\eta)+\eta(1-3\omega-\gamma)\right]-2p\eta \tag{15}$$

3 Result and discussion

3.1 Solution of the Field Equations

The average scale factor (a), spatial volume (V), Hubble parameter (H), scalar expansion (θ) shear scalar (σ) and mean anisotropic parameter are (A_m) given by

$$a^3=\chi_1\chi_2\chi_3 \tag{16}$$

$$V=a^3=\chi_1\chi_2\chi_3 \tag{17}$$

$$H=\frac{\dot{a}}{a}=\frac{1}{3}\left(\frac{\dot{\chi_1}}{\chi_1}+\frac{\dot{\chi_2}}{\chi_2}+\frac{\dot{\chi_3}}{\chi_3}\right) \tag{18}$$

$$\theta=3H \tag{19}$$

$$\sigma^2=\frac{1}{2}\left(\sum_{i=1}^{3}H_i^2-\frac{\theta^2}{3}\right) \tag{20}$$

$$A_m=\frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i-H}{H}\right)^2 \tag{21}$$

To find the deterministic solution, we use the following physical conditions,

1. The shear is proportional to scalar expansion (θ) which gives

$$\chi_2 = \chi_3^s \tag{22}$$

where $s > 1$ is a constant.

2. Here we choose the scale factor of the form

$$a = R_0 \left(\frac{t}{t_0}\right)^{\lambda_1} e^{\lambda_2 \left(\frac{t}{t_0} - 1\right)} \tag{23}$$

where $R_0 \neq 0$ and $\lambda_1, \lambda_2 > 0$

3. The EoS parameter ω is proportional to the skewness parameter γ such that

$$\omega + \gamma = 0 \tag{24}$$

Using the relation $\left(\frac{t}{t_0} - 1\right) = t$ in the above form of scale factor, we renovate the equation into a bouncing phase (regular bounce) at which the scale factor never vanishes.

$$a = R_0 (t+1)^{\lambda_1} e^{\lambda_2 t} \tag{25}$$

Now from **Equations (11), (16), (22), and (25)**, we obtain

$$\chi_1 = \chi_2 = R_0^{sk} (t+1)^{\lambda_1 sk} e^{\lambda_2 skt} \tag{26}$$

$$\chi_3 = R_0^k (t+1)^{\lambda_1 k} e^{\lambda_2 kt} \tag{27}$$

where $k = \frac{3}{2s+1}$

Hence the metric **(1)** with the suitable choice of coordinates and constants can be written as

$$ds^2 = dt^2 - R_0^{2sk} (t+1)^{2\lambda_1 sk} e^{2\lambda_2 skt} \left(dx^2 + e^{-2m_0 x} dy^2\right) - R_0^{2k} (t+1)^{2\lambda_1 k} e^{2\lambda_2 kt} dz^2 \tag{28}$$

3.2 Physical Aspects of the Model

Equation (28) represents the Bianchi type III dark energy model in $f(R,T)$ gravity with the following physical and kinematical parameters, which play a vital role in the discussion of cosmology.

The expression for Spatial volume, Expansion scalar, Hubble parameter and Shear scalar are given by

$$V = R_0^3 (t+1)^{3\lambda_1} e^{3\lambda_2 t} \tag{29}$$

$$\theta = 3\left(\frac{\lambda_1}{t+1} + \lambda_2\right) \tag{30}$$

$$H = \frac{\lambda_1}{t+1} + \lambda_2 \tag{31}$$

$$\sigma^2 = \frac{\left[\left(2s^2+1\right)k^2 - 3\right]}{2} \left[\frac{\lambda_1}{1+t} + \lambda_2\right]^2 \tag{32}$$

The expression for the deceleration parameter is expressed as

$$q = -1 + \frac{\lambda_1}{\left[\lambda_1 + \lambda_2 (t+1)\right]^2} \tag{33}$$

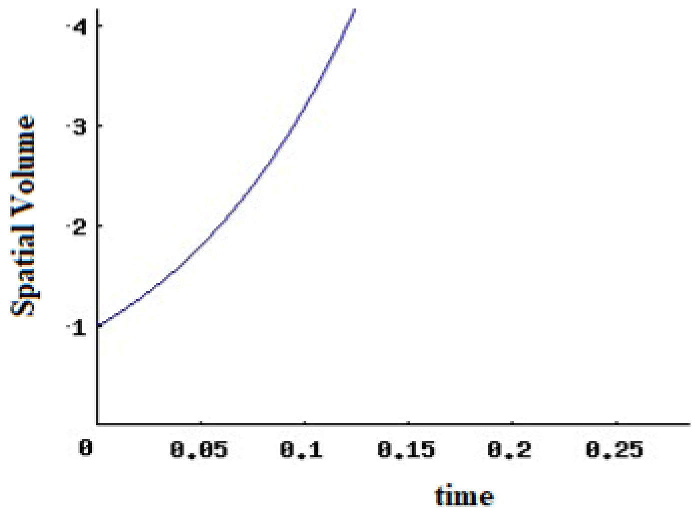

Fig 1. Spatial Volume Vs time

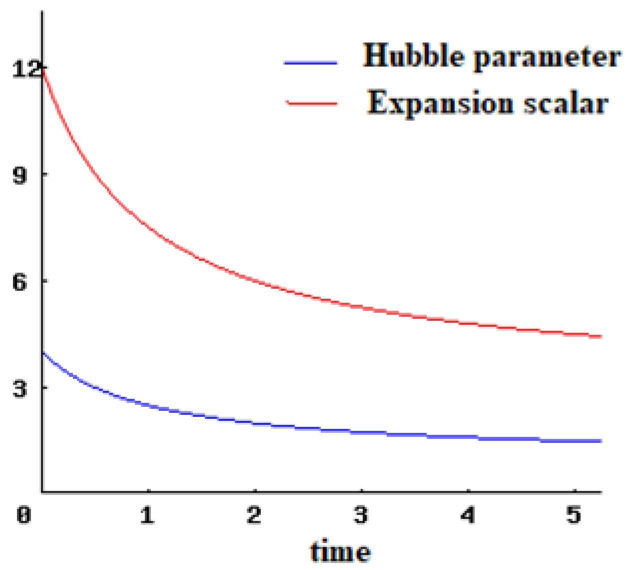

Fig 2. Hubble parameter & Expansion scalar Vs time

The expression for the mean anisotropic parameter is

$$A_m = \frac{(2s^2+1)k^2 - 2(2s+1)k + 3}{3} \tag{34}$$

Using **Equations (14), (15), and (25)**, the energy density is

$$\rho = -p = \frac{2sk}{(8\pi + 2\eta)} \left\{ (s-1)k \left[\frac{\lambda_1}{t+1} + \lambda_2 \right]^2 - \frac{\lambda_1}{(t+1)^2} \right\} \tag{35}$$

It has been observed that the spatial volume (V) is an increasing function of time, which is shown in Figure 1, whereas the Hubble parameter (H), expansion scalar (θ) & density (ρ) are decreasing functions of time, which can be seen in Figure 2, and Figure 4. Also from Figure 3, we can see as t tends to infinity, the deceleration parameter (q) approaches to -1.

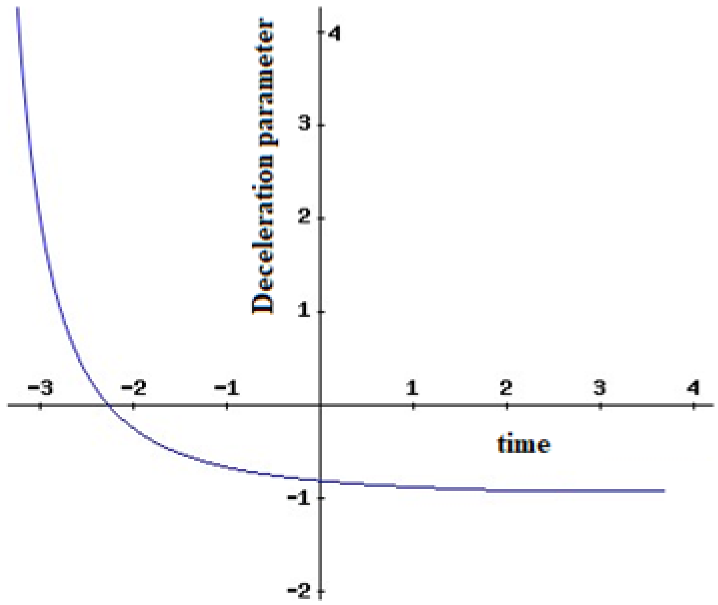

Fig 3. Deceleration parameter Vs time

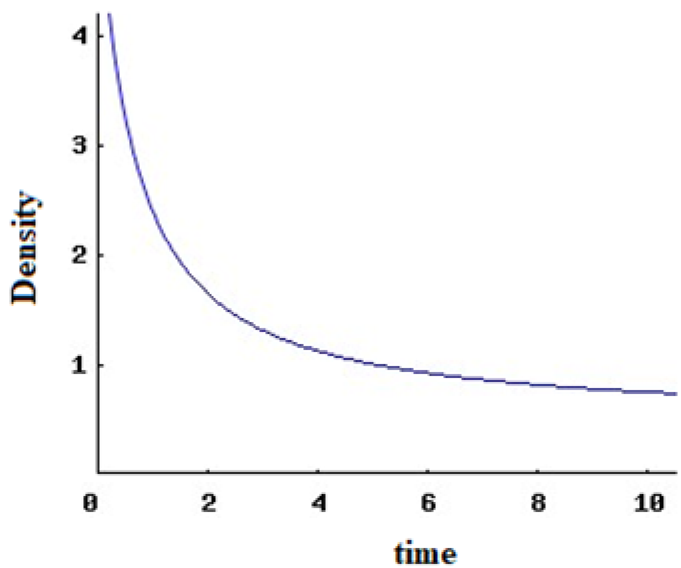

Fig 4. Density Vs time

With the help of **Equations (13), (15) and (35)**, the EoS and Skewness parameter are

$$\omega = -\gamma = -1 + \frac{1}{\rho(8\pi + 2\eta)} \left\{ (1-s)k^2 \left[\frac{\lambda_1}{t+1} + \lambda_2 \right]^2 - \frac{k(s+1)\lambda_1}{(t+1)^2} + \frac{m^2}{e^{2nk\lambda_2 t}(t+1)^{2nk\lambda_1}} \right\}$$

The relation between scale factor $a(t)$ and redshift parameter z may be written as

$$1 + z \equiv \frac{a(t_0)}{a(t)} \tag{36}$$

where $a(t_0)$ is the present value of scale factor, and it takes as unity. The scale factor $a(t)$ is stable under metric perturbation, so that the redshift parameter in terms of scale factor is given by

$$a = \frac{1}{1+z} \tag{37}$$

Therefore,

$$1 + z = \left[R_0 (1+t)^{\lambda_1} (1 + \lambda_2 t) \right]^{-1} \tag{38}$$

3.3 Key Observations

It is found that for large values of t, the spatial volume V tends to infinity. The volume increases as time increases, i.e., the model is expanding. If $t \to \infty$, then the scale factors also tend to infinity. It is also observed that for the suitable values of the constant, the expansion scalar (θ), shear scalar (σ), and Hubble parameter (H) become zero for large values of t. We noticed that the energy density is positive for the suitable values of constants. With the appropriate values of constants, we obtained q < 0, which favors accelerating models. For large values of t, the deceleration parameter approaches -1. Hence, we can say that the result is consistent with the current observations of SNeIa and CMBR. Since $\frac{\sigma}{\theta} \neq 0$, the model does not approach isotropy at any time. However, the model becomes isotropic for $s = 1$. The dynamics of the mean anisotropy parameter depend on the value of s. As $s = 1$, the anisotropy parameter vanishes. Also, we observed two different cases.

- $For s = 1$, the density becomes

$$\rho = -p = \frac{-2k\lambda_1}{(8\pi + 2\eta)(t+1)^2} \tag{39}$$

Hence, we can say that as time increases, density decreases.

- $For \lambda_1 = \lambda_2 = 1$ and R_0 = 1, we obtain the redshift and deceleration parameter as follows:

$$1 + z = \frac{1}{(1+t)^2} \tag{40}$$

$$\text{and } q = -1 + \frac{1}{(t+2)^2} \tag{41}$$

4 Conclusion

This study presents the anisotropic dark cosmological model with EOS parameter in the presence of perfect fluid in the framework of *f(R,T)* gravity. The physical and kinematical parameters, which are very important in the description of cosmological model has been obtained and discussed for $s \neq 1$ and $s = 1$. To solve the field equations, it is assumed an average scale factor (a) in the combination of time (t). It has been observed that the EoS parameter and skewness parameter are functions of time t. The model does not approach isotropy at any time. Through this study, it is attempted to present a better knowledge of the cosmological evolution of the present universe with the help of Bianchi–III universe in *f(R,T)* gravity. It is verified that the energy condition is satisfied, i.e. $\rho > 0$ for suitable values of constants. However, the model becomes isotropic for $s = 1$. Also, the mean anisotropic parameter and shear scalar vanish. Therefore, the resultant model becomes a shear free model. It is clear that the model is expanding, anisotropic for $\alpha \neq 1$, at the late universe, accelerating, non-shearing, and admits an initial singularity at $t = 0$, which also agrees with the present-day observational data. Many cosmologists have observed the results of the inhomogeneous equation of state and found it to be quite useful in describing various cosmological phenomena. Despite its simplicity, this model may be applicable for a deeper perception about the development of the universe.

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