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RESEARCH ARTICLE

Generalization of CD-Number for Power Graph of Some Special Types of Tree Graphs

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Abstract

Objectives: The main objective of the article is to finding the corona domination for the power graph of some special types of tree graph. **Method:** A dominating set S of a graph H is said to be a corona dominating set $CD-set$ if every vertex in $<S>$ is either a pendant vertex or a support vertex. The minimum cardinality of a corona dominating set is called the corona domination number and is denoted by $\gamma_{CD}(H)$. **Findings:** In this article, we study the CD-number for the k^{th} power of PVB-tree and $((OT_r)^k)$ where $2 \leq k \leq 7$ and identify their exact γ_{CD} values. **Novelty:** The corona domination was one of the recently developed domination parameter, along with this parameter we found the exact γ_{CD} value for some special types of graphs.

Keywords: Domination Number; Corona Domination Number; Total Domination; Power Graphs; Olive Tree And PVBTree

1 Introduction

For a graph $H = (X(H), Y(H))$, where $X(H)$ and $Y(H)$ are the vertex and edge set of H respectively. A dominating set [1] S is a set of vertices of H with the condition that every $x \in X - S$, $d(x, S) = 1$. If D is a dominating set of H with minimum cardinality, then the domination number $\gamma(H) = |D|$. The total dominating set was initiated by Kazemnejad et al. [2] and defined as the induced subgraph of a dominating set has no isolated vertex and the total domination number $\gamma_t(H)$ denotes the minimum cardinality of a total dominating set. The concept of corona domination, was introduced by G . Mahadevan et al. [3] and determined the γ_{CD} value for some unique graphs. The corona domination number(CD-number) γ_{CD} is a minimum cardinality of a dominating set S, with the subgraph induced by S having either pendant or support vertex only. G . Mahadevan et al. [4] discussed the value of γ_{CD} for some peculiar graphs. L. Praveenkumar et al. [5] characterized the γ_{CD} value for the Jahangir graph, Tadpole graph, etc.For convenience, all graphs H considered here are finite, undirected and simple.

Let H be a graph, then $\bar{H}$ denotes the complement of H. Let P_r and C_s denotes the path and cycle on r and s vertices respectively. For each $x_1, x_2 \in X(H)$, we define

$$d(x_1,x_2) = \left(\begin{array}{cc} length of the shortest x_1 - x_2 path & if\ exists, \\ \infty & otherwise. \end{array} \right.$$

If H is said to be a bipartite then its vertex can be separated into two nonempty subsets X_1 and X_2 such that each edge of H has one end in X_1 and other in X_2 and is denoted by $G(X_1,X_2)$. A bipartite graph $G(X_1,X_2)$ is said to be complete bipartite if each vertex of X_1 is adjacent to all the vertices of X_2. If $G(X_1,X_2)$ is complete with $|X_1| = r$ and $|X_2| = s$ then it is denoted by $K_{r,s}$. We call $K_{1,s}$ is a star graph. The power graph H^k of H with $X(H^k) = X(H)$ and two vertices $x_1, x_2 \in X(H^k)$ are adjacent if and only if $d(x_1,x_2) \leq k$, if $k = 2$, then H^2 is called the square graph of H. The authors in [6] have studied the characterization of the power graphs along with the domination parameter called complementary perfect corona domination number(CPCD-number). In [7] the characterization all quasi f-graphs and quasi f-ideals of degree 2 and all the minimal primes ideals of these ideals were determined. In [8] the authors proposed a general design pipeline for GNN models and discuss the variants of each component, also systematically categorize the applications. In [9] the spectral properties of varoius types of graphs were discussed and the spectrum for some were also determined. In [10] the sub-tree generating functions and the sub-tree number index of generalized book graphs, generalized fan graphs, and generalized wheel graphs were determined.In this article, we determined the CD-number for the k^{th} power of PVB-tree and $((OT_r)^k)$ where $2 \leq k \leq 7$.

2 Methodology

Although the study on domination theory have increasing with lot of new domination parameters and in many cases the results were determined for special types of graph and product graphs. This article is also contains one of the new domination parameter which is called corona domination and is defined by imposing a condition on the dominating set that every vertex in $< S >$ is either a pendant vertex or a support vertices. The case of distance in graphs has yet to be attempted. The distance between the vertices is increased for the considered graph and the increased distance is estimated by length less than or equal to k, where $k \geq 2$.This article, we determined the CD-number for the k^{th} power of PVB-tree and $((OT_r)^k)$ where $2 \leq k \leq 7$.

3 Results and Discussion

3.1 γ_{CD} value for the $k^{th}(2 \leq k \leq 7)$ power of olive tree

Theorem 3.1.1 If $r \geq 3$, then

$$\gamma_{CD}((OT_r)^2) = \left(\begin{array}{cc} \lceil \frac{r(r+1)}{7} \rceil & if r \equiv\ 1\ or\ 2\ or\ 3\ or\ 4\ or\ 5\ (mod\ 7), \\ \frac{r(r+1)}{7} + 1 & Otherwise. \end{array} \right.$$

Proof. Let $X((OT_r)^2) = \{v_0, P_1(v_1), P_2(v_1), P_2(v_2), P_3(v_1), P_3(v_2), P_3(v_3), \ldots, P_r(v_1), P_r(v_2), \ldots, P_r(v_r)\}$. For $1 \leq r \leq 2$, then $\gamma_{CD}((OT_r)^2) = 2$. If $r \geq 3$, consider the set $S = \{P_j(v_{j-i}) : i \equiv\ 2\ or\ 4\ (mod\ 7)\ and\ 3 \leq j \leq r\}$. Clearly S is a CD-set of $(OT_r)^2$. Thus

$$\gamma_{CD}((OT_r)^2) = |S| \leq \left(\begin{array}{cc} \lceil \frac{r(r+1)}{7} \rceil & if r \equiv\ 1\ or\ 2\ or\ 3\ or\ 4\ or\ 5\ (mod\ 7), \\ \frac{r(r+1)}{7} + 1 & Otherwise. \end{array} \right.$$

Suppose there exist a dominating set $D \subseteq X((OT_r)^3)$ of cardinality at most

$$d = \left(\begin{array}{cc} \left\lceil \frac{r(r+1)}{7} \right\rceil - 1 & if r \equiv\ 1\ or\ 2\ or\ 3\ or\ 4\ or\ 5\ \ (mod\ 7)\,, \\ \frac{r(r+1)}{7} & Otherwise. \end{array} \right.$$ then $< D >$ has an isolated vertex. Thus

$$\gamma_{CD}((OT_r)^3) \geq d + 1 = \left(\begin{array}{cc} \lceil \frac{r(r+1)}{7} \rceil & if r \equiv\ 1\ or\ 2\ or\ 3\ or\ 4\ or\ 5\ (mod\ 7), \\ \frac{r(r+1)}{7} + 1 & Otherwise. \end{array} \right.$$

Hence the theorem follows.

Theorem 3.1. 2 If $r \geq 4$, then

$$\gamma_{CD}((OT_r)^3) = \left(\begin{array}{cc} \lceil \frac{r(r+1)}{10} \rceil + 1 & if r \equiv\ 0\ or\ 9\ (mod\ 10), \\ \lfloor \frac{r(r+1)}{10} \rfloor & if r \equiv\ 3\ or\ 6\ (mod\ 10), \\ \lceil \frac{r(r+1)}{10} \rceil & Otherwise. \end{array} \right.$$

Proof. Let $X((OT_r)^3) = \{v_0, P_1(v_1), P_2(v_1), P_2(v_2), P_3(v_1), P_3(v_2), P_3(v_3), \ldots, P_r(v_1), P_r(v_2), \ldots, P_r(v_r)\}$. For $1 \leq r \leq 3$, then $\gamma_{CD}((OT_r)^3) = 2$.

If $r \geq 4$, consider the set $S = \{P_j(v_{j-i}) : i \equiv 3 \ or \ 6 \ (mod \ 10) \ and \ 4 \leq j \leq r\}$. Clearly S is a CD-set of $(OT_r)^3$. Thus

$$\gamma_{CD}((OT_r)^3) = |S| \leq \left(\begin{array}{ll} \lceil \frac{r(r+1)}{10} \rceil + 1 & if r \equiv \ 0 \ or \ 9 \ (mod \ 10), \\ \lfloor \frac{r(r+1)}{10} \rfloor & if r \equiv \ 3 \ or \ 6 \ (mod \ 10), \\ \lceil \frac{r(r+1)}{10} \rceil & \quad Otherwise. \end{array} \right.$$

Suppose there exist a dominating set $D \subseteq X((OT_r)^3)$ of cardinality at most

$$d = \left(\begin{array}{ll} \lceil \frac{r(r+1)}{10} \rceil & if r \equiv \ 0 \ or \ 9 \ (mod \ 10), \\ \lfloor \frac{r(r+1)}{10} \rfloor - 1 & if r \equiv \ 3 \ or \ 6 \ (mod \ 10), \\ \lceil \frac{r(r+1)}{10} \rceil - 1 & \quad Otherwise, \end{array} \right.$$

then $< D >$ has an isolated vertex. Thus

$$\gamma_{CD}((OT_r)^3) \geq d+1 = \left(\begin{array}{ll} \lceil \frac{r(r+1)}{10} \rceil + 1 & if r \equiv \ 0 \ or \ 9 \ (mod \ 10), \\ \lfloor \frac{r(r+1)}{10} \rfloor & if r \equiv \ 3 \ or \ 6 \ (mod \ 10), \\ \lceil \frac{r(r+1)}{10} \rceil & \quad Otherwise. \end{array} \right.$$

Hence the theorem follows.

Theorem 3.1. 3 If $r \geq 5$, then

$$\gamma_{CD}((OT_r)^4) = \left(\begin{array}{ll} \lceil \frac{r(r+1)}{13} \rceil + 1 & if r \equiv \ 0 \ or \ 12 \ (mod \ 13), \\ \lfloor \frac{r(r+1)}{13} \rfloor & if r \equiv \ 4 \ or \ 5 \ or \ 6 \ or \ 7 \ or \ 8 \ (mod \ 13), \\ \lceil \frac{r(r+1)}{13} \rceil & if r \equiv \ 1 \ or \ 2 \ or \ 3 \ or \ 9 \ or \ 10 \ (mod \ 13). \end{array} \right.$$

Proof. Let $X((OT_r)^4) = \{v_0, P_1(v_1), P_2(v_1), P_2(v_2), P_3(v_1), P_3(v_2), P_3(v_3), \ldots, P_r(v_1), P_r(v_2), \ldots, P_r(v_r)\}$. For $1 \leq r \leq 4$, then $\gamma_{CD}((OT_r)^4) = 2$.

If $r \geq 5$, consider the set $S = \{P_j(v_{j-i}) : i \equiv 4 \ or \ 8 \ (mod \ 13) \ and \ 5 \leq j \leq r\}$. Clearly S is a CD-set of $(OT_r)^4$. Thus

$$\gamma_{CD}((OT_r)^4) \leq \left(\begin{array}{ll} \lceil \frac{r(r+1)}{13} \rceil + 1 & if r \equiv \ 0 \ or \ 12 \ (mod \ 13), \\ \lfloor \frac{r(r+1)}{13} \rfloor & if r \equiv \ 4 \ or \ 5 \ or \ 6 \ or \ 7 \ or \ 8 \ (mod \ 13), \\ \lceil \frac{r(r+1)}{13} \rceil & if r \equiv \ 1 \ or \ 2 \ or \ 3 \ or \ 9 \ or \ 10 \ (mod \ 13). \end{array} \right.$$

Suppose there exist a dominating set $D \subseteq X((OT_r)^4)$ of cardinality at most

$$d = \left(\begin{array}{ll} \lceil \frac{r(r+1)}{13} \rceil & if r \equiv \ 0 \ or \ 12 \ (mod \ 13), \\ \lfloor \frac{r(r+1)}{13} \rfloor - 1 & if r \equiv \ 4 \ or \ 5 \ or \ 6 \ or \ 7 \ or \ 8 \ (mod \ 13), \\ \lceil \frac{r(r+1)}{13} \rceil - 1 & if r \equiv \ 1 \ or \ 2 \ or \ 3 \ or \ 9 \ or \ 10 \ (mod \ 13), \end{array} \right.$$

then $< D >$ an isolated vertex. Thus

$$\gamma_{CD}((OT_r)^4) \geq d+1 = \left(\begin{array}{ll} \lceil \frac{r(r+1)}{13} \rceil + 1 & if r \equiv \ 0 \ or \ 12 \ (mod \ 13), \\ \lfloor \frac{r(r+1)}{13} \rfloor & if r \equiv \ 4 \ or \ 5 \ or \ 6 \ or \ 7 \ or \ 8 \ (mod \ 13), \\ \lceil \frac{r(r+1)}{13} \rceil & if r \equiv \ 1 \ or \ 2 \ or \ 3 \ or \ 9 \ or \ 10 \ (mod \ 13). \end{array} \right.$$

Hence the theorem follows.

Theorem 3.1. 4 If $r \geq 6$, then

$$\gamma_{CD}((OT_r)^5) = \left(\begin{array}{ll} \frac{r(r+1)}{13} + 1 & if r \equiv \ 0 \ or \ 15 \ (mod \ 16), \\ \lceil \frac{r(r+1)}{16} \rceil & if r \equiv \ 1 \ or \ 2 \ or \ 3 \ or \ 12 \ or \ 13 \ or \ 14 \ (mod \ 16), \\ \lfloor \frac{r(r+1)}{16} \rfloor & \quad Otherwise. \end{array} \right.$$

Proof. Let $X((OT_r)^5) = \{v_0, P_1(v_1), P_2(v_1), P_2(v_2), P_3(v_1), P_3(v_2), P_3(v_3), \ldots, P_r(v_1), P_r(v_2), \ldots, P_r(v_r)\}$. For $1 \leq r \leq 5$, then $\gamma_{CD}((OT_r)^5) = 2$.

If $r \geq 6$, consider the set $S = \{P_j(v_{j-i}) : i \equiv 5 \ or \ 10 \ (mod \ 16) \ and \ 6 \leq j \leq r\}$. Clearly S is a CD-set of $(OT_r)^5$. Thus

$$\gamma_{CD}((OT_r)^5) \leq \left(\begin{array}{ll} \frac{r(r+1)}{13} + 1 & if r \equiv \ 0 \ or \ 15 \ (mod \ 16), \\ \lceil \frac{r(r+1)}{16} \rceil & if r \equiv \ 1 \ or \ 2 \ or \ 3 \ or \ 12 \ or \ 13 \ or \ 14 \ (mod \ 16), \\ \lfloor \frac{r(r+1)}{16} \rfloor & \quad Otherwise. \end{array} \right.$$

Suppose there exist a dominating set $D \subset X((OT_r)^5)$ of cardinality at most
$$d = \begin{cases} \frac{r(r+1)}{13} & if r \equiv 0 \text{ or } 15 \ (mod\ 16), \\ \lceil \frac{r(r+1)}{16} \rceil - 1 & if r \equiv 1 \text{ or } 2 \text{ or } 3 \text{ or } 12 \text{ or } 13 \text{ or } 14 \ (mod\ 16), \\ \lfloor \frac{r(r+1)}{16} \rfloor - 1 & Otherwise, \end{cases}$$
then $< D >$ has an isolated vertex. Thus
$$\gamma_{CD}((OT_r)^5) \geq d+1 = \begin{cases} \frac{r(r+1)}{13} + 1 & if r \equiv 0 \text{ or } 15 \ (mod\ 16), \\ \lceil \frac{r(r+1)}{16} \rceil & if r \equiv 1 \text{ or } 2 \text{ or } 3 \text{ or } 12 \text{ or } 13 \text{ or } 14 \ (mod\ 16), \\ \lfloor \frac{r(r+1)}{16} \rfloor & Otherwise. \end{cases}$$
Hence the theorem follows.

Theorem 3.1. 5 If $r \geq 7$, then
$$\gamma_{CD}((OT_r)^6) = \begin{cases} \frac{r(r+1)}{19} + 1 & if r \equiv 0 \text{ or } 18 \ (mod\ 19), \\ \lfloor \frac{r(r+1)}{19} \rfloor & if r \equiv 4 \text{ or } 5 \text{ or } 11 \text{ or } 13 \text{ or } 14 \ (mod\ 19), \\ \lceil \frac{r(r+1)}{19} \rceil & if r \equiv 1 \text{ or } 2 \text{ or } 3 \text{ or } 15 \text{ or } 16 \text{ or } 17 \ (mod\ 19), \\ \lfloor \frac{r(r+1)}{19} \rfloor - 1 & if r \equiv 6 \text{ or } 7 \text{ or } 8 \text{ or } 9 \text{ or } 10 \text{ or } 12 \ (mod\ 19). \end{cases}$$

Proof. Let $X((OT_r)^6) = \{v_0, P_1(v_1), P_2(v_1), P_2(v_2), P_3(v_1), P_3(v_2), P_3(v_3), \ldots, P_r(v_1), P_r(v_2), \ldots, P_r(v_r)\}$. For $1 \leq r \leq 6$, then $\gamma_{CD}((OT_r)^6) = 2$.

If $r \geq 7$, consider the set $S = \{P_j(v_{j-i}) : i \equiv 6 \text{ or } 12 \ (mod\ 19) \text{ and } 7 \leq j \leq r\}$. Clearly S is a CD-set of $(OT_r)^6$. Thus
$$\gamma_{CD}((OT_r)^6) \leq \begin{cases} \frac{r(r+1)}{19} + 1 & if r \equiv 0 \text{ or } 18 \ (mod\ 19), \\ \lfloor \frac{r(r+1)}{19} \rfloor & if r \equiv 4 \text{ or } 5 \text{ or } 11 \text{ or } 13 \text{ or } 14 \ (mod\ 19), \\ \lceil \frac{r(r+1)}{19} \rceil & if r \equiv 1 \text{ or } 2 \text{ or } 3 \text{ or } 15 \text{ or } 16 \text{ or } 17 \ (mod\ 19), \\ \lfloor \frac{r(r+1)}{19} \rfloor - 1 & if r \equiv 6 \text{ or } 7 \text{ or } 8 \text{ or } 9 \text{ or } 10 \text{ or } 12 \ (mod\ 19). \end{cases}$$

Suppose there exist a dominating set $D \subseteq X((OT_r)^6)$ of cardinality at most
$$d = \begin{cases} \frac{r(r+1)}{19} & if r \equiv 0 \text{ or } 18 \ (mod\ 19), \\ \lfloor \frac{r(r+1)}{19} \rfloor - 1 & if r \equiv 4 \text{ or } 5 \text{ or } 11 \text{ or } 13 \text{ or } 14 \ (mod\ 19), \\ \lceil \frac{r(r+1)}{19} \rceil - 1 & if r \equiv 1 \text{ or } 2 \text{ or } 3 \text{ or } 15 \text{ or } 16 \text{ or } 17 \ (mod\ 19), \\ \lfloor \frac{r(r+1)}{19} \rfloor & if r \equiv 6 \text{ or } 7 \text{ or } 8 \text{ or } 9 \text{ or } 10 \text{ or } 12 \ (mod\ 19), \end{cases}$$
then $< D >$ has an isolated vertex. Thus
$$\gamma_{CD}((OT_r)^6) \geq d+1 = \begin{cases} \frac{r(r+1)}{19} + 1 & if r \equiv 0 \text{ or } 18 \ (mod\ 19), \\ \lfloor \frac{r(r+1)}{19} \rfloor & if r \equiv 4 \text{ or } 5 \text{ or } 11 \text{ or } 13 \text{ or } 14 \ (mod\ 19), \\ \lceil \frac{r(r+1)}{19} \rceil & if r \equiv 1 \text{ or } 2 \text{ or } 3 \text{ or } 15 \text{ or } 16 \text{ or } 17 \ (mod\ 19), \\ \lfloor \frac{r(r+1)}{19} \rfloor - 1 & if r \equiv 6 \text{ or } 7 \text{ or } 8 \text{ or } 9 \text{ or } 10 \text{ or } 12 \ (mod\ 19). \end{cases}$$
Hence the theorem follows.

Theorem 3.1. 6 If $r \geq 8$, then
$$\gamma_{CD}((OT_r)^7) = \begin{cases} \lceil \frac{r(r+1)}{22} \rceil & if r \equiv 1 \text{ or } 2 \text{ or } 3 \text{ or } 4 \text{ or } 17 \text{ or } 18 \text{ or } 19 \text{ or } 20 \ (mod\ 22), \\ \lceil \frac{r(r+1)}{22} \rceil + 1 & if r \equiv 0 \text{ or } 21 \ (mod\ 22), \\ \lfloor \frac{r(r+1)}{22} \rfloor & if r \equiv 5 \text{ or } 6 \text{ or } 15 \text{ or } 16 \ (mod\ 22), \\ \lfloor \frac{r(r+1)}{22} \rfloor - 1 & if r \equiv 7 \text{ or } 8 \text{ or } 9 \text{ or } 10 \text{ or } 11 \text{ or } 12 \text{ or } 13 \text{ or } 14 \ (mod\ 22). \end{cases}$$

Proof. Let $X((OT_r)^7) = \{v_0, P_1(v_1), P_2(v_1), P_2(v_2), P_3(v_1), P_3(v_2), P_3(v_3), \ldots, P_r(v_1), P_r(v_2), \ldots, P_r(v_r)\}$.For $1 \leq r \leq 7$, then $\gamma_{CD}((OT_r)^7) = 2$.

If $r \geq 8$, consider the set $S = \{P_j(v_{j-i}) : i \equiv 7 \text{ or } 14 \ (mod\ 22) \text{ and } 8 \leq j \leq r\}$. Clearly S is a CD-set of $(OT_r)^7$. Thus
$$\gamma_{CD}((OT_r)^7) \leq \begin{cases} \lceil \frac{r(r+1)}{22} \rceil & if r \equiv 1 \text{ or } 2 \text{ or } 3 \text{ or } 4 \text{ or } 17 \text{ or } 18 \text{ or } 19 \text{ or } 20 \ (mod\ 22), \\ \lceil \frac{r(r+1)}{22} \rceil + 1 & if r \equiv 0 \text{ or } 21 \ (mod\ 22), \\ \lfloor \frac{r(r+1)}{22} \rfloor & if r \equiv 5 \text{ or } 6 \text{ or } 15 \text{ or } 16 \ (mod\ 22), \\ \lfloor \frac{r(r+1)}{22} \rfloor - 1 & if r \equiv 7 \text{ or } 8 \text{ or } 9 \text{ or } 10 \text{ or } 11 \text{ or } 12 \text{ or } 13 \text{ or } 14 \ (mod\ 22). \end{cases}$$
Suppose there exist a dominating set $D \subseteq X((OT_r)^7)$ of cardinality

$$d=\begin{cases}\left\lceil\frac{r(r+1)}{22}\right\rceil-1 & if r\equiv\ 1\ or\ 2\ or\ 3\ or\ 4\ or\ 17\ or\ 18\ or\ 19\ or\ 20\ (mod\ 22),\\ \left\lceil\frac{r(r+1)}{22}\right\rceil & if r\equiv\ 0\ or\ 21\ (mod\ 22),\\ \left\lfloor\frac{r(r+1)}{22}\right\rfloor-1 & if r\equiv\ 5\ or\ 6\ or\ 15\ or\ 16\ (mod\ 22),\\ \left\lfloor\frac{r(r+1)}{22}\right\rfloor & if r\equiv\ 7\ or\ 8\ or\ 9\ or\ 10\ or\ 11\ or\ 12\ or\ 13\ or\ 14\ (mod\ 22),\end{cases}$$

$$d=\begin{pmatrix}\left\lceil\frac{r(r+1)}{22}\right\rceil-1 & if r\equiv\ 1\ or\ 2\ or\ 3\ or\ 4\ or\ 17\ or\ 18\ or\ 19\ or\ 20\ (mod\ 22),\\ \left\lceil\frac{r(r+1)}{22}\right\rceil & if\ r\equiv\ 0\ or\ 21\ (mod\ 22),\\ \left\lfloor\frac{r(r+1)}{22}\right\rfloor-1 & if\ r\equiv\ 5\ or\ 6\ or\ 15\ or\ 16\ (mod\ 22),\\ \left\lfloor\frac{r(r+1)}{22}\right\rfloor & if\ r\equiv\ 7\ or\ 8\ or\ 9\ or\ 10\ or\ 11\ or\ 12\ or\ 13\ or\ 14\ (mod\ 22),\end{pmatrix}$$

then $<D>$ has an isolated vertex. Thus

$$\gamma_{CD}((OT_r)^7)\geq d+1=\begin{pmatrix}\lceil\frac{r(r+1)}{22}\rceil & if r\equiv\ 1\ or\ 2\ or\ 3\ or\ 4\ or\ 17\ or\ 18\ or\ 19\ or\ 20\ (mod\ 22),\\ \lceil\frac{r(r+1)}{22}\rceil+1 & if r\equiv\ 0\ or\ 21\ (mod\ 22),\\ \lfloor\frac{r(r+1)}{22}\rfloor & if r\equiv\ 5\ or\ 6\ or\ 15\ or\ 16\ (mod\ 22),\\ \lfloor\frac{r(r+1)}{22}\rfloor-1 & if r\equiv\ 7\ or\ 8\ or\ 9\ or\ 10\ or\ 11\ or\ 12\ or\ 13\ or\ 14\ (mod\ 22).\end{pmatrix}$$

Hence the theorem follows.

3.2 γ_{CD} value for the $k^{th}(k\geq 2)$ power of PVB-tree

Theorem 3.2 .1 [4] If H is PVB tree. Then $\gamma_{CD}(H)=2p(r-3), p\geq 1, v,b>1\ and\ r\geq 6$.

Theorem 3.2 .2 Let H be a PVB tree. Then

$$\gamma_{CD}(H^k)=\begin{pmatrix}\lceil\frac{2v}{3k-1}\rceil P & if v\equiv\ 0\ or\ 1\ or\ 2\ or\ 3\ or\ldots ork-1\ ork+1\ldots\\ & or\ 3k-1\ (mod\ 3k-1),\\ (\lceil\frac{2v}{3k-1}\rceil+1)P & if v\equiv k\ (mod\ 3k-1).\end{pmatrix}$$

Proof. Let $S_1=\{y_{ij}: j\equiv kor\ 2k\ (mod\ 3k-1): 1\leq i\leq P\}$. Then

$$S=\begin{pmatrix}S_1 & if v\equiv\ 0\ or\ 2kor\ 2k+2\ or\ldots\\ & or\ 3k-3\ (mod\ 3k-1),\\ S_1\cup\{y_{i(j-1)}: 1\leq i\leq P\} & if v\equiv j\ (mod\ 3k-1), 1\leq j\leq k-1,\\ (S_1\cup\{y_{i(k-1)}: 1\leq i\leq P\})-\{y_{iv}\} & if v\equiv k\ (mod\ 3k-1),\\ S_1\cup\{y_{iv}: 1\leq i\leq P\} & if v\equiv k+1\ ork+2\ or\ldots\\ & or\ 2k-1\ (mod\ 3k-1),\end{pmatrix}$$

is a CD-set of H^k. Thus

$$\gamma_{CD}(H^k)\leq|S|=\begin{pmatrix}\lceil\frac{2v}{3k-1}\rceil P & if v\equiv\ 0\ or\ 1\ or\ 2\ or\ 3\ or\ldots ork-1\ ork+1\ldots\\ & or\ 3k-1\ (mod\ 3k-1),\\ (\lceil\frac{2v}{3k-1}\rceil+1)P & if v\equiv k\ (mod\ 3k-1).\end{pmatrix}$$

Suppose there exist a dominating set $D\subseteq X$ of cardinality at most

$$d=\begin{pmatrix}\lceil\frac{2v}{3k-1}\rceil P-1 & if v\equiv\ 0\ or\ 1\ or\ 2\ or\ 3\ or\ldots ork-1\ ork+1\ldots\\ & or\ 3k-1\ (mod\ 3k-1),\\ (\lceil\frac{2v}{3k-1}\rceil+1)P-1 & if v\equiv k\ (mod\ 3k-1),\end{pmatrix}$$

then $<D>$ has an isolated vertex. Thus

$$|D|\geq d+1=\begin{pmatrix}\lceil\frac{2v}{3k-1}\rceil P & if v\equiv\ 0\ or\ 1\ or\ 2\ or\ 3\ or\ldots ork-1\ ork+1\ldots\\ & or\ 3k-1\ (mod\ 3k-1),\\ (\lceil\frac{2v}{3k-1}\rceil+1)P & if v\equiv k\ (mod\ 3k-1).\end{pmatrix}$$

Hence the theorem.

4 Conclusion

In this article, the exact γ_{CD} values for the k^{th} power of PVB-tree and $((OT_r)^k)$, where $2 \leq k \leq 7$ were determined. In the subsequent article the exact γ_{CD} values for the k^{th} power of $((OT_r)^k)$ will be determined for any k and the comparison of our parameter for the above-mentioned graph with another parameter will be discussed in the subsequent papers.

Declaration

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