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RESEARCH ARTICLE

Outer Triple Connected Corona Domination Number of Graphs

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Abstract

Background/Objective: Given a graph G, a dominating set $S \subseteq V(G)$ is said to be corona dominating set if every vertex $v \in S$ such that $d(v) = 1$ or there exist a vertex $u \in S$ if $d(u) = 1$ then $u, v \in S$. A corona dominating set $S \subseteq V(G)$ is said to be an outer triple connected corona dominating set if any three vertices in $< V - S >$ lie on a path. The minimum cardinality taken over all the outer triple connected corona dominating sets of G is called outer triple connected corona dominating number and it is denoted by $\gamma_{CD}(G)$. The study aims to find the outer triple connected corona domination number of some graphs. **Method:** To obtain outer triple connected corona domination number say m by proving $\gamma_{OTC}(G) \leq m$ and $\gamma_{OTC}(G) \geq m$. To prove $\gamma_{OTC}(G) \leq m$ for a graph G we find a outer triple connected corona dominating set of G with cardinality m and then to prove $\gamma_{OTC}(G) \geq m$ we prove by contradiction. **Findings:** We investigated the above parameter for some derived graphs of path, cycle and wheel graph. **Novelty :** Outer triple connected corona domination number is a new concept in which the conditions of corona domination and triple connected are linked together.

Keywords: Corona Domination; Pendent Vertex; Support Vertex; Triple Connected; Isolated Vertex; Pendant vertes

1 Introduction

This study considers simple, finite and undirected graphs $G(V, E)$, where V denotes the vertex set of the graph G and E denotes the edge set of the graph G. The set $S \subseteq V(G)$ is a dominating set if $N[S] = V(G)$. The least possible cardinality of the dominating sets is called the domination number of a graph and it is expressed as $\gamma(G)$ [(1)]. A graph is said to be a triple connected graph if any three vertices of the graph are lie on a path [(2)]. G. Mahadevan et.al introduced Corona domination number of graphs. A dominating set is a corona dominating set if every vertex $v \in S$ such that $d(v) = 1$ or there exist a vertex $u \in S$ if $d(u) = 1$ then $u, v \in S$. The least possible cardinality of the corona dominating sets is called the corona domination number of the graph and it is expressed by γ_{CD}(G) [(3)]. Motivated by the above concepts here we initiate a new parameter called outer triple connected corona domination number which will be explained in section 2. Consider P_n and $P_{2(n-1)}$ removal of all the even edges in $P_{2(n-1)}$, joining of all the odd vertex i of $P_{2(n-1)}$ to $\left(i-\lfloor \frac{i}{2} \rfloor\right)^{th}$ vertex of P_n and joining of all the even vertex i of $P_{2(n-1)}$ to $\left(\frac{i-2}{2}\right)^{th}$ vertex of P_n results in Quadrilateral snake (QS_n) [4]. Linking K_m to P_n with an edge is (m,n)-lollipop graph [5] ,[6]. The joining $wK_{v-1}+K_1$ is a windmill graph $wd\,(v,w)\,, v \geq 2$ and $w \geq 2$ [7]. Two K_n graphs with a cut edge is called n-barbell graph B_n [8]. Let K be a graph and K' be a replica of K then under the mapping $\chi: \ K \to K'$ defined by $\chi(u) = u'$ the mirror graph $M_r(K)$ is accomplished from $K \cup K'$ by joining the vertex $u \in K$ to the vertex $\chi(u)$ of K' for all $u \in K$ [9]. Let K be a graph and K' be a copy of K then under the mapping $\chi: \ K \to K'$ defined by $\chi(u) = u'$ the Shadow graph $D_2(K)$ is accomplished from $K \cup K'$ by joining the vertex $u \in K$ to the vertex neighborhood $\chi(u)$ of K' for all $u \in K$ [10]. The Middle graph $M(G)$ of a graph G is accomplished by splitting the edges of G into two edges by an vertex and linking the splitting vertices to the vertices at distance two. [11] [12]. The joining $C_n + K_1$ is a wheel graph W_n. Cone graph C(m,n) is the graph $C_m + K_n$ $.Throughoutthispaperthewhiteverticesarethedominatingset.$

2 Methodology

Definition 2.1. A Corona dominating set $S \subseteq V$ is said to be a outer triple connected corona dominating set (OTCD-set) if any three vertex in $< V - S >$ are lie on a path. The minimum cardinality of an OTCD-set is called the outer triple connected corona domination number and it is expressed by $\gamma_{OTC}(G)$.

Example 2.1. Here $S = \{u_1,\ u_2\}$ is a outer triple connected corona dominating set and hence $\gamma_{OTC}(G) = 2$

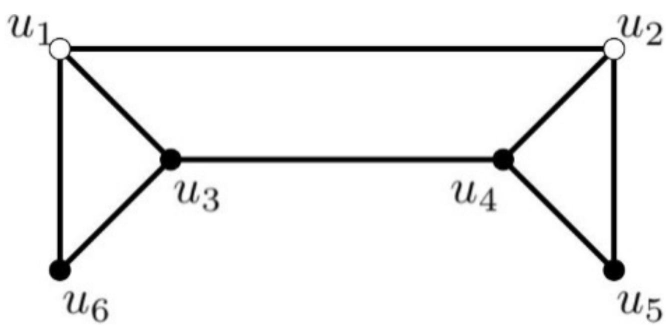

Fig 1.

To find an Outer triple connected corona domination number initially we assume a outer triple connected corona dominating set with cardinality m. Hence $\gamma_{OTC}(G) \leq m$. By contradiction it can be proved that $\gamma_{OTC}(G) \geq m$. Thus $\gamma_{OTC}(G) = m$.

3 Results and Discussion

Outer Triple Connected Corona Domination Number of a Graph

Observation 2.1. For a graph G, $\gamma(G) \leq \gamma_{CD}(G) \leq \gamma_{OTC}(G)$.

Observation 2.2. For a graph G of order n, $2 \leq \gamma_{OTC}(G) \leq n-3$.

Observation 2.3. For a graph G of order n, $\lceil \frac{n}{\Delta+1} \rceil \leq \gamma_{OTC}$.

Example 2.2. Here, $\gamma_{OTC} = 3$ and $\lceil \frac{n}{\Delta+1} \rceil = 2$.

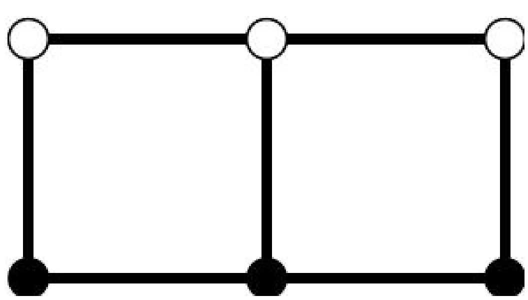

Fig 2.

Observation 2.4. For a $K_{g,h},\ g\ \geq 2,\ h \geq 3,\ \gamma_{OTC}(K_{g,h}) = 2$.
Example 2.3.

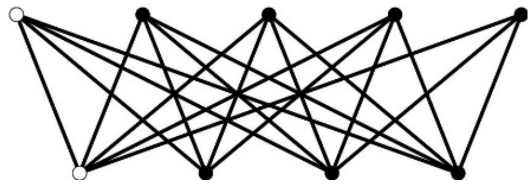

Fig 3. A graph with $\gamma_{\mathrm{OTC}}\ (K_{5,4}) = 2$

Observation 2.5. For a graph $G = W_n$ or $K_n,\ n \geq 5,\ \gamma_{OTC}(G) = 2$.
Observation 2.6. For a graph $G = Q_n, n \geq 3,\ \gamma_{OTC}(G) = 2n-2$.
Example 2.4.

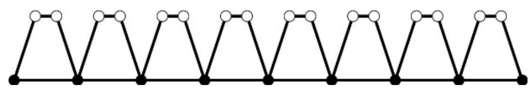

Fig 4. A graph with $\gamma_{OTC}\,(Q_9) = 16$

Observation 2.7. For a graph $= L(g,h)\ ,\ g \geq 4,\ h \leq 5,\ \gamma_{OTC}(G) = h+1$.
Example 2.5.

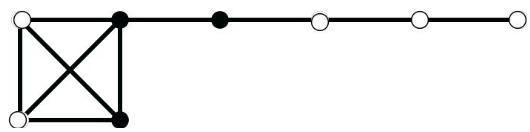

Fig 5. A graph with $\gamma_{\mathrm{OTC}}\ (L(4,4)) = 5$

Observation 2.8. For a graph $G = Wd(k,2), k \geq 4,\ \gamma_{OTC}(G) = 4$.
Observation 2.9. For a graph $G = B_n\ , n \geq 4, \gamma_{OTC}(G) = 2$.
Observation 2.10. For a graph $G = C(m,n), m \geq 3,\ n \geq 2,\ \gamma_{OTC}(G) = 2$.
Example 2.6.

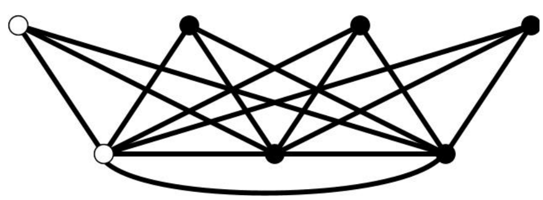

Fig 6. A graph with $\gamma_{OTC}(C(3,4)) = 2$

Observation 2.11. OTCD- number does not exist for path, cycle and Jahangir graph.
Theorem 2.1. For a cycle C_n,

$$\gamma_{OTC}(M_r(C_n)) = \begin{cases} 4\left\lfloor\frac{n}{6}\right\rfloor + 1 & if\ n \equiv 1\ (mod\ 6), \\ 4\left\lceil\frac{n}{6}\right\rceil - 1 & if\ n \equiv 3\ (mod\ 6), \\ 4\left\lfloor\frac{n}{6}\right\rfloor + 2 & if\ n \equiv 2\ (mod\ 6), \\ 4\left\lceil\frac{n}{6}\right\rceil & otherwise. \end{cases}$$

Proof: Let $V(M_r(C_n)) = \{v_1, v_2, \ldots, v_n, u_1, u_2, \ldots, u_n\}$ and

$$E(M_r(C_n)) = \{v_g u_g, v_1 v_n, u_1 u_n, v_h v_{h+1}, u_h u_{h+1}; 1 \le g \le n, 1 \le h \le n-1\}$$

Assume $S_1 = \{v_g, u_h; g \equiv 1\ or\ 2\ (mod\ 6), h \equiv 4\ or\ 5\ (mod\ 6)\}$

Let

$$S = \begin{cases} S_1 \cup \{v_n\} & \text{if } n \equiv 4(\bmod 6), \\ S_1 \cup \{u_{n-1}\} & \text{if } n \equiv 3(\bmod 6), \\ S_1 & \text{otherwise}. \end{cases}$$

Then S is a OTCD-set of $M_r(C_n)$ and hence

$$\gamma_{OTC}(M_r(C_n)) \le |S| = \begin{cases} 4\left\lfloor\frac{n}{6}\right\rfloor + 1 & if\ n \equiv 1\ (mod\ 6), \\ 4\left\lceil\frac{n}{6}\right\rceil - 1 & if\ n \equiv 3\ (mod\ 6), \\ 4\left\lfloor\frac{n}{6}\right\rfloor + 2 & if\ n \equiv 2\ (mod\ 6), \\ 4\left\lceil\frac{n}{6}\right\rceil & otherwise. \end{cases}$$

Let P' be a OTCD- set of $M_r(C_n)$. Suppose P is a dominating set of cardinality not more than

$$k = \begin{cases} 4\left\lfloor\frac{n}{6}\right\rfloor & \text{if } n \equiv 1(\bmod 6) \\ 4\left\lceil\frac{n}{6}\right\rceil - 2 & \text{if } n \equiv 3(\bmod 6), \\ 4\left\lfloor\frac{n}{6}\right\rfloor + 1 & \text{if } n \equiv 2(\bmod 6), \\ 4\left\lceil\frac{n}{6}\right\rceil - 1 & \text{otherwise} \end{cases}$$

Then either $< P >$ has a isolated vertex or $< V - P >$ is not triple connected, we have

$$k = \begin{cases} 4\left\lfloor\frac{n}{6}\right\rfloor & \text{if } n \equiv 1(\bmod 6) \\ 4\left\lceil\frac{n}{6}\right\rceil - 2 & \text{if } n \equiv 3(\bmod 6), \\ 4\left\lfloor\frac{n}{6}\right\rfloor + 1 & \text{if } n \equiv 2(\bmod 6), \\ 4\left\lceil\frac{n}{6}\right\rceil - 1 & \text{otherwise} \end{cases}$$

Hence proved.

Example 2.7.

Theorem 2.2 . For a path $P_n, n \ge 3$,

$$\gamma_{\mathrm{OTC}}(M_r(P_n)) = \begin{cases} 4\left\lfloor\frac{n}{6}\right\rfloor + 1 & \text{if } n \equiv 0(\bmod 6), \\ 4\left\lceil\frac{n}{6}\right\rceil - 1 & \text{if } n \equiv 3(\bmod 6), \\ 4\left\lfloor\frac{n}{6}\right\rfloor + 2 & \text{if } n \equiv 1 \text{ or } 2(\bmod 6), \\ 4\left\lceil\frac{n}{6}\right\rceil & \text{otherwise} \end{cases}$$

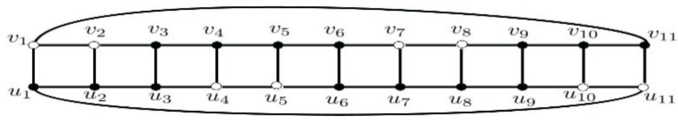

Fig 7. A graph with $\gamma_{OTC}(M_r(C_{11})) = 8$

Proof: Let $V(M_r(P_n)) = \{v_1, v_2, \ldots, v_n, u_1, u_2, \ldots, u_n\}$ and
$E(M_r(P_n)) = \{v_g u_g, v_h v_{h+1}, u_h u_{h+1}; 1 \le g \le n, 1 \le h \le n-1\}$
Assume $S_1 = \{v_g, u_h; g \equiv 1 \ or\ 2 \ (mod\ 6), h \equiv 4 \ or\ 5 \ (mod\ 6)\}$
Let

$$S = \begin{cases} S_1 \cup \{v_n\} & \text{if } n \equiv 3 \text{ or } 4 (\bmod 6), \\ S_1 \cup \{u_n\} & \text{if } n \equiv 0 \text{ or } 1 (\bmod 6), \\ S_1 & \text{otherwise .} \end{cases}$$

Then S is a OTCD-set of $M_r(P_n)$ and hence

$$\gamma_{\text{OTC}}(M_r(P_n)) \le |S| = \begin{cases} 4\left\lfloor \frac{n}{6} \right\rfloor + 1 & \text{if } n \equiv 0 (\bmod 6), \\ 4\left\lfloor \frac{n}{6} \right\rfloor - 1 & \text{if } n \equiv 3 (\bmod 6), \\ 4\left\lfloor \frac{n}{6} \right\rfloor + 2 & \text{if } n \equiv 1 \text{ or } 2 (\bmod 6), \\ 4\left\lceil \frac{n}{6} \right\rceil & \text{otherwise.} \end{cases}$$

Let $P^{'}$ be a OTCD- set of $M_r(P_n)$. Suppose P is a dominating set of cardinality at most

$$k = \begin{cases} 4\left\lfloor \frac{n}{6} \right\rfloor & \text{if } n \equiv 0 (\bmod 6), \\ 4\left\lceil \frac{n}{6} \right\rceil - 2 & \text{if } n \equiv 3 (\bmod 6), \\ 4\left\lfloor \frac{n}{6} \right\rfloor + 1 & \text{if } n \equiv 1 \text{ or } 2 (\bmod 6), \\ 4\left\lceil \frac{n}{6} \right\rceil \frac{n}{6} - 1 & \text{otherwise .} \end{cases}$$

Then either $< P >$ has a isolated vertex or $< V - P >$ is not triple connected, we have

$$\left|P^{'}\right| \ge k + 1 = \begin{cases} 4\left\lfloor \frac{n}{6} \right\rfloor + 1 & \text{if } n \equiv 0 (\bmod 6) \\ 4\left\lceil \frac{n}{6} \right\rceil - 1 & \text{if } n \equiv 3 (\bmod 6) \\ 4\left\lfloor \frac{n}{6} \right\rfloor + 2 & \text{if } n \equiv 1 \text{ or } 2 (\bmod 6) \\ 4\left\lceil \frac{n}{6} \right\rceil & \text{otherwise} \end{cases}$$

Hence $|S|$ is the outer triple connected corona dominating number.
Example 2.8.

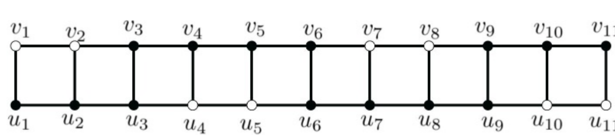

Fig 8. A graph with $\gamma_{OTC}(M_r(P_{11})) = 8$

Theorem 2.3 . For a path P_n,

$$\gamma_{\text{OTC}}(D_2(P_n)) = \begin{cases} \left\lceil \frac{n}{2} \right\rceil + 2 & \text{if } n \equiv 0 \text{ or } 1 (\bmod 4) \\ \left\lceil \frac{n}{2} \right\rceil + 1 & \text{otherwise} \end{cases}$$

Proof: Let $V(D_2(P_n)) = \{v_1, v_2, \ldots, v_n, u_1, u_2, \ldots, u_n\}$ and
$E(M_r(C_n)) = \{v_h v_{h+1}, u_h u_{h+1}, v_h u_{h+1}, v_g u_{g-1}; 2 \le g \le n, 1 \le h \le n-1\}$
Assume $S_1 = \left\{v_g, ; g \equiv 1 \ or\ 2 \ (mod\ 4)\right\}$ $and\ S_2 = \{v_g;\ g \equiv 2 \ or\ 3 \ (mod\ 4)\}$
Let

$$S = \begin{cases} S_1 \cup \{v_{n-1}, v_n\} & \text{if } n \equiv 0(\mod 4), \\ S_1 \cup \{v_n\} & \text{if } n \equiv 3(\mod 4), \\ S_1 \cup \{v_1, v_{n-1}, v_n\} & \text{if } n \equiv 2(\mod 4) \\ S_2 \cup \{v_1,\ v_{n-1}, v_n\} & \text{otherwise.} \end{cases}$$

Then S is a OTCD-set of $D_2(P_n)$ and hence

$$\gamma_{\text{OTC}}\ (D_2\,(P_n)) \leq |S| = \begin{cases} \left\lceil \frac{n}{2} \right\rceil + 2 & \text{if } n \equiv 0 \text{ or } 1(\mod 4), \\ \left\lfloor \frac{n}{2} \right\rfloor + 1 & \text{otherwise .} \end{cases}$$

Let $P^{'}$ be a OTCD- set of $D_2(P_n)$. Suppose P is a dominating set of cardinality at most

$$k = \begin{cases} \left\lceil \frac{n}{2} \right\rceil + 1 & \text{if } n \equiv 0 \text{ or } 1(\mod 4) \\ \left\lfloor \frac{n}{2} \right\rfloor & \text{otherwise} \end{cases}$$

Then either $< D >$ has a isolated vertex or

$< V - P >$ is not triple connected, we have

$$|P'| \geq k + 1 = \begin{cases} \left\lceil \frac{n}{2} \right\rceil + 2 & \text{if } n \equiv 0 \text{ or } 1(\mod 4) \\ \left\lfloor \frac{n}{2} \right\rfloor + 1 & \text{otherwise .} \end{cases}$$

Hence $|S|$ is the outer triple connected dominating number.

Example 2.9.

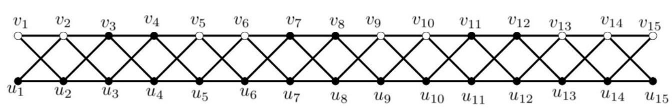

Fig 9. A graph with $\gamma_{OTC}\,(D_2\,(P_{15})) = 9$

Theorem 2.4. For a cycle C_n,

$$\gamma_{\text{OTC}}\ (D_2\,(C_n)) = \begin{cases} \left(\frac{n}{2}\right) + 1 & \text{if } n \equiv 2(\mod 4), \\ \left\lceil \frac{n}{2} \right\rceil & \text{otherwise .} \end{cases}$$

Proof: Let $V\,(D_2\,(C_n)) = \{v_1,\ v_2,\ \ldots,\ v_n,\ u_1,\ u_2,\ \ldots,\ u_n\}$ and

$$E\,(M_r\,(C_n)) = \{v_h v_{h+1},\ u_h u_{h+1},\ v_h u_{h+1},\ v_g u_{g-1},\ v_1 v_n,\ u_1 u_n; 2 \leq g \leq n,\ 1 \leq h \leq n-1\}$$

Assume $S = \left\{v_g,\ ; g \equiv 1 \ or\ 2\ \ (mod\ 4)\right\}$

Then S is a OTCD-set of $D_2(C_n)$ and hence

$$\gamma_{\text{OTC}}\ (D_2\,(C_n)) \leq |S| = \begin{cases} \left(\frac{n}{2}\right) + 1 & \text{if } n \equiv 2(\mod 4), \\ \left\lceil \frac{n}{2} \right\rceil & \text{otherwise. .} \end{cases}$$

Let $P^{'}$ be a OTCD- set of $D_2(C_n)$. Suppose P is a dominating set of cardinality at most

$$k = \begin{cases} \left(\frac{n}{2}\right) & \text{if } n \equiv 2(\mod 4), \\ \left\lceil \frac{n}{2} \right\rceil - 1 & \text{otherwise.} \end{cases}$$

Then either $< P >$ has a isolated vertex or

$< V - P >$ is not triple connected, we have

$$|P'| \geq k + 1 = \begin{cases} \left(\frac{n}{2}\right) & \text{if } n \equiv 2(\mod 4) \\ \left\lceil \frac{n}{2} \right\rceil & \text{otherwise.} \end{cases}$$

Hence $|S|$ is the outer triple connected domination number.

Example 2.10.

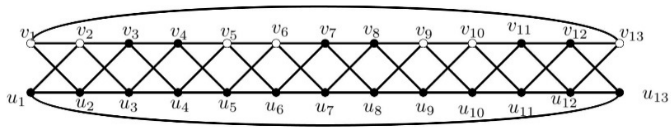

Fig 10. A graph with $\gamma_{OTC}(D_2(C_{13})) = 7$

Theorem 2.5. For a wheel W_n,

$$\gamma_{\mathrm{otC}}(M(\mathrm{W_n})) = \begin{cases} n-1 & \text{if } n \text{ is even} \\ n & \text{otherwise.} \end{cases}$$

Proof: Let $V(M(W_n)) = \{v, v_1, v_2, \ldots, v_n, u_1, u_2, \ldots, u_{2n}\}$ and $E(M(W_n)) = \{v_h v, v_g v_h, u_k u_{k+1}, u_{2n} u_1, u_l u_{l+2}, u_1 u_{2n-1}, v_m u_p, v_n u_1; 1 \le g, h, m \le n, g \ne h, 1 \le k \le 2n-1, 2m-1 \le 2n, 1 \le l \le 2n-3$ and l is odd $\}$

Assume $S_1 = (v_1, v_3, u_g, ; g \equiv 1 \; or \; 2 \; (mod\, 4), \; 5 \le g \le 2n-3 \; and \; g \ne 6\}$

Let

$$P = \begin{cases} S_1 \cup \{u_{2n-1}\} & \text{if } n \equiv 0 \text{ or } 2(\bmod 4) \\ S_1 \cup \{u_{2n}, u_{2n-1}\} & \text{if } n \equiv 1 \text{ or } 3(\bmod 4) \end{cases}$$

Then P is a OTCD-set of $M(W_n)$ and hence

$$\gamma_{\mathrm{OTC}}(M(W_n)) \le |P| = \begin{cases} n-1 & \text{if } n \text{ is even,} \\ n & \text{otherwise.} \end{cases}$$

Let P' be a OTCD- set of $M(W_n)$. Suppose D is a dominating set of cardinality not more than

$$k = \begin{cases} n-2 & \text{if } n \text{ is even,} \\ n-1 & \text{otherwise .} \end{cases}$$

Then either $< D >$ has a isolated vertex or $< V - D >$ is not triple connected, we have

$$|P'| \ge k+1 = \begin{cases} n-1 & \text{if } n \text{ is even} \\ n & \text{otherwise.} \end{cases}$$

Hence $|P|$ is the outer triple connected corona domination number.

Example 2.11.

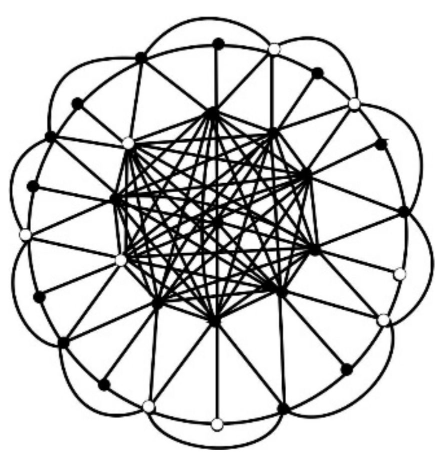

Fig 11. A graph with $\gamma_{OTC}(M(W_{10})) = 9$

4 Conclusion

This study has gone through the new parameter called outer triple connected corona domination number. Also, it obtained the results of outer triple connected corona domination number for standard graphs.

Declaration

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