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ORIGINAL RESEARCH

Solution Of Nonlinear Integro-Differential Equations by Using Laplace Decomposition Method and Series Solution Method

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Abstract

Objectives: To investigate two mutually beneficial methods: One is the Laplace Adomian decomposition method, and the second is the series solution method. This study intends to investigate the efficiency of both approaches in resolving nonlinear integral-differential equations. **Methods:** Each strategy's main ideas and theoretical underpinnings are methodically described, along with each approach's advantages and uses. By contrasting their performances, this work sheds light on the effectiveness and practicality of the series solution method and the Laplace decomposition method. **Findings**: This study reveals that combining these techniques leads to a very successful approach to solving challenging nonlinear situations. Additionally, the results show that this dual strategy increases computing efficiency and improves solution correctness. Overall, the results indicate that the Laplace decomposition method and the series solution method provide a strong framework for resolving nonlinear integro-differential equations, making them an important addition to computational mathematics. **Novelty:** This study is unique in that it combines the Laplace decomposition method with the series solution method, resulting in a novel dual technique for solving nonlinear integro-differential equations. The integrated technique improves computing efficiency and solution accuracy. The paper substantially adds to computational mathematics by developing a strong framework for dealing with complicated nonlinear issues.

Keywords: Laplace Decomposition Method; Adomian Polynomials; Series Solution Method; Nonlinear IntegroDifferential Equations; Analytical Solution

1 Introduction

When an unknown function's integrals and derivatives are merged into a single equation, such an equation is considered integral-differential. These equations are fundamental in science and engineering since they are used to simulate various technical and physical processes. They are used in many different disciplines, especially when it's necessary to comprehend and analyse complicated systems. Nonlinear integro-differential equations aid in the description of complex system behaviours and interactions in both scientific and industrial settings[1]. These equations are frequently used to explain nonlinear occurrences in fields including chemical kinetics, fluid dynamics, solid-state physics, plasma physics, and mathematical biology[2]. These equations, for instance, represent the rate of reactions in chemical kinetics while considering temperature and concentration. They described how fluids flow when subjected to different forces in fluid dynamics[3]. These equations are essential for comprehending how particles interact at the atomic level in materials or high-energy settings in plasma and solid-state physics. They are used in mathematical biology to simulate disease transmission and population dynamics[1] and solving such equations is a challenging task. Many researchers have solved these types of Integro differential equations using numerical methods. In[2], Bharathi, SD et al. solved Volterra-Fredholm integro-differential equations using the modified Laplace decomposition method. In[4], Rani et al. introduce a powerful tool for solving Volterra integral and integro-differential equations using a modified Laplace decomposition method. Elsaid in[5] introduces a powerful tool for iterative methods of series solutions of nonlinear equations. Some other related works for the solution of integro differential equations are available in[6]. In this work, we adopted two analytic methods for the solution integro-differential equations. The first method is the Laplace decomposition method introduced in[2]. This method is applied there for Volterra-Fredholm integro-differential[7] or partial differential equations[8]. Here, we apply the Laplace decomposition method for the solution of Volterra nonlinear integral-differential equations. Further, we demonstrated the solution of Volterra nonlinear integral-differential equations using the series solution method introduced in[9].

Nonlinear integro-differential equations occur in two primary forms. The first and second kinds of nonlinear Fredholm integro-differential equations and the initial and secondary kinds of Volterra integro-differential are of a nonlinear type. Here, we are employing second-kind Volterra integro-differential equations, which are nonlinear.

$$v^{(i)}(x) = g(x) + \int_0^x \kappa(x,t) G(v(t)) dt \tag{1}$$

where $v^i = \frac{d^i v}{dx^i}$, and $G(v(t))$ is a function in $v(t)$.

The following is an outline of this paper's structure:

Section 2: Describes the theoretical foundations and application of the LDM and SSM. Section 3: Showcases the performance and efficacy of the techniques described by analysing two examples and presenting comparison data. Section 4: Summarises the main conclusions and revelations from the analysis and comparisons, as well as any possible ramifications for further study or applications.

2 Methodology

2.1 Laplace Decomposition Method:

When we combine the Laplace transform and the Adomian decomposition method (ADM), it is called the Laplace decomposition method (LDM)[2]. This method is often employed to solve complex nonlinear differential equations. The capacity of the LDM to produce precise or extremely accurate approximation solutions to these equations is one of its biggest benefits. The technique makes a nonlinear problem more comprehensible by dissecting it into several smaller, more manageable parts. Every one of these elements is methodically resolved, and a combination of these resolutions yields the entire answer to the problem. Since its development by Suheil A. Khuri, LDM has shown to be incredibly successful in solving a wide range of differential equations. The method's adaptability and usefulness have been demonstrated by its successful applications in various fields, including fluid dynamics, chemical processes, biological systems, and more. In both theoretical research and real-world applications, the LDM remains a crucial technique for resolving nonlinear differential and integral equations[8,10]. When we apply Laplace transformation to both sides of equation (1) then, we obtain

$$sj\, L\{v(x)\} - sj-1\, v(0) - sj-2\, v'(0) - - \ldots\ldots v(j-1)\,(0) = L\{g(x)\} + L\{\kappa(x-t)\} L\{G(v(t))\}, \tag{2}$$

on solving above equation (2), we get,

$$\mathrm{L}\{\mathrm{v(x)}\} = \frac{1}{s}\mathrm{v}(0) + \frac{1}{s^2}\mathrm{v}'(0) + \ldots + \frac{1}{s^j}\mathrm{v}^{(\mathrm{j}-1)}(0) + \frac{1}{s^j}\,\mathrm{L}\{\,\mathrm{g(x)}\} + \frac{1}{s^j}\,\mathrm{L}\{\kappa(\mathrm{x}-\mathrm{t})\}\mathrm{L}\{\mathrm{G(v(t))}\} \tag{3}$$

To achieve this, we begin by representing the left-hand linear expression, v(x), using a limitless sequence of elements provided by,

$$v(x) = \sum_{n=0}^{\infty} v_n(x) \tag{4}$$

where a recursive determination of the elements $v_n(x)$, for $n \geq 0$ is being performed. The nonlinear term $G(v(x))$ is difficult to handle, so we use the Adomian polynomials. The nonlinear term $G(v(x))$ shall be represented through Adomian polynomials introduced in [7,11],

$$G(v(x)) = \sum_{n=0}^{\infty} A_n(x) \tag{5}$$

where $A_n = \frac{1}{n!}\frac{d^n}{d\lambda^n}\left\{\left(\sum_{j=0}^{n}\lambda^j v_j\right)\right\}_{\lambda=0,n=0,1,2,3,4,\ldots\ldots}$, is achievable for all types of nonlinearities. The result of substituting Eq. (4) and Eq. (5) into Eq. (3) is

$$\mathrm{L}\left\{\sum_{n=0}^{\infty} v_n(x)\right\} = \frac{1}{s}v(0) + \frac{1}{s^2}v'(0) + \cdots + \frac{1}{s^j}v^{(j-1)}(0) + \frac{1}{s^j}L[g(x)] + \frac{1}{s^j}L[\,\mathrm{K}(x- t)]L\left\{\sum_{n=0}^{\infty} A_n(x)\right\}. \tag{6}$$

The Adomian decomposition approach permits the use of the following recursive connection.

$$L\{v_0(x)\} = \frac{1}{s}v(0) + \frac{1}{s^2}v'(0) + \cdots + \frac{1}{s^j}v^{(j-1)}(0) + \frac{1}{s^j}L\{g(x)\}, \tag{7}$$

and

$$\mathrm{L}\{v_{n+1}(\mathrm{x})\} = \frac{1}{s^j}\,\mathrm{L}\{\,\mathrm{K}(\mathrm{x}-\mathrm{t})\}\mathrm{L}\{A_n(x)\}, \quad \mathrm{n} \geq 1 \tag{8}$$

where, $v_0(x)$, which defines A_0, is produced by transforming (7) with the inverse Laplace transform. Eq. (8) will ultimately result in the complete determination of the elements in Eq. (4) by recursive relation.

2.2 Series Solution Method:

Taylor's series for analytic functions is the primary source of inspiration for the series solution method [12]. This technique is a potent tool for solving differential equations and is frequently called the Taylor series or series solution method [13]. This method is popular because it provides a simple means of representing complicated functions and may be used in several fields, such as engineering, physics, and mathematics [14]. Analytical function expression is essential for resolving issues where other approaches might not yield precise answers.

$$v_k(x) = \sum_{n=0}^{k} \frac{v^{(n)}(b)}{n!}(x-b)^n, \tag{9}$$

approaches $v(x)$ in convergence in the vicinity of the Taylor series in its general form when the expression for $x = 0$ is as,

$$v(x) = \sum_{n=0}^{\infty} a_n x^n \tag{10}$$

The solution to the Volterra nonlinear integro-differential equations is indicated as $v(x)$.

$$v^{(n)}(x) = g(x) + \chi\int_0^x K(x,t)G(v(t))dt, v^k(0) = k!a_k, \quad 0 \leq \mathrm{k} \leq (\mathrm{n}-1) \tag{11}$$

The coefficients a_n will be found recurrently in Eq. (10). Indeed, using the specified initial conditions, one may find the first few coefficients, a_k as

$$a_0 = v(0), a_1 = v'(0), a_2 = \frac{1}{2!}v''(0), a_3 = \frac{1}{3!}v'''(0), a_4 = \frac{1}{4!}v^{iv}(0), a_5 = \frac{1}{5!}v^{v}(0) \tag{12}$$

etc.

The SSM will be applied to Volterra integro-differential equations, which is a nonlinear type (11) in order to ascertain the remaining coefficient a_k of (10). When (10) is substituted into both sides of (11) it yields

$$\sum_{k=0}^{\infty} a_k {x^k}^{(n)} = T(g(x)) + \int_0^x \kappa(x,t)G\sum_{k=0}^{\infty} x^k dt \tag{13}$$

Once the right hand side of integral (11) has been integrated and the coefficients of the same powers of x have been gathered, the integro-differential Eq. (11) will be transformed into a conventional integral Eq. (13). The mapping $T(x)$ have a Taylor series expansion for $g(x)$. To identify a recurrent relationship in $a_j, j \geq 0$, we examine the two sides of the equation and compare the coefficients of the same power of x. The coefficients can be entirely determined by solving the recurrent relation and initial conditions [6,15]. After determining the coefficients $a_j, j \geq 0$, the series solution can be obtained by replacing the determined coefficients in Eq. (10). Researchers may find similar type of work in [9,15].

3 Application of Methods:

We used the series solution method (SSM) and the Laplace decomposition method (LDM) to solve the nonlinear integro-differential equations. When these approaches are integrated, a solid basis for dealing with complicated nonlinear problems is created. Using the benefits of both methodologies, the primary purpose of this work is to effectively solve the integro differential equation using LDM and SSM. While SSM allows for an analytic series representation of the solution, increasing accuracy and convergence, LDM simplifies nonlinear equations without the need for linearization. When these approaches are coupled, nonlinear integro-differential equations may be solved more accurately and efficiently.

Example 3.1:

Consider the nonlinear integro-differential equation,

$$v^{'}(x) = \frac{-1}{3} + \frac{5}{3} cosx - \frac{1}{3} cos^2 x + \int_0^x sin\ (x-t)\ v^2(t)\ \ dt, v(0) = 0 \tag{14}$$

i. Employing the Laplace Decomposition Method:

Applying the Laplace transform to Eq. (14), and using the given initial conditions we obtain

$$\begin{aligned} \mathrm{L}\left[v'(x)\right] &= L\left[\frac{-1}{3} + \frac{5}{3}\cos x - \frac{1}{3}\cos^2 x + \int_0^x \sin(x-t)v^2(t)dt\right] \\ \mathrm{L}\left[v'(x)\right] &= \frac{-1}{3s^2} + \frac{5}{3}\frac{1}{s^2+1} - \frac{1}{6s^2} - \frac{1}{6(s^2+4)} + \frac{1}{(s^2+1)} L\left[v^2(x)\right] \end{aligned} \tag{15}$$

By performing the inverse Laplace transform on Eq. (15), we acquire

$$v(x) = x - \frac{1}{6}x^3 - \frac{1}{120}x^5 + \frac{1}{560}x^7 + L^{-1}\frac{1}{(s^2+1)}L[v^2(x)] \tag{16}$$

We decompose the solution; the Adomian polynomial is represented as an infinite sum and a nonlinear term as follows:

$$v(x) = \textstyle\sum_{n=0}^{\infty} v_n(x) \text{ and } v^2(x) = \sum_{n=0}^{\infty} A_n \tag{17}$$

By substituting Eq. (17) in Eq. (16), we obtain,

$$\textstyle\sum_{n=0}^{\infty} v_n(x) = x - \frac{1}{6}x^3 - \frac{1}{120}x^5 + \frac{1}{560}x^7 + \ldots L^{-1}\ \frac{1}{(s^2+1)}\ L[\sum_{n=0}^{\infty} A_n] \tag{18}$$

Comparing both sides of the Eq. (18), we get,

$v_0(x) = x - \frac{x^3}{6} - \frac{x^5}{120} + \frac{x^7}{560}$,

$v_1(x) = L^{(-1)}\left(\frac{1}{(s^2+1)}L[A_0]\right]$,

$v_2(x) = L^{(-1)}\left(\frac{1}{(s^2+1)}L[A_1]\right]$,

.

.

.

where $A_0 = v_0^2,\ \ A_1 = 2v_0 v_1,\ \ A_2 = 2v_0\, v_2 + v_1^2 \ldots$. and so on.

We get the following recursive relation

$v_0(x) = x - \frac{1}{3!}x^3 - \frac{1}{5!}x^5 + \frac{1}{560}x^7$,

$v_1(x) = \frac{x^5}{60} - \frac{x^7}{504} + \ldots$,

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Therefore, the series solutions are provided by using (17).

$v(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ldots$.

This brings us to the precise resolution, $v(x) = sinx$, which is the required precise solution of example 3.1.

ii. Employing Series Solution Method:

Substituting $v(x)$ and $v^{'}(x)$ in Eq. (14) as follows,

$$\text{Since, } v(x) = \sum_{n=0}^{\infty} a_n x^n. \tag{19}$$

Differentiating Eq. (19) w. r. t. x, we get,

$$\begin{aligned} v'(x) &= \sum_{n=1}^{\infty} n a_n x^{n-1} \\ v'(x) &= a_1 + 2a_2x + 3a_3x^2 + 4a_4x^3 + 5a_5x^4 + 6a_6x^5 + 7a_7x^6 + \cdots \end{aligned} \tag{20}$$

Evaluating the first half of the right-hand side of Eq. (14), we get

$$\frac{-1}{3} + \frac{5}{3}cosx - \frac{1}{3}cos^2x = 1 - \frac{x^2}{2} - \frac{1}{24}x^4 + \frac{1}{80}x^6 + \ldots \tag{21}$$

Evaluating the integral appeared in the second half of the RHS of Eq. (14) using the given initial condition

$$\begin{aligned} &v(0) = 0 = a_0 \\ &\int_0^x \sin(x-t)v^2(t)dt = \frac{a_1^2}{12}x^4 - \frac{a_1^2}{360}x^6 + \frac{a_1^2}{20160}x^8 + \cdots + \frac{a_2^2}{30}x^6 - \frac{a_2^2}{1680}x^8 + \ldots + \frac{a_3^2}{56}x^8 \\ &\ldots + \frac{a_1a_2}{10}x^5 - \frac{a_1a_2}{420}x^7 + \cdots + \frac{a_1a_3}{15}x^6 - \frac{a_1a_3}{840}x^8 + + \ldots + \frac{a_1a_4}{21}x^7 + \cdots + \frac{a_1a_5}{28}x^8 + \cdots \end{aligned} \tag{22}$$

After combining Eq. (21) and Eq. (22) and contrasting the like powers of x in Eq. (20), the resulting system of equations may be solved. we get

$a_1 = 1!,\ \ a_2 = 0,\ \ a_3 = \frac{-1}{3!},\ \ a_4 = 0,\ \ a_5 = \frac{1}{5!},\ \ a_6 = 0,\ \ a_7 = \frac{-1}{7!}.$

Then, we obtain the final result

$v(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ldots.$

Thus, in the limiting case, we get $v(x) = sinx$, which is an exact solution of example 3.1.

Example 3.2:

Consider the integro-differential equation, which is a nonlinear type,

$$v^{''}(x) = 2 + 2x + x^2 - x^2e^x - e^{2x} + \int_0^x e^{x-t}v^2(t)\,dt,\ \ v(0) = 1, v^{'}(0) = 2 \tag{23}$$

i. Employing the Laplace Decomposition Method:

Taking the Laplace transforms of Eq. (23), and using the given initial condition, we have,

$$L\left(v^{''}(x)\right] = L\ \left[2 + 2x + x^2 - x^2e^x - e^{2x} + \int_0^x e^{x-t}v^2(t)\ dt\right]$$

$$v(s) = \frac{1}{s} + \frac{2}{s^2} + \frac{2}{s^3} + \frac{2}{s^4} + \frac{2}{s^5} - \frac{2}{s.s(s-1)^3} - \frac{1}{s.s(s-2)} + \frac{1}{s.s(s-1)}L[v^2(x)] \tag{24}$$

By performing the inverse Laplace transform on Eq. (24), we acquire

$$v(x) = 1 + 2x + \frac{1}{2}x^2 - \frac{1}{6}x^4 - \frac{7}{60}x^5 - \frac{7}{180}x^6 + \cdots + L^{-1}\ \left(\frac{1}{s^2(s-1)}L(v^2(x))\right) \tag{25}$$

We decompose the solution, the Adomian polynomial is represented as an infinite sum and a nonlinear term as an infinite sum by,

$$v(x) = \sum_{n=0}^{\infty} v_n(x) \text{ and the nonlinear term by } v^2(x) = \sum_{n=0}^{\infty} A_n \tag{26}$$

substituting Eq. (26) into Eq. (25), we get

$$\sum_{n=0}^{\infty} v_n(x) = 1 + 2x + \frac{1}{2}x^2 - \frac{1}{6}x^4 - \frac{7}{60}x^5 - \frac{7}{180}x^6 + \cdots + L^{-1}\,\frac{1}{s^2(s-1)}L\left[\sum_{n=0}^{\infty} A_n\right] \tag{27}$$

Comparing both sides of above equation, we get,

$$v_0(x) = 1 + 2x + \frac{1}{2}x^2 - \frac{1}{6}x^4 - \frac{7}{60}x^5 - \frac{7}{180}x^6 + \ldots$$

$$v_1(x) = L^{-1}\,\frac{1}{s^2(s-1)}L\left[\sum_{n=0}^{\infty} A_0\right]$$

$$v_2(x) = L^{-1}\,\frac{1}{s^2(s-1)}L\left[\sum_{n=0}^{\infty} A_1\right]$$

.

.

.

where $A_0 = v_0^2,\; A_1 = 2v_0v_1\,,\; A_2 = 2v_0v_2 + v_1^2 \ldots$ and so on. Hence, we have the following recursive relation.

$$v_0(x) = 1 + 2x + \frac{1}{2}x^2 - \frac{1}{6}x^4 - \frac{7}{60}x^5 - \frac{7}{180}x^6 + \ldots$$

$$v_1(x) = \frac{1}{6}x^3 + \frac{5}{24}x^4 + \frac{1}{8}x^5 + \frac{3}{80}x^6 + \ldots$$

$$v_2(x) = \frac{1}{360}x^6 + \ldots$$

.

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Using (26), thus, the series solutions are provided by,

$$v(x) = 1 + 2x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!} + \ldots$$

This brings us to the precise solution $v(x) = x + e^x$, at the limiting case.

ii. Employing Series Solution Method:

Substituting the infinite series sum $v(x)$ in Eq. (23),

$$v(x) = \sum_{n=0}^{\infty} a_n x^n \tag{28}$$

Differentiating Eq. (28) w. r. t. x, we get,
$v^{'}(x) = \sum_{n=1}^{\infty} n.a_n x^{n-1}$.
Again, by differentiating the above equation, we get

$$v''(x) = \sum_{n=2}^{\infty} n(n-1)a_n x^{n-2}$$
$$v''(x) = 2a_2 + 6a_3 x + 12a_4 x^2 + 20a_5 x^3 + 30a_6 x^4 + 42a_7 x^5 + 56a_8 x^6 + \cdots \tag{29}$$

Evaluating the first half on the right-hand side of Eq. (23) we get,

$$2 + 2x + x^2 - x^2 e^x - e^{2x} = 1 - 2x^2 - \frac{7}{3}x^3 - \frac{7}{6}x^4 - \frac{13}{30}x^5 - \frac{141}{1080}x^6 \tag{30}$$

Evaluating the integral in the second half at the RHS of Eq. (23) using the given initial condition
$v(0) = 1 = a_0, \quad v^{'}(0) = 2 = a_1,$

$$\int_0^x e^{(x-t)} v^2(t)\,dt = x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!} + \cdots + \frac{4}{3}x^3 + \frac{x^4}{3} + \frac{x^5}{11} + \frac{x^6}{90} + \frac{x^7}{630} + \cdots +$$

$$\frac{a_2^2}{5}x^5 + \frac{a_2^2}{30}x^6 + \cdots + \frac{a_3^2}{7}x^7 + \cdots + 2x^2 + \frac{2}{3}x^3 + \frac{x^4}{6} + \frac{x^5}{30} + \cdots + \frac{2a_2}{3}x^3 +$$

$$\frac{a_2}{6}x^4 + \frac{a_2}{30}x^5 + \frac{a_2}{180}x^6 + \cdots + \frac{a_3}{2}x^4 + \frac{a_3}{10}x^5 + \frac{a_3}{60}x^6 + \cdots + \frac{2a_4}{5}x^5 +$$

$$\frac{a_4}{15}x^6 + \cdots + \frac{a_5}{3}x^6 + \frac{a_5}{21}x^7 + \cdots + \frac{2a_6}{7}x^7 + \cdots + a_2 x^4 + \frac{a_2}{5}x^5 + \frac{a_2}{30}x^6 +$$

$$\cdots + \tfrac{4a_3}{3}x^5 + \tfrac{2a_3}{15}x^6 + \cdots + \tfrac{2a_4}{3}x^6 + \tfrac{2a_4}{21}x^7 + \cdots + \tfrac{4a_5}{7}x^7 + \ldots \tag{31}$$

Combining Eq. (30) & Eq. (31) and contrasting the like powers of x in Eq. (29), we solve the resulting system of equations. Hence, we obtain
$a_2 = \frac{1}{2!}, \ a_3 = \frac{1}{3!}, \ a_4 = \frac{1}{4!}, \ a_5 = \frac{1}{5!}, \ a_6 = \frac{1}{6!}, \ a_7 = \frac{1}{7!}, \ldots$.
Then, we obtain the final result as

$$v(x) = 1 + 2x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!} + \frac{x^6}{6!} + \frac{x^7}{7!} + \ldots$$

Thus, in the liming case, we get $v(x) = x + e^x$, which is an exact solution of example 3.2.

4 Results and Discussion

In [1,2], Umut Ö and Bharathi, SD et al. applied techniques related to the Adomian decomposition method for initial value problems of integral, differential, and integro-differential equations. Maleknejad, K., and Georgiev SG. et. al. applied the Taylor series method to differential and integral equations [12,13]. Indeed, in [2], Bharathi, SD, and Kamali applied the Modified Laplace Discrete Adomian decomposition method to solve nonlinear Volterra-Fredholm integro-differential equations. Further, Rani D and Mishra V, in [10] applied a modified Laplace Adomian decomposition method based on Bernstein polynomials for the solution nonlinear Volterra integral and integro-differential equations. Moreover, Goldfine, A. applied Taylor series methods to solve Volterra integral and integro-differential equations in [15]. All these articles discuss the approximate solution, not the exact solution. In this work, we applied both methods, i.e., the Laplace decomposition method and the series solution method, for the solution of nonlinear Volterra integro-differential equations, which were unsolved by these techniques. Indeed, in Examples 3.1 and 3.2, we considered nonlinear integro-differential equations with specific initial conditions. Using the Laplace decomposition method, we utilised Adomian polynomials to manage the nonlinear terms effectively, ensuring an accurate analytical solution. The series solution method's coefficients were derived through Taylor series expansion, leading to precise solutions. Both methods demonstrated their efficacy and reliability by providing exact solutions to the integro-differential equations. Both methods are accurate and provide the exact solution. This dual approach highlights their robustness and reinforces their practical value in solving complex problems across scientific and engineering disciplines.

5 Conclusion:

This study solves nonlinear Integro differential equations using both LDM and SSM. Both methods are reliable and effective, yielding very precise approximations or closed-form answers when feasible. The LDM is a flexible technique that may be used for both linear and nonlinear integro-differential equations, even though it necessitates the employment of Adomian polynomials. However, nonlinear integro-differential equations have historically been resolved using the series solution technique. Surprisingly, for the circumstances studied, both methods provide the same outcomes. Finally, when solving nonlinear integral-differential equations, we show that the series solution approach is easier to utilize than the Laplace decomposition method.

6 Contributory statement

The authors contributed equally to the conception, development, and execution of this work.

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