

## RESEARCH ARTICLE



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# To Solve Pythagorean Fuzzy Decagonal Transportation Problem Using Bipartite Graph with Optimal Solution

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## Abstract

**Objectives:** This study aims to solve the Pythagorean fuzzy transportation problem by means of providing a novel approach. **Methods:** To determine the optimum decision-making, we applied the MODI approach. To illustrate the proposed approach, we applied several types of numerical issues. An algorithm for solving the suggested technique is described. **Findings:** This study presents a bipartite graph-based strategy to the Pythagorean fuzzy transportation problem. Applying TORA statistical software, the data analysis discovered that the crisp model becomes linear when the membership function is applied. **Novelty:** This study has paved a way for better optimal results to obtain when the Pythagorean fuzzy set is converted using the score function instead of the standard techniques for Defuzzification.

**Keywords:** Score Function; Pythagorean Fuzzy; Defuzzification; Pythagorean Fuzzy Transportation Problem; Decagonal Fuzzy Number; Ranking Method; Initial Basic Feasible Solution; and Optimal Solution

## 1 Introduction

One specific type of linear programming problem-solving is the transportation problem. The goal of the transportation problem is to reduce the cost of a certain commodity by working with several providers to reach different locations. Every destination has a demand that must be satisfied, and every supplier has a limited quantity to provide<sup>(1)</sup>. A helpful method for illustrating the ambiguity of issues requiring multiple attribute decision-making (MADM) is the Pythagorean fuzzy model. PFSSs are helpful in many different domains<sup>(2)</sup>. PyFDG, together with its crucial procedure and grading system, have been developed to solve application concerns in healthcare institutions. A unique solution to the Job Sequencing issue utilising PFN was proposed by Harpreet Kaur<sup>(3)</sup>. Several remote measurements of sophisticated Pythagorean fuzzy sets were constructed by KifayatUllah et al.<sup>(4)</sup> and used to a problem involving the recognition of construction materials. For the purpose of developing novel goods, Goçer and Büyükoçkan<sup>(5)</sup> developed an original

modification of the Pythagorean fuzzy Multi-objective Optimisation by Ratio Analysis (MULTIMOORA) technique. Jayapriya and Sophia Porchelvi<sup>(6)</sup> try to provide a variety of Pythagorean fuzzified and defuzzified functions in the Pythagorean uncertain environment to simulate real-world situations. Following this, Divya Bharathi and Kanmani<sup>(7)</sup> demonstrated that their suggested methods produced the best outcome while presenting an original PyFTP algorithm. For decagonal integers, Saranya and Charles Rabinson<sup>(8)</sup> developed the ideal PyFTP method. Priyanka Nagar et al. created a technique that minimises the chance of missing any kind of information while still providing the best response in fuzzy form<sup>(9)</sup>. A geometric mean technique was devised by Krishna Prabha et al.<sup>(10)</sup> to address an H. K and V. B Heliyon 9 (2023) e20775 3 PyFTP. This study's main objective is to lower overall transit costs in a Pythagorean fuzzy setting. Finding novel, simultaneous algorithms for ranking and IBFS to solve TP in a Pythagorean uncertain environment is not feasible in light of past presentations. This research gap motivated the authors to develop a novel ranking and IBFS technique that may optimum the transportation problem in a Pythagorean uncertain condition.

## 2 Methodology

### Definition 2.1

Assuming that  $X$  is a fixed set, a Pythagorean fuzzy set is any object with the form  $P = \{x, (\theta_p(x), \tau_p(x)) \mid x \in X\}$ , where the significance is  $\theta_p(x): X \rightarrow [0,1]$  &  $\tau_p(x): X \rightarrow [0,1]$  the matching levels of membership and non-membership for each of the components  $x \in X$  to  $P$ . Furthermore, for each  $x \in X$ , it explains that  $(\theta_p(X))^2 + (\tau_p(X))^2 \leq 1$ .

### Definition 2.2

Let  $\check{a}_1^p = (\theta_i^p, \tau_s^p)$  and  $\check{b}_1^p = (\theta_o^p, \tau_f^p)$  It is composed of two PFNs, or Pythagorean fuzzy numbers. Thus, the resulting mathematical operations are as follows:

$$(i) \text{ Additive Functions: } \check{a}_1^p \oplus \check{b}_1^p = \sqrt{((\theta_i^p)^2 + (\theta_o^p)^2 - (\theta_i^p)^2 (\theta_o^p)^2, \tau_s^p \tau_f^p)}$$

(ii) Multiplicative functions:

$$\check{a}_1^p \otimes \check{b}_1^p = \left( \theta_i^p \cdot \theta_o^p \sqrt{(\tau_s^p)^2 + (\tau_f^p)^2 - (\tau_s^p)^2 (\tau_f^p)^2} \right)$$

Where  $k$  non negative constant,  $k > 0$

$$(iii) \text{ The } \mathcal{S} \text{ calar Product: } k\check{a}_1^p = \left( \sqrt{1 - (1 - \theta_i^p)^k}, (\tau_s^p)^k \right)$$

### Definition 2.3

**Two Pythagorean fuzzy numbers are compared (PFNs)**

Let  $\check{a}_1^p = (\theta_i^p, \tau_s^p)$  and  $\check{\beta}_1^p = (\theta_o^p, \tau_f^p)$  Two Pythagorean Fuzzy Numbers should be used, and the following accuracy and scoring functions should be used:

(i) The Score Component:

$$\mathcal{S}(\check{\alpha}_1^p) = \frac{1}{2} \left( 1 - (\theta_i^p)^2 - (\tau_s^p)^2 \right)$$

(ii) The Accuracy Component:

$$\mathcal{H}(\check{\alpha}_1^p) = (\theta_i^p)^2 + (\tau_s^p)^2 \alpha_1^p$$

After that, the following five scenarios take place:

- If  $\check{\alpha}_1^p > \check{\beta}_1^p$  iff  $\mathcal{S}(\check{\alpha}_1^p) > \mathcal{S}(\check{\beta}_1^p)$
- If  $\check{\alpha}_1^p < \check{\beta}_1^p$  iff  $\mathcal{S}(\check{\alpha}_1^p) < \mathcal{S}(\check{\beta}_1^p)$
- If  $\mathcal{S}(\check{\alpha}_1^p) = \mathcal{S}(\check{\beta}_1^p)$  &  $\mathcal{H}(\check{\alpha}_1^p) < \mathcal{H}(\check{\beta}_1^p)$ ,  $\check{\alpha}_1^p < \check{\beta}_1^p$
- If  $\mathcal{S}(\check{\alpha}_1^p) = \mathcal{S}(\check{\beta}_1^p)$  &  $\mathcal{H}(\check{\alpha}_1^p) > \mathcal{H}(\check{\beta}_1^p)$ ,  $\check{\alpha}_1^p > \check{\beta}_1^p$
- If  $\mathcal{S}(\check{\alpha}_1^p) = \mathcal{S}(\check{\beta}_1^p)$  &  $\mathcal{H}(\check{\alpha}_1^p) = \mathcal{H}(\check{\beta}_1^p)$ , then  $\check{\alpha}_1^p = \check{\beta}_1^p$

### Definition 2.3

A decision-making process that meets praise, linearity, and additive qualities provide outcomes that are motivated by human nature. If  $d_i^p$  is a fuzzy number, then that location is distinguished by

$$R(d_i^p) = \frac{1}{d_i^p} \sum d_i^p, i = 1 \text{ to } 10$$

### 3 Results and Discussion

The idea that there are  $m$  sources and  $n$  destinations must also be taken into account. Given the following assumptions and limitations, the goal of the transportation issue is to reduce the cost of moving a good from these sources to the destinations, even in cases when supply and demand for the good are obvious:

- When considering the entire network, there are  $m$  sources.
- $n$  is the whole number of destination nodes
- $i$  is the database of sources for all  $m$
- $j$  is the database of destination for all  $n$
- $C_{ij}^p$  is the Average Pythagorean expense of transportation from the  $i^{th}$  the original source of the  $j^{th}$  destination of a unit quantity of the item.
- $C_{ij}$  is the approximate cost of one quantity unit.
- $a_{ij}$  is the amount of supply that is available from each source in the good environment
- $a_{ij}^p$  is the product's Pythagorean fuzzy availability at the  $i^{th}$  source
- $b_{ij}$  is the market demand quantity from each region in a good environment
- $b_{ij}^p$  is the product's fuzzy Pythagorean demand at the  $j^{th}$  destination
- To minimize the overall fuzzy transportation cost,  $X_{ij}$  is the fuzzy amount of the product that has to be conveyed from the  $i^{th}$  origin to the  $j^{th}$  final destination.

As stated by, the Pythagorean fuzzy transportation problem

Minimize  $\sum_{i=1}^m \sum_{j=1}^n C_{ij}^p * X_{ij}$

Subject to  $\sum_{j=1}^n X_{ij} = a_i^p = \text{supply}$ , where  $i = 1, 2, 3, \dots, m$

$\sum_{i=1}^m X_{ij} = b_j^p = \text{Demand}$ , where  $j = 1, 2, 3, \dots, n$   $\sum_{i=1}^m a_i^p = \sum_{j=1}^n b_j^p$

#### 3.1 The Fuzzy Number derived from the Pythagorean Decagonal

The fuzzy number of Pythagorean Decagonal  $(d_1^p, d_2^p, d_3^p, d_4^p, d_5^p, d_6^p, d_7^p, d_8^p, d_9^p, d_{10}^p)$  And the membership function is specified as

$$\mu_{d_p}(x) = \begin{cases} \frac{1}{4} \frac{(x-d_1^p)}{(d_2^p-d_1^p)}, & d_1^p \leq x \leq d_2^p \\ \frac{1}{4} + \frac{1}{4} + \frac{(x-d_2^p)}{(d_3^p-d_2^p)}, & d_2^p \leq x \leq d_3^p \\ \frac{1}{2} + \frac{1}{4} + \frac{(x-d_3^p)}{(d_4^p-d_3^p)}, & d_3^p \leq x \leq d_4^p \\ \frac{3}{4} + \frac{1}{4} + \frac{(x-d_4^p)}{(d_5^p-d_4^p)}, & d_4^p \leq x \leq d_5^p \\ 1, & d_5^p \leq x \leq d_6^p \\ 1 - \frac{1}{4} \frac{(x-d_6^p)}{(d_7^p-d_6^p)}, & d_6^p \leq x \leq d_7^p \\ \frac{3}{4} - \frac{1}{4} \frac{(x-d_7^p)}{(d_8^p-d_7^p)}, & d_7^p \leq x \leq d_8^p \\ \frac{1}{2} - \frac{1}{4} \frac{(x-d_8^p)}{(d_9^p-d_8^p)}, & d_8^p \leq x \leq d_9^p \\ 1 - \frac{1}{4} \frac{(x-d_9^p)}{(d_{10}^p-d_9^p)}, & d_9^p \leq x \leq d_{10}^p \\ 0 & \text{otherwise} \end{cases}$$

#### An Algorithmic Approach to the Pythagorean Fuzzy Decagonal Transportation Problem

**Step 1:** The cost table should be framed in accordance with the given Pythagorean fuzzy decagonal transportation problem.

- Verify whether the overall supply and demand are equal. If so, move on to step 2.
- If not, make an artificial row or column that represents supply and demand with all of its cost components set to zero by adding a (+ve) difference between supply and demand.

**Step 2:** The rank of every cell has to be found in the chosen Pythagorean fuzzy cost matrix  $y$  using the ranking technique previously provided.

**Step 3:** Using the scoring technique, on each value, converts all of the Pythagorean fuzzy numbers into crisp numbers.

- Step 4:** A bipartite graph will be created from the transportation table.  
**Step 5:** The highest bipartite edge value is selected from the sum of the edges.  
**Step 6:** The maximum number of resultant bipartite vertices is found.  
**Step 7:** If more than one maximum result vertices value is found, it can choose any one.  
**Step 8:** The smallest edge value is to be determined and allocated.  
**Step 9:** Step 4 is repeated until there is no edge left.  
**Step 10:** The prerequisites for the rim have been met.

### 3.3 Numerical Illustration

The Pythagorean fuzzy decagonal transportation problem's input data are listed in Table 1. The process's ideal goal is to maximize profit while minimizing transportation costs.

**Table 1.** Pythagorean Fuzzy Decagonal Transportation Problem

	$O_1$	$O_2$	$O_3$	$O_4$	Supply
$S_1$	(-5, -4, -3, -2, 1, 2, 3, 4, 5, 6)	(-21, -20, -18, -16, 0, 4, 16, 18, 20, 21)	(-18, -16, -14, -11, 0, 8, 11, 14, 16, 18)	(-11, -10, -9, -8, 0, 6, 8, 9, 10, 11)	(-1, 1, 5, 20, 25, 30, 35, 40, 45, 50)
$S_2$	(-21, -20, -18, -16, 0, 4, 16, 18, 20, 21)	(-13, -12, -11, -9, 2, 5, 9, 11, 12, 13)	(-7, -6, -5, 0, 2, 3, 4, 5, 6, 7)	(-13, -12, -11, -9, 0, 5, 9, 11, 12, 13)	(-10, -7, 12, 17, 18, 19, 21, 22, 33, 35)
$S_3$	(-11, -10, -9, -8, 0, 6, 8, 9, 10, 11)	(-16, -14, -13, -8, 1, 7, 8, 13, 14, 16)	(-13, -12, -11, -9, 2, 5, 9, 11, 12, 13)	(-7, -6, -5, 0, 2, 3, 4, 5, 6, 7)	(-1, 0, 1, 2, 3, 8, 17, 29, 40, 51)
Demand	(-1, 0, 10, 13, 15, 23, 30, 35, 50, 55)	(-23, -21, 13, 19, 21, 23, 30, 35, 36, 37)	(0, 1, 2, 3, 4, 5, 7, 8, 9, 11)	(-4, 0, 4, 5, 10, 16, 17, 19, 20, 23)	

Using the ranking function, each cell  $C_{ij}$  of represents the selected Pythagorean fuzzy cost matrix  $y$ .

**Table 2.** Pythagorean fuzzy transportation problem using the scoring technique

	$O_1$	$O_2$	$O_3$	$O_4$	Supply
$S_1$	(0.70, 0.30)	(0.40, 0.60)	(0.80, 0.20)	(0.60, 0.40)	25
$S_2$	(0.40, 0.60)	(0.70, 0.30)	(0.90, 0.10)	(0.50, 0.50)	16
$S_3$	(0.60, 0.40)	(0.80, 0.20)	(0.70, 0.30)	(0.90, 0.10)	15
Demand	23	17	5	11	

In the Table 2, the sum of the totals of both supply and demand is 80. Hence, there is a balanced approach to the TP.

Step 1: Analyze the cost table in the provided issue. Since the entire supply and total demand in this situation are equal, we may go on to step 2. After the scoring procedure is applied to each value, all of the Pythagorean fuzzy numbers are transformed into crisp numbers. In accordance with the scoring function

Definition 2.2,

$$\mathcal{S} = \frac{1}{2} \left( \left( 1 - (\theta_i^p)^2 \right) - (\tau_s^p)^2 \right)$$

$$\mathcal{S}(\alpha_1^p) = \frac{1}{2} \left( \left( 1 - (\theta_i^p)^2 \right) - (\tau_s^p)^2 \right)$$

$$\mathcal{S}(C_{11}) = (0.7, 0.3) = \frac{1}{2} \left( (1 - (0.7)^2) - (0.3)^2 \right) = 0.21$$

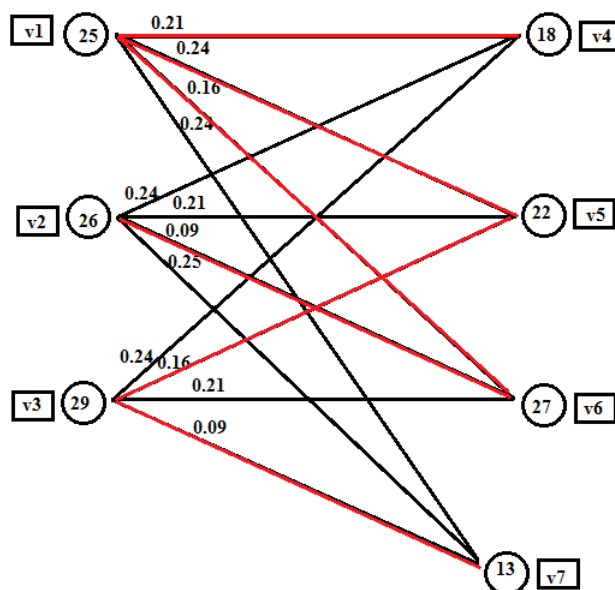
$$\mathcal{S}(C_{12}) = (0.40, 0.60) = \frac{1}{2} \left( (1 - (0.40)^2) - (0.60)^2 \right) = 0.24$$

$$\text{Similarly, } \mathcal{S}(C_{34}) = (0.90, 0.10) = \frac{1}{2} \left( (1 - (0.90)^2) - (0.10)^2 \right) = 0.09$$

Step 3 Convert the transportation table into bipartite graph and Step 4 We select the highest each bipartite edges value divided into total number of edges  $V = \{V_1, V_2, V_3, V_4, V_5, V_6, V_7\}$  SUPPLY =  $\{V_1, V_2, V_3\}$  DEMAND =  $\{V_4, V_5, V_6, V_7\}$

**Table 3.** Pythagorean fuzzy transportation problem that has been Defuzzified

	$O_1$	$O_2$	$O_3$	$O_4$	Supply
$S_1$	0.21	0.24	0.16	0.24	25
$S_2$	0.24	0.21	0.09	0.25	26
$S_3$	0.24	0.16	0.21	0.09	29
Demand	18	22	27	13	


**Fig 1.**

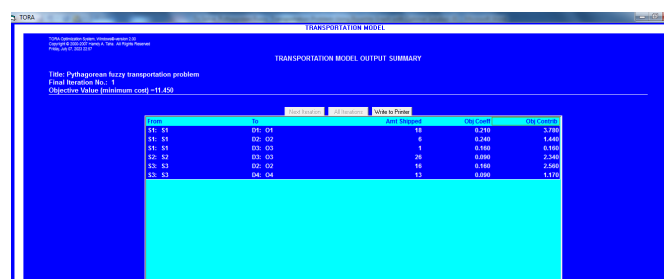
In the graph, the requirements for the rim have been satisfied.

The previous table's  $(m+n-1)$  the rim requirement is satisfied by non-negative allocations at distinct sites. The allocation that is presented below is thus the best one:

$$X_{11} = 18, X_{12} = 6, X_{13} = 1, X_{23} = 26, X_{32} = 16, X_{34} = 13$$

The proposed strategy results in a transportation cost of Total Cost =  $(0.21 \times 18 + 0.24 \times 6 + 0.16 \times 1 + 0.09 \times 26 + 0.16 \times 16 + 0.09 \times 13 = 11.45$

The above mentioned outcome was verified by TORA.


**Fig 2.** Simulation result using TORA Software

*NorthWestCornerMethod(NWC)*, *LeastCostMethod(LCM)*, *HeuristicMethod(HM)*, *GeometricMethod*<sup>(7)</sup>(*GM*), and to demonstrate the effectiveness of the recommended solution, the identical problem is solved using both the Modi technique

and other approaches.

The proposed technique is compared to the other methods. A better optimal answer is provided by the proposed method, obviously compared to other approaches already in use, this approach is more effective.

## 4 Conclusion

We examine the Pythagorean fuzzy transportation problem in this work. In the present research, we provide a strategy for resolving a Pythagorean fuzzy transportation issue. The Pythagorean fuzzy transportation problem-solving may be transformed into a crisp transportation problem by using the scoring approach. The suggested algorithm offers the best optimum result when compared to other methods that are currently in use. The suggested approach can be used to solve shortest path issues, future assignments, work schedules, and other issues.

## Relevance

The transportation problem (TP) is now the most popular domain for optimisation. Pythagorean Fuzzy Decagonal Transportation Problem Solved Using Bipartite Graph with Optimal Solution. Even if precise data have been utilised in the majority of cases, the values are genuinely imprecise and unclear. A significant issue with all decision-making processes is imprecision. Many techniques and instruments have been developed to handle the ambiguous environment of collective decision-making. The Pythagorean fuzzy set is a fuzzy set extension that effectively handles fuzziness and ambiguity.

## Declaration

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