INDIAN JOURNAL OF SCIENCE AND TECHNOLOGY

ORIGINAL ARTICLE

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OPEN ACCESS
**Received:** 01/09/2024
**Accepted:** 27/01/2025
**Published:** 02/05/2025

**Citation:** Manjumari M (2025) Decision Making Problem with Multiple Attribute Group Employing an Array of Singularly Perturbed Differential Equations. Indian Journal of Science and Technology 18(14): 1147-1154. https://doi.org/10.17485/IJST/v18i14.2820

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**Funding:** None

**Competing Interests:** None



Published By Indian Society for Education and Environment (iSee)

**ISSN**
Print: 0974-6846
Electronic: 0974-5645

# Decision Making Problem with Multiple Attribute Group Employing an Array of Singularly Perturbed Differential Equations

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## Abstract

**Objectives:** This study deals with an ensemble of linear singularly perturbed differential equations that has been employed to solve Multiple Attribute Group Decision Making issues on that the weights of the Decision Makers (DMs) are unknown. **Methods:** Two sophisticated operators-the Intuitionistic Fuzzy Generalised Hybrid Weighted Averaging (IFGHWA) operator as well as the Intuitionistic Fuzzy Weighted Averaging (IFWA) operator—are employed to aid in the course of decision-making. **Findings:** In order to determine the best course of action, these operators are used to combine intuitionistic fuzzy decision matrices into a collective decision matrix. We can use the newly proposed correlation coefficient method and score function for ranking the best alternative from the available alternatives. **Novelty:** To demonstrate the efficiency and applicability of the suggested method in resolving MAGDM situations with ambiguous decision maker weights, a computational instance is provided. Numerical illustration is given to show the effectiveness of the proposed approach.

**Keywords:** Intuitionistic Fuzzy Sets (IFSs); Singular Perturbation Problem; IFGHWA operator; IFWA operator; MAGDM

## 1 Introduction

The Multiple Attribute Group Decision Making (MAGDM) Problem is a substantial challenge in modern decision research. It has been used in many fields, including engineering technology, economics, management, and society. The idea of interval-valued IFSs that utilise means and variances and a new aggregation operator has been developed[1]. Upon the weighted reduced cosine sets coined[2], IFSs have been extensively used in multi-attribute decision-making methods. The CPT-TODIM approach for interval-valued IFMAGDM was expanded also used for urban environmental valuation[3], also proposed few IF Einstein hybrid aggregation methods and demonstrated the way of applying them to several attribute decision-making issues. In an interval-valued IF environment[4] presented a unique MAGDM approach depends on the correlation coefficient (CC) and hesitation degree. It is characterised as two functions that indicate the degree of membership, the level of non-membership, correspondingly.

Linguistic IFSs (LIFSs), where LIFNs indicate the membership and non-membership grades of every parameter in the discourse universe that belongs to a LIFS[5]. Singularly perturbed differential difference equations (SPDDE) have been solved through the utilisation of the Liouville Green Transformation[6]. Consistency analysis for IFPRs and IVIFPR[7] were carried out. An extended intuitionistic hesitant fuzzy model was developed for determining order preference by concise to an ideal solution strategy, whose efficacy was demonstrated by a practical application. Entropy measure is used to figure out criteria weights in multi-criteria decision-making problems[8].

When it comes to assessing uncertainty and avoiding paradoxical situations, the suggested distance measure of interval-valued IFSs performs better than alternative metrics. The usefulness and excellent discriminating capabilities of the suggested distance measure are demonstrated by a few real-world instances of multi-attribute decision making[9]. A real-world mathematical scenario provides evidence to illustrate the tactic's superiority and usefulness, and the findings are compared to those of many other techniques already in utilisation[10].

When compared to the various aggregation operators as well accuracy functions presently in consumption, multi-criteria group decision making remains a widespread and successful decision-making technique that enhances decision quality and is far more comprehensive and adaptable[11]. an innovative operator learning method of Component Fourier Neural Operator builds upon Fourier Neural Operator (FNO), while simultaneously incorporating valuable prior knowledge obtained from asymptotic analysis[12]. Yet there is no study to investigates the MAGDM problem.

In order to investigate the MAGDM problem, the study shows that our novel distance measures are less sensitive to the permitted changes in the weight vectors obtained from the kinds of suggested distance measures in uncertain scenarios. The unknown weights in this study are derived by perturbing an array of differential equations separately. Here we discussed the MAGDM issue having an IFSs for option ranking in addition to the IFWA and IFGHWA operators. The feasibility of the suggested approach is illustrated using a numerical example.

## 2 Methodology

A few fundamental ideas regarding IFSs and the various classes of aggregating operators are covered in this section.

**Definition 2.1**

The IFS, is described as $\tilde{R}$ here $\alpha^{+}$,$\alpha^{-}$, under the provided constraint $x_i(i=1,2,...,7)$, is an IFS A in X.

The elements

$$\tilde{r}_{ij}=\left(\begin{array}{c}\left[1-\prod_{k=1}^{p}\left(1-\tilde{\mu}^{L}_{\tilde{r}_{ij}(t_k)}\right)^{\lambda(t_k)},1-\prod_{k=1}^{p}\left(1-\tilde{\mu}^{U}_{\tilde{r}_{ij}(t_k)}\right)^{\lambda(t_k)}\right],\left[\prod_{k=1}^{p}\left(\tilde{\nu}^{L}_{\tilde{r}_{ij}(t_k)}\right)^{\lambda(t_k)},\prod_{k=1}^{p}\left(\tilde{\nu}^{U}_{\tilde{r}_{ij}(t_k)}\right)^{\lambda(t_k)}\right],\\ \left[\prod_{k=1}^{p}\left(1-\tilde{\mu}^{U}_{\tilde{r}_{ij}(t_k)}\right)^{\lambda(t_k)}-\prod_{k=1}^{p}\left(\tilde{\nu}^{U}_{\tilde{r}_{ij}(t_k)}\right)^{\lambda(t_k)},\prod_{k=1}^{p}\left(1-\tilde{\mu}^{L}_{\tilde{r}_{ij}(t_k)}\right)^{\lambda(t_k)}-\prod_{k=1}^{p}\left(\tilde{\nu}^{L}_{\tilde{r}_{ij}(t_k)}\right)^{\lambda(t_k)}\right]\end{array}\right)$$

and $\lambda(t)=(0.101662862, 0.398345634, 0.4999915)^{T}$ demonstrates membership and non-membership degrees to A are denoted by the integers and, duly.

**Definition 2.2**

For every IFS *A* of *X*, suppose $\pi_A(x)=1-\mu_A(x)-\gamma_A(x),\ \forall x\in X$, then $\pi_A(x)$ is known to be the degree of indeterminacy or hesitancy of *x* to *A*, here $0\le\pi_A(x)\le 1, \forall xX$.

**Definition 2.3**

Let $\tilde{a}_j=(\mu_j,\gamma_j), \forall j=1,2,\ldots,n$ be a class of intuitionistic fuzzy values. IFWA operator, IFWA: $Q^n\rightarrow Q$ is described as:

$IFWA_{\omega}\left(\tilde{a}_1,\tilde{a}_2,......,\tilde{a}_n\right)=\sum_{j=1}^{n}\omega_j\tilde{a}_j=$

here $\omega=$ is the weight vector of $\tilde{a}_j\forall j=1,2,\ldots\ldots,n\ni\omega_j>0$ and $\sum_{j=1}^{n}\omega_j=1$.

**Definition 2.4**

Let $\widetilde{a}_j = (\mu_j, \gamma_j), \forall j = 1,2,\ldots\ldots,n$ be IF Sets. A map $f^{GH}_{\omega,w} : F^n \to F$ is an IFGHAO if it holds

$$f^{GH}_{\omega.w}(A_1, A_{2,\ldots\ldots}, A_n) = \sqrt[q]{\sum_{k=1}^{n} w_k \acute{B}_k^q},$$

$$= \left\langle \sqrt[q]{1 - \prod_{k=1}^{n} (1-\acute{\mu}_k^q)^{w_k}}, 1 - \sqrt[q]{1 - \prod_{k=1}^{n} \left[1-(1-\acute{\gamma}_k)^q\right]^{w_k}} \right\rangle$$

whereas $w = (w_1, w_2, \ldots\ldots, w_n)^T$ a weight vector relevant to the map $f^{GH}_{\omega,w}$;$\omega = (w_1, w_2, \ldots\ldots, w_n)^T$ is a weight vector of IFS $A_j (j = 1,2,\ldots.,n)$; $\acute{B}_k$ is the k$^{th}$ largest of the *n* IFS $\acute{A}_j = n\omega_j A_j (j = 1,2,\ldots.,n)$, this is ascertained by applying a ranking technique, such the scoring function ranking approach mentioned above; $q > 0$ is a control parameter.

**Definition 2.5**

Let $\widetilde{a}_j = (\mu_j, \gamma_j), \forall j = 1,2,\ldots\ldots,n$ be IF numbers. A score function has described by

$$S(A) = \mu_j - \gamma_j.$$

**Definition 2.6**

The correlation coefficient of IFSs based on Robinson & Indhumathi's technique(?) is provided in this work. Let $X = \{x_1, x_2, \ldots., x_n\}$ be the finite universal class and let $A, B \in IFS(X)$ as

$A = \{\langle x, (\mu_A(x), ((1-\gamma_A(x)), \pi_A(x)), \rangle / x \in X\} B = \{\langle x, (\mu_B(x), ((1-\gamma_B(x)), \pi_B(x)), \rangle / x \in X\}$.

The correlation of $A, B \in IFS(X)$ is defined as follows:

$$C_{IFS}(B,B) = \frac{1}{n}\sum_{i=1}^{n}\left(\mu_A^2(x) + (1-\gamma_A^2(x)) + \pi_A^2(x)\right), C_{IFS}(B,B) = \frac{1}{n}\sum_{i=1}^{n}(\mu_B^2(x) + (1-\gamma_B^2(x)) + \pi_B^2(x)),$$

and

$$C_{IFS}(A,B) = \frac{1}{n}\sum_{i=1}^{n}\left[u_A(x_i)u_B(x_i) + (1-\gamma_A(x_i))(1-\gamma_B(x_i)) + \pi_A(x_i)\pi_B(x_i)\right].$$

Also, the correlation coefficient of $A, B \in IFS(X)$ is

$$\rho_{IFS}(A,B) = \frac{C_{ZL}(A,B)}{\sqrt{C_{ZL}(A,A)C_{ZL}(B,B)}}.$$

# 3 Results and Discussion

## 3.1 A method of employing IF information in group decision making

**Step 1:** To create a collective IF decision matrix $R = (r_{ij})_{\text{mxn}}$, combine all of the individual IF decision matrices utilized the IFWA operator.

**Step 2:** Apply the IFGHA operator to get the alternative Ai's collective overall preference IF values, where $\omega = (w_1, w_2, \ldots\ldots, w_n)^T$ refer the decision makers' weighting vector and $\omega_k \in [0,1], \sum_{k=1}^{t} \omega_k = 1;\quad w = (w_1, w_2 \ldots w_n)$ is the corresponding weighting vector of the IFGHWA operator with $w_j \in [0,1], \sum_{j=-1}^{n} w_j = 1$.

**Step 3:** Determine correlation coefficient and score function between the positive ideal value and the total preference values.

**Step 4:** Sort through all of the options and choose one based on the score function.

## 3.2 Singular Perturbation Problems

Numerous fields, such as fluid mechanics, optimal control, aerodynamics, reaction-diffusion processes, etc incorporate singularly perturbed differential equations. A tiny parameter in such problems multiplies to the largest derivative. A solution to this kind of issue exhibits distinct internal layers or boundaries when the singular perturbation parameter $\epsilon$ is extremely small. These solitary perturbation parameters are presumed to be one of a kind. Solutions' structural elements exhibit overlapping layers. To formulate a mathematical model for this problem, Shishkin piecewise-uniform meshes were developed and employed alongside a standard finite difference discretization. The numerical approximation generated by this model is shown to be uniformly second order convergent for every parameter. The ensuing two-point border value problem is

$$-Eu'' + A(x)u\,(x) = f(x), x \in (0,1), u(0) \text{ and } u(1) \text{ given.}$$

$\forall x \in [0,1], \vec{u}(x) = (u_1(x), u_2(x))^T$ and $\vec{f}(x) = (f_1(x), f_2(x))^T$.

$E, A(x)$ are $2 \times 2$ matrices. $E = diag(\vec{\varepsilon}), \vec{\varepsilon} = (\varepsilon_1, \varepsilon_2)$ with $0 \prec \varepsilon_1 \leq \varepsilon_2 \leq 1$.

The $\varepsilon_i$ are thought to be distinct as well as for ease of use, to have the ordering $\varepsilon_1 \prec \varepsilon_2$.

#### *3.2.1. Shishkin mesh*

A piecewise uniform shishkin mesh $\overrightarrow{\Omega}^N$ having N mesh-intervals is constructed on $\overrightarrow{\Omega} = [0,1]$ as given. $[0,1]$ has been divided into 5 sub-intervals

$$[0,\tau_1] \cup [\tau_1,\tau_2] \cup [\tau_2, 1-\tau_2] \cup [1-\tau_2, 1-\tau_1] \cup [1-\tau_1, 1]$$

where,

$$\tau_2 = \min\left\{\frac{1}{4}, 2\sqrt{\frac{\varepsilon_2}{\alpha}}\ln N\right\},$$

$$\tau_1 = \min\left\{\frac{\tau_2}{2}, 2\sqrt{\frac{\varepsilon_1}{\alpha}}\ln N\right\}.$$

Trivially $0 \prec \tau_1 \prec \tau_2 \leq \frac{1}{4}$. Implies that, on the subinterval $\tau_2, 1-\tau_2$ a uniform mesh with $\frac{N}{2}$ mesh points has been located also on each sub-intervals $[0,\tau_1], (\tau_1,\tau_2], (1-\tau_2, 1-\tau_1]$ and $[1-\tau_1, 1]$, a uniform mesh of $\frac{N}{8}$ points is located. Keep it in mind that, when both objects $\tau_r, r = 1,2$ take on their left-hand value, the shishkin mesh yields a classical uniform mesh on $[0,1]$.

In the case,$\varepsilon_1 = \varepsilon_2$ a simple construction with just one parameter $\tau$ is sufficient.

#### *3.2.2. Discrete problem*

The current study extends the definition of the grid-based discrete two-point boundary value problem by the finite difference methodology. $-E\delta^2 U + A(x)U = f(x), u(0) = u(0)$ and $u(1) = u(1)$.

This serves to calculate a numerical approximation of the precise answer and may be represented in operator form.

$$L^N U = f, U(0) = u(0), U(1) = u(1)$$

here $L^N = -E\delta^2 + A(x)$

also $\delta^2, D^+, D^-$ are denotes the difference operators

$$\delta^2 U(x_j) = \left(\frac{D^+U(x_j) - D^-U(x_j)}{\bar{h}_j}\right), \bar{h}_j = \frac{h_j + h_{j+1}}{2}, h_j = x_j - x_{j-1}.$$

$$D^+U(x_j) = \frac{U(x_{j+1}) - U(x_j)}{h_{j+1}} \text{ and } D^-U(x_j) = \frac{U(x_j) - U(x_{j-1})}{h_j}$$

### 3.3 MAGDM Problem Deterrer Weight Determination Employing a Singular Perturbation Problem System

#### *3.3.1. Issue raised by the decision-maker*

The weighting vector is represented by the decision maker using the subsequent array of second order SPDE:

$$-E\vec{u}''(x) + A(x)\vec{u}(x) = \vec{f}(x), for x \in (0,1), \vec{u}(0) = \vec{0}, \vec{u}(1) = \vec{0}$$

where, $E = diag(\varepsilon_1, \varepsilon_2), A = \begin{pmatrix} 4 & -2 \\ 2 & 4 \end{pmatrix}, \vec{f} = (1, e^y)$.

The numerical solution, which is provided above, could be computed utilising the traditional finite difference model. Tables 1 & 2 provide the weighting vectors' normalisation, and the weighting vectors may be computed with the aid of various toa values.

Therefore, the decision maker's provided weighting vectors are computed simply $\omega = (0.20361, 0.34305, 0.20044, 0.25290)^T$, $w = (0.14538, 0.45220, 0.14209, 0.26029)^T$

**Table 1.** Numerical solution of $-E\vec{u}''(x)+A(x)\vec{u}(x)=\vec{f}(x)$

| X | $u_1(x)$ | Normalize | $u_2(x)$ | Normalize |
|---|---|---|---|---|
| 0.12500 | 0.33468 | 0.20361 | 0.16232 | 0.14538 |
| 0.03125 | 0.56387 | 0.34305 | 0.50475 | 0.45220 |
| 0.25000 | 0.32947 | 0.20044 | 0.15860 | 0.14209 |
| 0.06250 | 0.41570 | 0.25290 | 0.29054 | 0.26029 |

**Table 2.** Numerical solution of $-E\vec{u}''(x)+A(x)\vec{u}(x)=\vec{f}(x)$

| X | $u_1(x)$ | Normalize | $u_2(x)$ | Normalize |
|---|---|---|---|---|
| 0.87500 | 0.45324 | 0.29869 | 0.40204 | 0.39107 |
| 0.21875 | 0.32655 | 0.21520 | 0.15243 | 0.14827 |
| 0.75000 | 0.41258 | 0.27189 | 0.32490 | 0.31603 |
| 0.18750 | 0.32505 | 0.21421 | 0.14867 | 0.144615 |

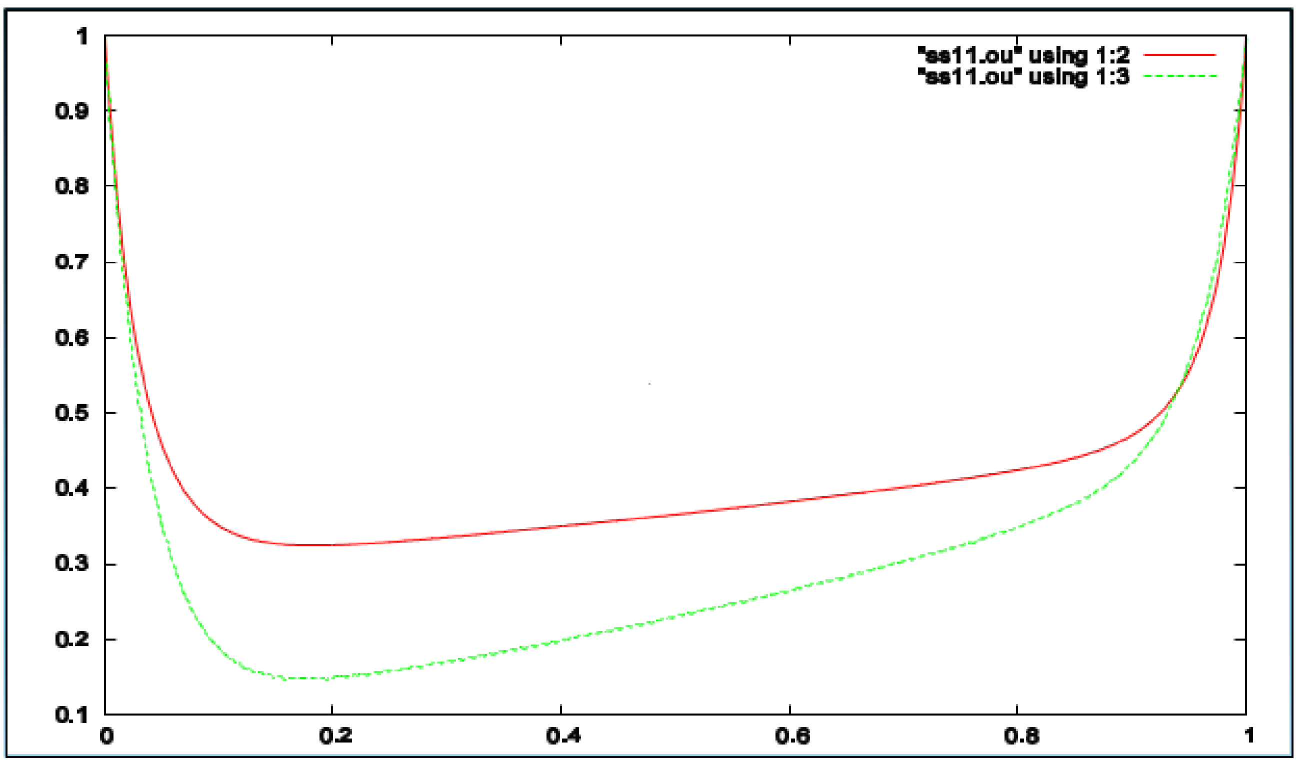


**Fig 1.** Solution of $-E\vec{u}''(x)+A(x)\vec{u}(x)=f(x)$

### *3.3.2. Numerical Illustration*

Assume that a wealth management organisation employs a fund manager. Risk (a1), growth (a2), socio-political challenges (a3), and ecological consequences (a4) are the four potential investment possibilities that the corporation expects the fund manager to look at. As an IFS, the fund manager might only feel satisfied offering his or her evaluation of all possibilities with regard to every attribute, whereby the decision criteria are formulated as follows:

$$R^1=\begin{pmatrix}(0.4210,0.4239) & (0.3708,0.4415) & (0.4759,0.3255) & (0.4511,0.3377)(0.4123,0.3124)\\ (0.4635,0.3470) & (0.4711,0.2856) & (0.4127,0.3364) & (0.4087,0.2674)(0.4010,0.2561)\\ (0.3722,0.3243) & (0.4191,0.1999) & (0.1930,0.5455) & (0.2640,0.3041)(0.1923,0.3141)\\ (0.2707,0.1672) & (0.1482,0.1737) & (0.1409,0.5869) & (0.1374,0.5935)(0.1200,0.1654)\end{pmatrix}$$

$$R^2 = \begin{pmatrix} (0.3700, 0.3931) & (0.5100, 0.4101) & (0.4694, 0.1164) & (0.4032.0.1570)(0.4132, 0.1758) \\ (0.4441, 0.1923) & (0.4087, 0.2850) & (0.3275, 0.4381) & (0.1509, 0.1323)(0.3457, 0.1487) \\ (0.3275, 0.1509) & (0.4381, 0.3461) & (0.2831, 0.1323) & (0.4215, 0.3425)(0.4512, 0.2438) \\ (0.3554, 0.5425) & (0.5054, 0.1122) & (0.1823, 0.1239) & (0.1181, 0.6254)(0.1542, 0.5652) \end{pmatrix}$$

$$R^3 = \begin{pmatrix} (0.4356, 0.5100) & (0.5253, 0.2761) & (0.4535, 0.1270) & (0.2876, 0.2486)(0.2050, 0.2810) \\ (0.4422, 0.2255) & (0.3911, 0.2469) & (0.3185, 0.2804) & (0.7503, 0.1100)(0.5730, 0.2145) \\ (0.3637, 0.1664) & (0.3268, 0.1877) & (0.5906, 0.3322) & (0.4268, 0.3422)(0.3867, 0.2233) \\ (0.4152, 0.3211) & (0.2144, 0.2328) & (0.3896, 0.4722) & (0.1321, 0.2571)(0.3965, 0.2333) \end{pmatrix}$$

$$R^4 = \begin{pmatrix} (0.4039, 0.1788) & (0.4329, 0.3482) & (0.3419, 0.2057) & (0.4254, 0.2866)(0.4542, 0.2686) \\ (0.4471, 0.3233) & (0.5334, 0.1291) & (0.5050, 0.1567) & (0.4835, 0.2008)(0.4587, 0.2798) \\ (0.5157, 0.1160) & (0.4629, 0.1123) & (0.5184, 0.1161) & (0.4421, 0.1937)(0.4124, 0.1739) \\ (0.2776, 0.1194) & (0.4008, 0.2776) & (0.1257, 0.4388) & (0.2863, 0.1808)(0.2369, 0.1989) \end{pmatrix}$$

Employing the weighting vector $\omega = (0.20361, 0.34305, 0.20044, 0.25290)^T$ we obtain the collective matrix in step 1 as

$$R = \begin{bmatrix} (0.40265, 0.34460)(0.46839, 0.36898)(0.43779, 0.16866)(0.39788, 0.23428)(0.38750, 0.24168) \\ (0.44848, 0.25533)(0.45237, 0.22676)(0.39297, 0.29272)(0.45572, 0.16351)(0.43767, 0.20975) \\ (0.39642, 0.16825)(0.42007, 0.20596)(0.40644, 0.20541)(0.39909, 0.28938)(0.38232, 0.23157) \\ (0.33281, 0.26207)(0.36369, 0.17851)(0.20775, 0.30617)(0.17048, 0.37831)(0.22359, 0.28301) \end{bmatrix}$$

Employing step 2 with $w = (0.14538, 0.45220, 0.14209, 0.26029)^T$ and $\gamma = (0.29869, 0.21520, 0.27189, 0.21421)^T$ we get:

Similarly, we may determine the remaining values of $S_i, i = 2, 3, ..., 5$. Ranking the best alternative according to the score function $S_1$.

By using step 3, we can calculate the score function and correlation coefficient for different values of $q$.

**For** $q = 2$

$s_1 = (0.70704, 0.17931); s_2 = (0.72208, 0.07423); s_3 = (0.66716, 0.08934);$

$s_4 = (0.68514, 0.07139); s_5 = (0.68361, 0.07687).$

**For** $q = 1$

$S_1 = (0.49991, 0.32646); S_2 = (0.52140, 0.14295); S_3 = (0.44511, 0.17071);$

$S_4 = (0.46943, 0.13769); S_5 = (0.46732, 0.14783).$

**For** $q \to 0$

$S_1 = (0.42419, 0.35978); S_2 = (0.45577, 0.24703); S_3 = (0.38158, 0.21169);$

$S_4 = (0.36339, 0.26060); S_5 = (0.38244, 0.23537).$

**For** $q \to \infty$

$s_1 = (0.73958, 0.45652); s_2 = (0.74371, 0.03490); s_3 = (0.67652, 0.06218);$

$s_4 = (0.74724, 0.01666); s_5 = (0.72789, 0.02926).$

Utilising the above results, the best alternative by score function has been discussed in Table 3 and the corresponding correlation has been illustrated in Table 4.

**Table 3.** Ranking by Score Function

| **Methods** | **Ranking the alternatives** |
|---|---|
| When $q = 2$ | $s_2 > s_4 > s_5 > s_3 > s_1$ - Best alternative is $s_2$ |
| When $q = 1$ | $s_2 > s_4 > s_5 > s_3 > s_1$ - Best alternative is $s_2$ |
| When $q \to 0$ | $s_2 > s_3 > s_5 > s_4 > s_1$ - Best alternative is $s_2$ |
| When $q \to \infty$ | $s_4 > s_2 > s_5 > s_3 > s_1$ - Best alternative is $s_4$ |

For second set of weighting vectors $\omega = (0.20361, 0.34305, 0.20044, 0.25290)^T w = (0.14538, 0.45220, 0.14209, 0.26029)^T$ and $\lambda = (0.39107, 0.14827, 0.31603, 0.144615)$ the same above procedure is followed and the overall values are as below:

**Table 4.** Ranking with Correlation Coefficient

| Methods | Ranking the alternatives |
|---|---|
| When $q=2$ | $s_1 > s_2 > s_3 > s_5 > s_4$ - Best alternative is $s_1$. |
| When $q=1$ | $s_1 > s_2 > s_3 > s_5 > s_4$ - Best alternative is $s_1$. |
| When $q \to 0$ | $s_1 > s_2 > s_5 > s_3 > s_4$ - Best alternative is $s_1$. |
| When $q \to \infty$ | $s_2 > s_4 > s_5 > s_1 > s_3$ - Best alternative is $s_2$. |

**For** $q=2$
$S_1 = (0.73335, 0.18793)$;$S_2 = (0.74460, 0.06295)$;$S_3 = (0.69240, 0.07561)$;
$S_4 = (0.71699, 0.05225)$; $S_5 = (0.71639, 0.06314)$.
**For** $q=1$
$S_1 = (0.53781, 0.34054)$;$S_2 = (0.55443, 0.12194)$;$S_3 = (0.47942, 0.14550)$;
$S_4 = (0.51409, 0.10178)$; $S_5 = (0.51322, 0.12231)$.
**For** $q \to 0$
$S_1 = (0.45039, 0.36935)$;$S_2 = (0.47734, 0.22180)$;$S_3 = (0.40439, 0.18588)$;
$S_4 = (0.38862, 0.22292)$; $S_5 = (0.41792, 0.21790)$.
**For**$q \to \infty$
$s_1 = (0.73958, 0.45652)$;$s_2 = (0.74371, 0.03490)$;$s_3 = (0.67652, 0.06218)$;
$s_4 = (0.74724, 0.01666)$; $s_5 = (0.72789, 0.02926)$.
Illustrate the above results as Table 5 and its correlation by Table 6.

**Table 5.** Ranking Comparison in terms of Score Function

| Methods | Ranking the Alternatives |
|---|---|
| When $q=2$ | $s_2 > s_4 > s_5 > s_3 > s_1$ - Best alternative is $s_2$. |
| When $q=1$ | $s_2 > s_4 > s_5 > s_3 > s_1$ - Best alternative is $s_2$. |
| When $q \to 0$ | $s_2 > s_3 > s_5 > s_4 > s_1$ - Best alternative is $s_2$. |
| When $q \to \infty$ | $s_4 > s_2 > s_5 > s_3 > s_1$ - Best alternative is $s_4$. |

**Table 6.** Ranking Comparison Using Correlation Coefficient

| When $q=2$ | $s_1 > s_2 > s_3 > s_5 > s_4$ - Best alternative is $s_1$. |
|---|---|
| When $q=1$ | $s_1 > s_2 > s_3 > s_5 > s_4$ - Best alternative is $s_1$. |
| When $q \to 0$ | $s_1 > s_2 > s_5 > s_3 > s_4$ - Best alternative is $s_1$. |
| When $q \to \infty$ | $s_2 > s_4 > s_5 > s_1 > s_3$ - Best alternative is $s_2$. |

The result obtained from the proposed IFGHWA operator is same as the results obtained by using the IFGHA operator the method of Liu & Jiang[9]. Hence from the table, it can be easy in identifying the most desirable alternative.

## 4 Conclusion

This study focused on exploring weighted averaging operators, methods for determining appropriate weights, and generalized hybrid weighted averaging operators. The main objective was to enhance decision-making processes by addressing the challenge of uncertainty in decision maker weights. To achieve this, it utilized a set of differential equations with singular perturbations to model and determine these uncertain weights, which are key to the aggregation of decision inputs. This approach serves a dual purpose: firstly, it minimizes the impact of unfair or biased arguments that could distort the final decision, ensuring that the decision process remains fair and unbiased. Secondly, it preserves the integrity of the original decision knowledge, preventing it from being lost or altered during the aggregation process. By introducing this novel methodology, we provide a more robust and reliable framework for decision-making that maintains the authenticity of the decision inputs while mitigating the influence of unreliable or imbalanced contributions. This work advances the field by offering a comprehensive approach to weighted aggregation in uncertain and dynamic decision-making environments. Future research will focus on the weight determining methods for SPP in different domains of fuzzy set to real-world decision-making problems.

## References


1) Ruipu Y, Huijuan G. A Multiattribute group decision-making method based on a new aggregation operator and the means and variances of interval-valued intuitionistic fuzzy values. *Information Sciences*. 2022. https://doi.org/10.1038/s41598-022-27103-z.
2) Verma R, Merig JM. A new decision-making method using interval-valued intuitionistic fuzzy cosine similarity measure based on the weighted reduced intuitionistic fuzzy sets. *Informatica*. 2020;31(2):399–433. https://doi.org/10.15388/20-INFOR405.
3) Zhao M, Wei G, Wei C. Extended CPT-TODIM method for interval-valued intuitionistic fuzzy MAGDM and its application to urban ecological risk assessment. *Journal of Intelligent & Fuzzy Systems*. 2021;40(3):4091–4106. https://doi.org/10.3233/JIFS-200534.
4) You P, Liu XH, Sun JB. A multi-attribute group decision making method considering both the correlation coefficient and hesitancy degrees under interval-valued intuitionistic fuzzy environment. *Information Sciences*. 2021;p. 104–107187. https://doi.org/10.1016/j.asoc.2021.107187.
5) Kumar K, Chen SM. Group decision making based on weighted distance measure of linguistic intuitionistic fuzzy sets and the TOPSIS method. *Information Sciences*. 2022;611:660–676. https://doi.org/10.1016/j.ins.2022.07.184.
6) Adilaxmi M, Swamy DK. Solution of singularly perturbed differential difference equations using Liouville Green transformation. *Journal of mathematical and computational science*. 2021;11(1):970–990. https://doi.org/10.28919/jmcs/5054.
7) Zhang ZM, Chen SM. Optimization-based group decision making using interval-valued intuitionistic fuzzy preference relations. *Information Sciences*. 2021;561:352–370. https://doi.org/10.1016/j.ins.2020.12.047.
8) Chen X, Suo CF, Li YG. Distance measures on intuitionistic hesitant fuzzy set and its application in decision-making. *Computational and Applied Mathematics*. 2021;40(3):63–84. https://doi.org/10.1007/s40314-021-01478-7.
9) Liu Y, Jiang W. A new distance measure of interval-valued intuitionistic fuzzy sets and its application in decision making. *Soft Computing*. 2020;24(9):6987–7003. https://doi.org/10.1007/s00500-019-04332-5.
10) Garg H, Kumar K. A novel exponential distance and its based TOPSIS method for interval-valued intuitionistic fuzzy sets using connection number of SPA theory. *Artificial Intelligence Review*. 2020;53(1):595–624. https://doi.org/10.1007/s10462-018-9668-5.
11) Priyadarshini J, Balasubramaniam P. Multi-criteria decision making method based on interval-valued intuitionistic fuzzy sets. *The Journal of Analysis*. 2019;27(1):259–276. https://doi.org/10.1007/s41478-018-0122-5.
12) Li Y, Du T, Pang Y, Huang Z. Component Fourier Neural Operator for Singularly Perturbed Differential Equations. *Proceedings of the AAAI Conference on Artificial Intelligence*. 2024;38(12):13691–13699. Available from: https://ojs.aaai.org/index.php/AAAI/article/view/29274.