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ORIGINAL ARTICLE

An Economic Reliability Test Plan for Weighted Weibull Distribution

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Abstract

Objectives: Statistical Quality Control is crucial for ensuring product quality in manufacturing environments. This study aims to formulate an economic reliability test plan based on the assumption that the product lifetime adheres to the Weighted Weibull distribution. The goal is to optimize resource allocation by minimizing experimental costs and time while safeguarding both producers and consumers. **Methods:** The termination time corresponding to the predefined producer's risk is determined. Additionally, the Operating Characteristic (OC) curve and minimum total cost are derived and presented. The required minimum sample sizes and acceptance numbers are ascertained using the parameters of the proposed plan. **Findings:** Numerical illustrations applicable to real-world scenarios are provided to facilitate a better understanding of the proposed methodology. These examples highlight the efficiency and practicality of the economic reliability test plan in various manufacturing contexts. **Novelty :** The proposed plan offers a systematic approach to reliability testing, ensuring quality control while being economical and efficient. It stands out by integrating the Weighted Weibull distribution into the reliability sampling plans, providing a method for assessing product lifetime and quality.

Keywords: Acceptance Sampling Plan; Weighted Weibull Distribution; Termination Time; Operating Characteristic Curve; Minimum Total Cost

1 Introduction

In the international business arena, assessing quality is crucial for identifying the distinct characteristics of different manufactured products. It's essential to grasp its importance and the existing methods for quality assessment, while also ensuring continuous improvement in this area. Statistical process control and statistical product control are two approaches utilized through acceptance sampling plans for this purpose. These plans are vital for the inspection of the final product. Their benefit lies in the ability to accept or reject assembled products based on the quality of the inspected sample. Reliability sampling tests are essential and fundamental to verify a product's trustworthiness concerning its lifetime. Its objectives are to find a confident and perfect limit with regard to the mean/median life of an item. In this way, it becomes easier to arrive at a definite decision as to whether the submitted lot may be accepted or rejected on the basis of the mean/median life of the product. The submitted lot is accepted in case the mean life of a product is found above the desired standard and if not, the same is rejected. Such tests are therefore deemed to be very economical. Since an exponential distribution is a central distribution in reliability studies, it necessitates the specifications of a probability model governing the products' life. Kantam et al. [1] (2006) introduced an Economic reliability test plan for Log logistic distribution. Rao et al. [2] (2011) explained the Economic reliability test plan on the basis of Marshall-Olkin extended exponential distribution. Kantam and Sriram. [3] (2013) developed an Economic reliability test plan based on Rayleigh distribution. Rosaiah et al. [4] (2018) has developed an Economic reliability test plan for Exponential pareto distribution. Rosaiah et al. [5] (2018) developed Odds Exponential log logistic distribution for an Economic reliability test plan. Ravikumar et al. [6] (2019) developed Economic reliability test plan for Burr Type X distribution. Naidu C.H. et al [7] (2020) developed Economic reliability test plan for exponentiated half logistic distributed lifetimes. Gillariose et al. [8] (2022) has developed an Economic reliability test plan based on truncated life tests for Marshall-Olkin power lomax distribution. Mugahal et al. [9] (2010) introduced Economic reliability group acceptance sampling plans for Marshall-Olkin extended distribution. KinzaTanveer and YaseenArbabKhattak [10] (2022) developed Economical group and modified group chain sampling plans for Weibull distribution. The primary objectives of acceptance sampling plans are to reduce both time and cost. Among these, cost is particularly significant in the economic model. Hsu [11] (2009) developed an economic design of a single sampling plan. Malathi and Muthulakshmi [12] (2016) studied truncated life test acceptance sampling plans assuring percentile life under Gompert distribution incorporated with the minimum total cost. This paper deals with the construction of a sampling plan and the operating characteristic is presented. The results are illustrated by an example.

2 Methodology

2.1 Weighted Weibull Distribution

Assume for the moment that a product's life distribution follows the Weighted Weibull distribution, with the probability density function and cumulative distribution function (Nasiru, 2015) [13] provided by,

$$f(t;\sigma,\theta,\lambda) = (1+\lambda^{\theta})\,\sigma\theta t^{\theta-1} e^{-(\sigma t^{\theta}+\sigma(\lambda t)^{\theta})} \quad ,t>0;\theta,\sigma,\lambda>0 \tag{1}$$

and

$$F(t;\sigma,\theta,\lambda) = 1-e^{-(\sigma t^{\theta}+\sigma(\lambda t)^{\theta})} \quad ,t>0;\theta,\sigma,\lambda>0 \tag{2}$$

Where σ is a scale parameter and, θ and λ are shape parameters.

2.2 An Economic Reliability Test Plan for Weighted Weibull Distribution

Let n be the number of items put on test taken from an infinite lot and r be the terminating number for the experiment. The experiment terminates if r failures are detected among the n sample items within the termination time t, or when the termination time elapses. In the case of the former scenario, the lot is deemed rejected; otherwise, it is accepted. Choosing the right sample size is a crucial aspect of life test experiments. We constructed the minimum sample size (Table 1). The choice of sample size is inherently influenced by cost factors and the anticipated decision-making time. Larger sample sizes may decrease expected waiting times but increase the cost of experimentation. As a balance between these two aspects let us consider that the sample size be a multiple of the termination number. We know that the probability of $r$failures out of n tested items are given by

$$\sum_{i=0}^{c} \binom{n}{i} p^{i}(1-p)^{n-i} \tag{3}$$

Where $p = F(t;\ \sigma_0,\theta,\lambda)$. If α^{*} is a prefixed risk probability this means

$$\sum_{i=0}^{c} \binom{n}{i} p^{i}(1-p)^{n-i} \geq 1-\alpha^{*} \tag{4}$$

The minimum values of n satisfying the inequality (4) for $\theta = 2.5$ and $\lambda = 0.2$ (Nasiru,S. 2015), α=0.05, 0.01 and t/σ_0=0.241, 0.361, 0.482, 0.602, 0.903, 1.204, 1.505, 1.806 by Rao et.al. 2011.

In the present study, inequality (4) can be considered in a different way. Let us fix n and let r be a natural number less than n, so that as soon as the rth (r=c+1) failure is observed, the process is stopped and the lot is rejected. Given $\sigma = \sigma_0$, the probability of such a rejection should be as small as possible. That is

$$\sum_{i=0}^{c}\binom{n}{i}p^{i}(1-p)^{n-i}<\alpha^{*} \tag{5}$$

Specified n as a multiple of r , say $kr[k=1,2,\ldots]$, inequality (5) can be regarded as an inequality in a single unknown in terms of t/σ with known α. With the choice of r,k,α^*, inequality (5) can be solved at the earliest p, say p_0, from which the value t/σ_0 can be obtained by inverting the $F(t;\,\sigma,\theta,\lambda)$ given by 2. The specified population average in terms of σ_0 can be used here to get the value of t called the termination time. These are presented in Table 2 at $\alpha^*=0.05,\ 0.01$.

Table 1. Minimum sample size necessary to assert that the average lifeσ_0, with probability $1-\alpha^*$ and the corresponding acceptance number c, using binomial probability for $\theta=2.5$ and $\lambda=0.2$

		$\frac{t}{\sigma_0}$							
$1-\alpha^*$	C	0.241	0.361	0.482	0.602	0.903	1.204	1.505	1.806
0.95	0	39	15	9	6	3	2	1	1
0.95	1	54	23	13	9	5	3	2	2
0.95	2	69	31	17	12	7	4	3	3
0.95	3	72	40	21	15	8	5	5	4
0.95	4	86	45	24	17	9	6	6	5
0.95	5	98	48	27	19	10	7	7	6
0.95	6	110	53	30	21	11	9	8	7
0.95	7	133	59	33	23	12	10	9	8
0.95	8	152	65	36	25	13	11	10	9
0.95	9	160	71	39	27	14	12	11	10
0.95	10	175	77	42	28	15	13	12	11
0.99	0	42	24	10	7	4	2	1	1
0.99	1	63	28	14	10	6	3	2	2
0.99	2	74	38	19	13	8	4	3	3
0.99	3	88	47	24	17	9	5	4	4
0.99	4	103	56	29	18	10	6	5	5
0.99	5	115	63	33	20	11	8	6	6
0.99	6	126	70	36	23	12	9	7	7
0.99	7	143	77	38	25	13	11	8	8
0.99	8	172	78	39	27	14	12	9	9
0.99	9	180	80	41	29	15	13	11	10
0.99	10	198	81	43	30	16	14	12	11

2.3 Minimum Total Cost

The costs associated with a sampling plan include sampling, inspection, warranty, prevention, and rejection costs. When developing an acceptance sampling plan, the goal is to minimize the total cost by considering appraisal costs, internal failure costs, and external costs. The process of the sampling plan begins with inspecting samples drawn from the population and identifying defective items. The likelihood of detecting defective items differs across samples. Henceforth, in the life test sampling plan, let D_d denote the defective item detected, D_n denote the defective item not detected. Hence for any sampling inspection plan,

where, $D_d=ASN*p$and $D_n=(N-ASN)*p*L(p)$

The producer's total cost of the inspection for any lot includes the appraisal cost C_a, internal failure cost C_o and external failure cost C_e. Hence the mathematical model of designing an acceptance sampling life test plan in the economic design is given as,

$$Minimize\,TC=C_aASN+C_o\,D_d+C_eD_n \tag{6}$$

Table 2. Life test termination in units of scale parameter for Weighted Weibull distribution with$\theta = 2.5$ and $\lambda = 0.2$

n	2r	3r	4r	5r	6r	7r	8r	9r	10r
r					α^*=0.05				
1	0.11603	0.09302	0.08071	0.07306	0.06733	0.06291	0.05946	0.05660	0.05414
2	0.23249	0.18703	0.16289	0.14738	0.13560	0.12690	0.11990	0.11404	0.10900
3	0.29408	0.23716	0.20694	0.18713	0.17232	0.16132	0.15202	0.14459	0.13834
4	0.33336	0.26948	0.23522	0.21245	0.19596	0.18312	0.17305	0.16472	0.15722
5	0.36032	0.29120	0.25333	0.22941	0.21176	0.19790	0.18673	0.17772	0.17011
6	0.38152	0.30768	0.26869	0.24273	0.22396	0.20942	0.19796	0.18810	0.18003
7	0.39755	0.32057	0.27950	0.25288	0.23334	0.21834	0.20605	0.19590	0.18745
8	0.41092	0.33213	0.28936	0.26120	0.24100	0.22555	0.21296	0.20268	0.19372
9	0.42005	0.33927	0.29612	0.26751	0.24676	0.23100	0.21820	0.20736	0.19823
10	0.44033	0.35535	0.31002	0.28012	0.25834	0.24151	0.22809	0.21692	0.20758
r					α^*=0.01				
1	0.08300	0.06564	0.05718	0.05181	0.04810	0.04448	0.04206	0.04004	0.03829
2	0.18133	0.14566	0.12829	0.11476	0.10646	0.09876	0.09333	0.08877	0.08484
3	0.24100	0.19398	0.16985	0.15315	014080	0.13185	0.12459	0.11884	0.11408
4	0.27856	0.22711	0.19808	0.17926	0.16514	0.15517	0.14570	0.13872	0.13271
5	0.30864	0.25100	0.21814	0.19765	0.18232	0.17020	0.16138	0.15271	0.14619
6	0.33265	0.26751	0.23477	0.21274	0.19659	0.18349	0.17283	0.16502	0.15702
7	0.34913	0.28334	0.24728	0.22316	0.20623	0.19275	0.18180	0.17340	0.16537
8	0.36356	0.29661	0.25687	0.23377	0.21578	0.20085	0.19026	0.18057	0.17325
9	0.37567	0.30518	0.26406	0.24100	0.22172	0.20769	0.19591	0.18601	0.17836
10	0.39559	0.31955	0.28005	0.25147	0.23419	0.21891	0.20687	0.19635	0.18739

Subject to,

$$L(p) \leq 1 - p^* \tag{7}$$

Where C_a=1.0, Co=2.0, Ce=10, and N=500 as in (Hus 2009)· The minimum total costs are shown in Table 3.

Table 3. Minimum Total Cost values of sampling plans for $\theta = 2.5$ and $\lambda = 0.2$

n	2 r	3 r	4 r	5 r	6 r	7 r	8 r	9 r	10 r
r					$\alpha^* = 0.05$				
1	0.11603	0.09302	0.08071	0.07306	0.06733	0.06291	0.05946	0.05660	0.05414
	0.16082	0.13136	0.11341	0.10183	0.09307	0.08576	0.08050	0.07595	0.07211
2	0.23249	0.18703	0.16289	0.14738	0.13560	0.12690	0.11990	0.11404	0.10900
	0.32118	0.25536	0.21809	0.19357	0.17642	0.16294	0.15165	0.14323	0.13555
3	0.29408	0.23716	0.20694	0.18713	0.17232	0.16132	0.15202	0.14459	0.13834
	0.40841	0.32118	0.27311	0.24214	0.21954	0.20292	0.18927	0.17785	0.16861
4	0.33336	0.26948	0.23522	0.21245	0.19596	0.18312	0.17305	0.16472	0.15722
	0.46375	0.36269	0.30755	0.27236	0.24727	0.22753	0.21230	0.19932	0.18856
5	0.36032	0.29120	0.25333	0.22941	0.21176	0.19790	0.18673	0.17772	0.17011
	0.50159	0.39092	0.33186	0.29325	0.26569	0.24507	0.22826	0.21447	0.20292

2.4 Operating Characteristic Function

The probability of acceptance of the lot should be significantly altered if the true but unknown life of the product differs from the specified life, as determined by the sampling plan. Hence the probability of acceptance can be regarded as a function of the deviation of the specified average from the true average. This function is called the Operating Characteristic (O.C.) function of the sampling plan. Specifically if $F(t)$ is the cumulative distribution function of the lifetime random variable of the product, σ_0 corresponds to specified life, we can write

$$F(t/\sigma) = F[(t/\sigma_0).(\sigma_0/\sigma)] \tag{8}$$

Where σ represents the true but unknown average life. The ratio σ_0/σ on the right side of the above equation serves as a measure of the differences between actual and specified lifetime. In quality control, Operating Characteristic (OC) Curves are pivotal in determining the likelihood of accepting manufactured lots or batches with various sampling plans. They illustrate the relationship between the percentage of defects and the probability of acceptance in a lifetime experiment. OC curves assist in choosing acceptance sampling plans and mitigating risks. The various behaviors of OC curves are shown in Table 4 and Figure 1.

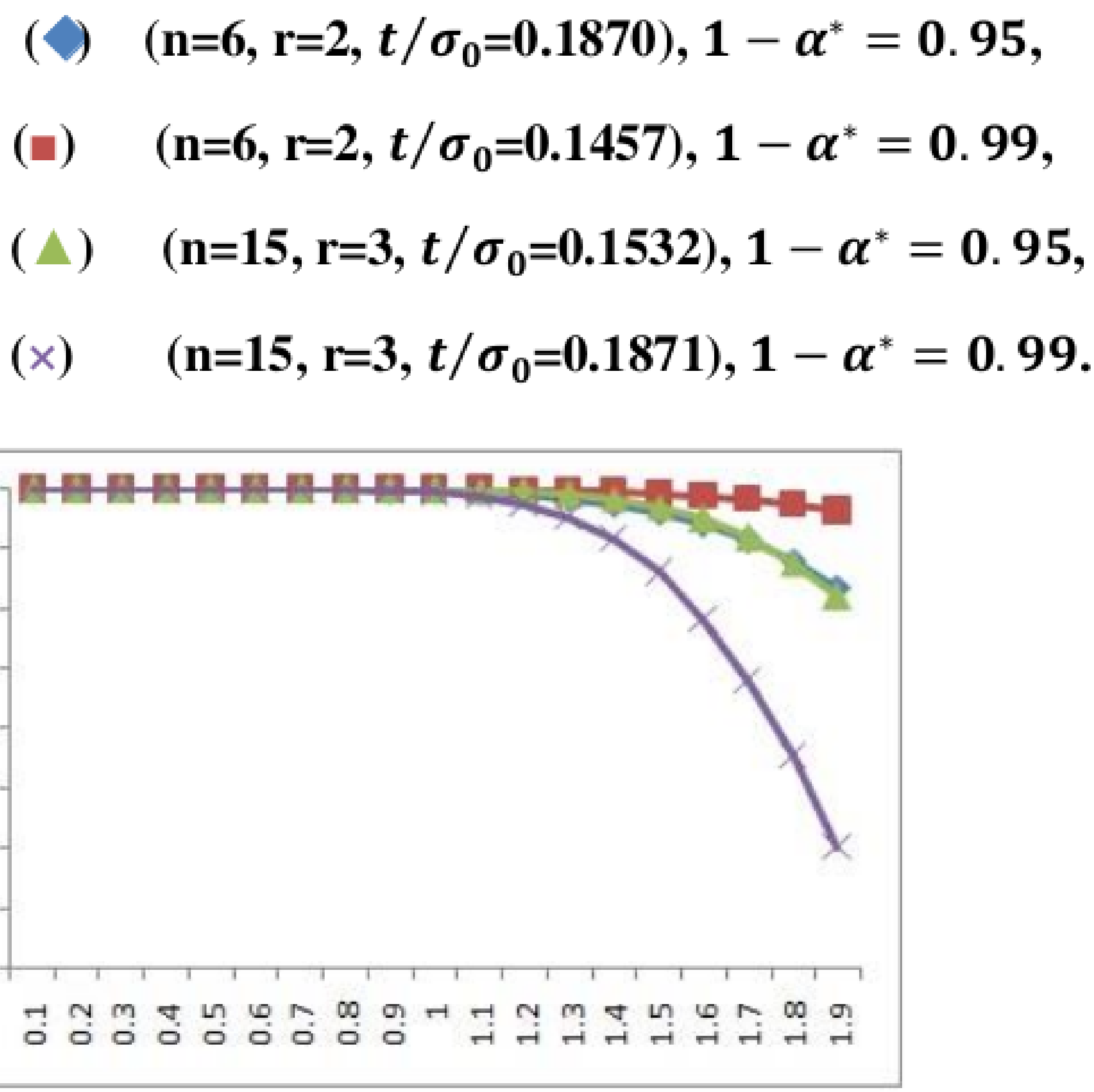

Fig 1. OC Curves for the sampling plans

3 Results and Discussion

3.1 Example

Consider the following ordered failure times of the release of a software given in terms of hours from the starting of the execution of the software denoting the times at which the failure of the software is experienced (Wood, 1996) [14]. This data can be regarded

Table 4. Operating Characteristic values of sampling plans for $\theta = 2.5$ and $\lambda = 0.2$

	$n = 6, r = 2$		$n = 15, r = 3$	
	t/σ_0		t/σ_0	
	0.1870	0.1457	0.1532	0.1871
σ_0/σ	$1-\alpha^* = 0.95$	$1-\alpha^* = 0.99$	$1-\alpha^* = 0.95$	$1-\alpha^* = 0.99$
0.2	1.0000	1.0000	1.0000	1.0000
0.3	0.9999	1.0000	1.0000	1.0000
0.4	0.9999	1.0000	1.0000	1.0000
0.5	0.9999	1.0000	1.0000	1.0000
0.6	0.9999	0.9999	0.9999	0.9999
0.7	0.9999	0.9999	0.9999	0.9999
0.8	0.9998	0.9999	0.9999	0.9999
0.9	0.9994	0.9999	0.9999	0.9996
1.0	0.9988	0.9999	0.9998	0.9990
1.1	0.9976	0.9998	0.9996	0.9974
1.2	0.9957	0.9996	0.9990	0.9941
1.3	0.9925	0.9993	0.9978	0.9877
1.4	0.9877	0.9987	0.9956	0.9764
1.5	0.9807	0.9978	0.9916	0.9581
1.6	0.9708	0.9964	0.9851	0.9305
1.7	0.9575	0.9944	0.9749	0.8916
1.8	0.9401	0.9915	0.9599	0.8402
1.9	0.9180	0.9876	0.9387	0.7761

as an ordered sample of size n = 12 with observations:

$$(x_i : i = 1, 2, \ldots 12\} = \{519,\ 968,\ 1430,\ 1893,\ 2490,\ 3058,\ 3625,\ 4422,\ 5218,\ 5823,\ 6539,\ 7083\}$$

Case I:

Let the required average lifetime be 1000 hours and the testing time be t = 602 hours, this leads to ratio of t/σ_0= 0.602 with a corresponding sample size n = 12 and an acceptance number c = 2, which are obtained from Table 1 for $1 - \alpha^*$ = 0.95. Therefore, the sampling plan for the above sample data is (n = 12, c = 2, t/σ_0= 0.602). Based on the observations, we have to decide whether to accept the product or reject it. We accept the product only if the number of failures before 602 hours is less than or equal to 2. In the above sample of 12 only one failure occurred at 519 hours before t = 602 hours.

Case II:

From Table 2, the entry against r =3(r = c + 1) under the column 4r is 0.20694. Since the acceptable mean life is given to be 1000 hours for Weighted Weibull Distribution. If the termination time is given by 't_0' the table value says that t_0= 0.20694 that is t_0= 0.20694 × 1000 = 206.94 = 207 hours (approx.).

Using the current sampling plan, the test plan will be executed as follows: Select 12 items from the submitted lot for testing. If the 3rd failure occurs before the 207th hour, reject the lot; otherwise, accept the lot. In both cases, the experiment is terminated as soon as the 3rd failure happens or the 207th hour is reached, whichever comes first. If the lot is accepted, it ensures that the average life of the submitted products is at least 1000 hours. In this scenario, since there are no failures among the 12 items before the 207th hour, the product is accepted.

3.2 Comparative study

To compare the current sampling plan with that of Kantam and Sriram (2013), entries shared by both approaches are presented in Table 5 for 1-α^*=0.95. The entries in the first row correspond to the present test plan, while those in the second row are from Kantam and Sriram (2013). All the entries in Table 3 indicate that for a given n and r (where r=c+1), the values of t/σ_0, the scaled termination time, are consistently smaller for the current reliability test plan compared to those of Kantam and Sriram (2013), leading to a reduction in experimental time.

Table 5. Proportions of life test termination time for the present sampling plan and sampling plan of Kantam and Sriram (2013) with producer's risk 's risk 1-α^*=0.95

n	2r	3r	4r	5r	6r	7r	8r	9r	10r
r	α^*=0.05								
1	0.11603	0.09302	0.08071	0.07306	0.06733	0.06291	0.05946	0.05660	0.05414
	0.16082	0.13136	0.11341	0.10183	0.09307	0.08576	0.08050	0.07595	0.07211
2	0.23249	0.18703	0.16289	0.14738	0.13560	0.12690	0.11990	0.11404	0.10900
	0.32118	0.25536	0.21809	0.19357	0.17642	0.16294	0.15165	0.14323	0.13555
3	0.29408	0.23716	0.20694	0.18713	0.17232	0.16132	0.15202	0.14459	0.13834
	0.40841	0.32118	0.27311	0.24214	0.21954	0.20292	0.18927	0.17785	0.16861
4	0.33336	0.26948	0.23522	0.21245	0.19596	0.18312	0.17305	0.16472	0.15722
	0.46375	0.36269	0.30755	0.27236	0.24727	0.22753	0.21230	0.19932	0.18856
5	0.36032	0.29120	0.25333	0.22941	0.21176	0.19790	0.18673	0.17772	0.17011
	0.50159	0.39092	0.33186	0.29325	0.26569	0.24507	0.22826	0.21447	0.20292

4 Conclusion

This work presents a Reliability Test Plan based on the lifetime of the testers using the Weighted Weibull distribution. The optimal number of testers, the OC function, termination time and the minimum total cost are determined. The proposed Acceptance Sampling Plan is a cost-effective mechanism, as it requires a small sample size. To summarize, it can help industrial companies reduce testing time, research costs, energy, and labor.

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