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RESEARCH ARTICLE

On the Construction of the Properties of t - Norms in the Q-Fuzzy T-Sub algebra and Ideals in the BP-Algebra

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Abstract

In the present piece, we propound t- norms of the QFTSA $\grave{\omega}(*,q) \geq (min\ (\grave{\omega}(,q),\ \grave{\omega}(,q)\}\}$ in BP-Algebra and also t- norms of the QFTI $\grave{\omega}(0,q) \geq \grave{\omega}(,q)\ and \grave{\omega}(*,q) \geq\ (min(\ min(\grave{\omega}((*,q)*), \grave{\omega}(,q)\}, \grave{\omega}(,q)\}\}, \forall\ ,, \in\ and\ q \in Q$ of the BP-Algebra and we assay some of their features. Likewise, we define Cartesian products of QFTSAs and QFTI of BP algebras. These, as well as other algebraic features, are bandied in depth. **Objective:** This article generally aimed at investigating t- norms concepts. It can also be applied to the algebraic properties of Q-Fuzzy T-Ideals, Q-Fuzzy T-Sub algebra and also defined for Cartesian products of QFTSAs and QFTI of BP algebras.**Methods:** t -norms concepts can apply for the algebraic properties of Q-Fuzzy T-Ideals, and T-Sub algebra related to the theorems, lemma, and examples we find the proof.**Findings:** In the present study, we propound 𝑡- norms of the QFTSA $\grave{\omega}(*,q) \geq (min\ (\grave{\omega}(,q),\ \grave{\omega}(,q)\}\}$ in BP-Algebra and also 𝑡- norms of the QFTI $\grave{\omega}(0,q) \geq \grave{\omega}(,q)\ and \grave{\omega}(*,q) \geq\ (min(\ min(\grave{\omega}((*,q)*), \grave{\omega}(,q)\}, \grave{\omega}(,q)\}\}, \forall\ ,, \in\ and\ q \in Q$ of the BP-Algebra and we assay some of their features. Likewise, we define Cartesian products of QFTSAs and QFTI of BP algebras. These, as well as other algebraic features, are bandied in depth. **Novelty:** This study has defined new concepts of t-norms that apply to Q-fuzzy T-Ideals and Q fuzzy T-sub-algebra in BP algebras.

Keywords: Fuzzy Set (FS); Q-Fuzzy Sets (QFS); Q-Fuzzy Subset (QFSb); Q-Fuzzy T-Ideal (QFTI); Q-Fuzzy T-Subalgebra (QFTSA)

1 Introduction

Jefferson [1] and [2] created L-FBPA in 2016 and designed the FTI notation in BP-Algebras the same year. Gulzar [3] provided a new notation for certain characterization of Q-complex fuzzy sub-rings in 2022. In 2020, Indhira created FPI in INK-Algebra[4]. In 1991, M. M. Gupta and Qi. J[5] looked into the theory of T-norms and fuzzy inference. Mohamed Ismail[6] described by the Product of Doubt $\psi - \bar{Q}-$ Fuzzy Subgroup in 2022. Osama[7] proposed the Bipolar FαI of BP-algebra in 2020. Prem[8] &[9] offered the novel concept of On Algebraic Characteristics of FTSA in T-Algebra under Normalization in 2023 and also On Characteristics of $\kappa - Q-$ FT and FM in T-Ideals in T-Algebra in 2022. In 2022 Girija Bai[10], A Doubt $K - Q$ −Bipolar Fuzzy BCI-Ideals and Doubt $K - Q$ −Bipolar Fuzzy BCI-Implicative Ideals in BCI-Algebra. Kumar, M. P[11] discovered algebraic properties for FT and FM in BP-algebras in 2020 invented the notation for fuzzy congruence on product lattices in 2021, Fd-A under T-norms were developed in 2022, following R. Rasuli[12,13] and t-norms over FI of BCK-algebras in 2022[14]. Solai[15] discussed the concept of A New Structure and Construction of Q-FGs. Zadeh established the code for FSs in 1965[16].

This work introduces the notation for the Characteristics of t-Norms under the QFTSA and QFTI in BP-Algebra. It likewise constructs the cartesian end product of the QFTSA and QFTI in BP-Algebra and analyses a number of properties.

2 Methodology

Description: 2.1[9]

Let, X be a non-empty set . A FSb of the set, Ĝ is $\ddot{\mathbf{V}}X \rightarrow [0, 1]$.

Description: 2.2[3]

A FS, $\tilde{V}$ in a BP-algebra, Ĝ is named a FTI of, Ĝ

(i) $\tilde{V}(0) \geq \tilde{V}(ƚ)$

(ii) $\tilde{V}(ƚ * ɉ) \geq min\{\tilde{V}((ƚ * ɳ) * ɉ), \tilde{V}(ɳ)\}, \forall ƚ, ɳ, ɉ \in Ĝ.$

Result: 2.3

A ŧ-‖‖, Ŧ is Ŧ $from$ [0,1] × [0,1] → [0,1]

(i) Ŧ(ƚ, 1) = ƚ

(ii) Ŧ(ƚ, ɳ) ≤ Ŧ(ƚ, ɉ)

(iii) Ŧ(ƚ, ɳ) = Ŧ(ɳ, ƚ)

(iv) Ŧ(ƚ, Ŧ(ɳ, ɉ)) = Ŧ(Ŧ(ƚ, ɳ), ɉ), ∀ƚ, ɳ, ɉ ∈ [0,1].

Description: 2.3[8]

Let Q̣ & Ğ be a set & Ğ correspondingly. A Ậ from Ğ × Q̣ → [0,1] is Q̣ - FS in Ğ. For any, Q̣ -FS Ậ in Ğ and t ∈ [0,1], we defined the set $U(Ậ; t) = \{x \in G \ / \mu(x, q) \geq t, q \in Q̣\}$.

3 Results and Discussion

On the Construction of the Properties of t -Norms in the Q-FTSA and Q-FTIs in the BP-Algebra

Definition: 3.1

Let, $\tilde{\tilde{\omega}}$ be the $QFSb$ of the T-Algebra Ĝ. Then, $\tilde{\tilde{\omega}}$ is christened $QFTSA$ of, Ĝ concealed by ŧ −norm, Ŧ($QFTSA$ Ŧ(Ĝ)) iff $\tilde{\tilde{\omega}}(ƚ * ɳ, q) \geq$ Ŧ$\{min\{\tilde{\tilde{\omega}}(ƚ, q), \tilde{\tilde{\omega}}(ɳ, q)\}\}$, ∀ ƚ, ɳ ∈ Ĝ and q ∈ Q.

Example: 3.1.1

Let, $Ĝ = \{0, 1, 2, 3\}$ be a set given by

Then, $(\hat{G}, *, 0)$ is a T-Algebra.

Define QFSb $\tilde{\tilde{\omega}} : (Ĝ, *, 0) \rightarrow [0, 1]$ as

$$\tilde{\tilde{\omega}}(ƚ, q) = \begin{cases} 0.45, & if\ ƚ = 0 \\ 0.50, & if\ ƚ \neq 0 \end{cases}$$

Ŧ$((a, b), q) =$ Ŧ$_p((a, b), q) = ab, \forall\ a, b \in [0,1] and\ q \in Q$ then $\tilde{\tilde{\omega}}$ is$QFTSA$ Ŧ(Ĝ).

*	0	1	2	3
0	0	1	2	3
1	1	0	1	1
2	2	2	0	2
3	3	3	3	0

Definition: 3.2

Let, ω̃ be the *QFS* of a T-Algebra, Ĝ and call ω̃ ∈ [0,1] *and* $q \in Q$ a *QFTI* of, Ĝ concealed bythe ƫ −norm, Ŧ(*QFTI* Ŧ(Ĝ)).

(i) ω̃(0, q) ≥ ω̃(ɭ, q)

(ii) ω̃(ɭ ∗ ɉ, q) ≥ Ŧ{*min*{ω̃((ɭ ∗ ɳ, q) ∗ ɉ), ω̃(ɳ, q)}}, ∀ ɭ, ɳ, ɉ ∈ Ĝ *and* $q \in Q$

Definition: 3.3

Let, ω̃ be a *QFSb* in T-Algebra of, Ĝ and, ω̃ as *QFTI* Ŧ(Ĝ)then

(i) ω̃, is *QFTSA* Ŧ(Ĝ).

(ii) ω̃(0, q) ≥ ω̃(ɭ, q) *and* ω̃(ɭ ∗ ɉ, q) ≥ Ŧ{*min*{Ŧ*min*{ω̃((ɭ ∗ ɳ, q) ∗ ɉ), ω̃(ɳ, q)}, ω̃(ɳ, q)}}, ∀ ɭ, ɳ, ɉ ∈ Ĝ *and* $q \in Q$

Example: 3.3.1

Let, Ğ = (0, 1, 2, 3} be a set given by

∗	0	1	2	3
0	0	0	0	0
1	1	0	0	1
2	2	2	0	0
3	3	3	3	0

(Ĝ, ∗, 0) is a T-Algebra.

*QFS*ω̃ : (Ĝ, ∗, 0) → [0, 1] as

$$\tilde{\omega}(ɭ, q) = \begin{cases} 0.85, & if\ ɭ = 0 \\ 0.25, & if\ ɭ \neq 0 \end{cases}$$

Ŧ((a, b)) = ab = $T_p(a, b)$ ∀ a, b ∈ [0,1] *and* $q \in Q$, then ω̃, is *QFTI* Ŧ(Ĝ).

Definition: 3.4

Let, ω̃ ∈ [0,1], ß∈ [0,1] *and* $q \in Q$. The Cartesian product of, ω̃ and ß is represented by, ω̃ × ß: Ĝ × Ĩ → [0,1]and is represented by

(ω̃ × ß)((ɭ, ɳ), q) = Ŧ{*min*{ω̃(ɭ, q), ß(ɳ, q)}}, ∀ ɭ, ɳ ∈ Ĝ × Ĩ and $q \in Q$.

Proposition: 3.5

Let, ω̃ ∈ [0,1], $q \in Q$ and, Ŧ be idempotent. Then, ω̃ is *QFTSA* Ŧ(Ĝ) iff the highest tier, ω̃ƫ is a SA of, Ĝ, ∀ Ŧ ∈ [0,1] *and* $q \in Q$.

Proof:

Let, $\tilde{\omega}$ is $QFTSA$ Ŧ(Ĝ), ∀ ḷ, ṇ ∈ $\tilde{\omega}_{ṭ}$ and $q \in Q$.

Then

$\tilde{\omega}((ḷ, ṇ), q) \geq$ Ŧ $\{min\{\tilde{\omega}(ḷ, q), \tilde{\omega}(ṇ, q)\}\} \geq$ Ŧ$((ṭ, ṭ), q) =$ ṭ

Thus, ḷ ∗ ṇ ∈ $\tilde{\omega}_{ṭ}$, $q \in Q$ and so, $\tilde{\omega}_{ṭ}$ ill be a QSA of, Ĝ , ∀ Ŧ ∈ [0,1] and $q \in Q$.

Converse

Let, $\tilde{\omega}_{ṭ}$ is a QSA of Ĝ, ∀ ṭ ∈ [0,1]and $q \in Q$

Let, ṭ = Ŧ $\{min\{\tilde{\omega}(ḷ, q), \tilde{\omega}(ṇ, q)\}\}$ and ḷ, ṇ ∈ $\tilde{\omega}_{ṭ}$ and $q \in Q$.

As $\tilde{\omega}_{ṭ}$ is a QSA of Ĝ, so ḷ ∗ ṇ ∈ $\tilde{\omega}_{ṭ}$,$q \in Q$and thus, $\tilde{\omega}((ḷ, ṇ), q) \leq$ ṭ =

Ŧ $\{min\{\tilde{\omega}(ḷ, q), \tilde{\omega}(ṇ, q)\}\}$

Then, $\tilde{\omega}$ is $QFTSA$ Ŧ(Ĝ).

Proposition: 3.6

Let, $\tilde{\omega}$ be a algebra of a T-Algebra Ĝ, and Ṡ ∈ [0,1]and $q \in Q$ such that

Ṡ(ḷ) = $\begin{cases} ṭ \text{ , } if \text{ ḷ} \in \text{Ṡ} \\ 0 \text{ , } if \text{ ḷ} \neq \text{Ṡ} \end{cases}$ since, ṭ ∈ [0,1]and $q \in Q$. If, Ŧ be idem then, Ṡ is $QFTSA$ Ŧ(Ĝ).

Proof:

W.K.T, $\tilde{\omega}$ = $Ṡ_{ṭ}$.

Let, ḷ, ṇ ∈ Ĝ and $q \in Q$

(i) Let, ḷ, ṇ ∈ $\tilde{\omega}$ and $q \in Q$ then, ḷ ∗ ṇ ∈ Ṡ and so

$\tilde{\omega}$(ḷ ∗ ṇ, q) = ṭ ≥ ṭ = Ŧ((ṭ, ṭ), q) = Ŧ $\{min\{\tilde{\omega}(ḷ, q), \tilde{\omega}(ṇ, q)\}\}$

(ii) Let, ḷ ∈ Ṡ , ṇ ∉ Ṡ and $q \in Q$ then, $\tilde{\omega}$(ḷ, q) = ṭ and, $\tilde{\omega}$(ṇ, q) = 0 and so

$\tilde{\omega}$(ḷ ∗ ṇ, q) ≥ 0 = Ŧ((ṭ, 0), q) = Ŧ $\{min\{\tilde{\omega}(ḷ, q), \tilde{\omega}(ṇ, q)\}\}$

(iii) If, ḷ ∉ Ṡ,ṇ ∈ Ṡ and $q \in Q$ then, $\tilde{\omega}$(ḷ, q) = 0 and $\tilde{\omega}$(ṇ, q) = ṭ and so, $\tilde{\omega}$(ḷ ∗ ṇ, q) ≥ 0 = Ŧ((0, ṭ), q) = Ŧ $\{min\{\tilde{\omega}(ḷ, q), \tilde{\omega}(ṇ, q)\}\}$

(iv) Let, ḷ ∉ Ṡ,ṇ ∉ Ṡand $q \in Q$then, $\tilde{\omega}$(ḷ, q) = 0 and $\tilde{\omega}$(ṇ, q) = 0 and so, $\tilde{\omega}$(ḷ ∗ ṇ, q) ≥ 0 = Ŧ((0,0), q) = Ŧ $\{min\{\tilde{\omega}(ḷ, q), \tilde{\omega}(ṇ, q)\}\}$

Thus (i) and (ii) we get, $\tilde{\omega}$ is $QFTSA$ Ŧ(Ĝ).

Proposition: 3.7

Let, $\breve{\varepsilon}$ be a $QFTSA$ Ŧ(Ĝ)and $\tilde{\eta}$ be a $QFTSA$ Ŧ(Ī). Then, $\breve{\varepsilon} \times \tilde{\eta}$ in Q-fuzzy T sub algebra Ĝ under ṭ −norm, Ŧ($QFTSA$ Ŧ(Ĝ × Ī)).

Proof:

Let, (ḷ$_1$, ṇ$_1$), (ḷ$_2$, ṇ$_2$) ∈ Ĝ × Ī and $q \in Q$.

Then, $(\breve{\varepsilon} \times \tilde{\eta})$(((ḷ$_1$, ṇ$_1$) ∗ (ḷ$_2$, ṇ$_2$)), q) = $\breve{\varepsilon} \times \tilde{\eta}$((ḷ$_1$, ṇ$_1$, ḷ$_2$, ṇ$_2$), q)

= Ŧ $\{min\{$ȷ((ḷ$_1$ ∗ ḷ$_2$), q), $\tilde{\eta}$((ṇ$_1$ ∗ ṇ$_2$), q)$\}\}$

≥ Ŧ $\{$ Ŧ$\{min(\breve{\varepsilon}$(ḷ$_1$, q), $\breve{\varepsilon}$(ḷ$_2$, q))$\}$, Ŧ$\{min(\tilde{\eta}$(ṇ$_1$, q), $\tilde{\eta}$(ṇ$_2$, q))$\}\}$

= Ŧ $\{$ Ŧ $\{min$ $(\breve{\varepsilon}$(ḷ$_1$, q), $\tilde{\eta}$(ṇ$_1$, q))$\}$,

Ŧ$\{min(\breve{\varepsilon}$(ḷ$_2$, q), $\tilde{\eta}$(ṇ$_2$, q))$\}\}$

= Ŧ $\{min\{(\breve{\varepsilon} \times \tilde{\eta})$((ḷ$_1$, ṇ$_1$), q), $(\breve{\varepsilon} \times \tilde{\eta})$((ḷ$_2$, ṇ$_2$), q)$\}\}$

⇒$(\breve{\varepsilon} \times \tilde{\eta})$(((ḷ$_1$, ṇ$_1$) ∗ (ḷ$_2$, ṇ$_2$)), q) ≥ Ŧ $\{min\{(\breve{\varepsilon} \times \tilde{\eta})$((ḷ$_1$, ṇ$_1$), q), $(\breve{\varepsilon} \times \tilde{\eta})$((ḷ$_2$, ṇ$_2$), q)$\}\}$

and so, $\breve{\varepsilon} \times \tilde{\eta}$ in $QFTSA$ Ŧ(Ĝ × Ī).

Proposition: 3.8

Let, $\breve{\varepsilon}$ be a $QFTI$ Ŧ(Ĝ) and $\tilde{\eta}$ be $QFTI$ Ŧ(Ī). Then, $(\breve{\varepsilon} \times \tilde{\eta})$ be a $QFTI$, Ĝ under ṭ −norm, Ŧ($QFTI$ Ŧ(Ĝ × Ī)).

Proof:

(i) Let, $(ɭ, ɳ) \in \hat{G} \times \tilde{I}$ and $q \in Q$

Then, $(\breve{\varepsilon} \times \tilde{\eta})((0,0), q) = Ŧ\left\{min\{\breve{\varepsilon}(0,q), \tilde{\eta}(0,q)\}\right\}$

$$\geq Ŧ\left\{min\{\breve{\varepsilon}(ɭ,q), \tilde{\eta}(ɭ,q)\}\right\}.$$

(ii) $(\breve{\varepsilon} \times \tilde{\eta})\left(((ɭ_1, ȷ_1) * (ɭ_1, ȷ_1)), q\right) = (\breve{\varepsilon} \times \tilde{\eta})((ɭ_1 * ɭ_2, ȷ_1 * ȷ_2), q)$

$$\geq Ŧ\left\{min\left(\left\{\breve{\varepsilon}\left(((ɭ_1 * ɳ_1) * ȷ_1), q\right), \tilde{\eta}\left(((ɭ_2 * ɳ_2) * ȷ_2), q\right)\right\}\right), (\{\breve{\varepsilon}(ɳ_1, q), \tilde{\eta}(ɳ_2, q)\})\right\}$$

$$= Ŧ\left\{min\left(\left\{(\breve{\varepsilon} \times \tilde{\eta})\left(((ɭ_1 * ɳ_1) * ȷ_1, (ɭ_2 * ɳ_2) * ȷ_2), q\right)\right\}, (\breve{\varepsilon} \times \tilde{\eta})((ɳ_1, ɳ_2), q)\right)\right\}$$

$(\breve{\varepsilon} \times \tilde{\eta})\left(((ɭ_1, ȷ_1) * (ɭ_1, ȷ_1)), q\right)$

$$\geq Ŧ\left\{min\left(\left\{(\breve{\varepsilon} \times \tilde{\eta})\left(((ɭ_1 * ɳ_1) * ȷ_1, (ɭ_2 * ɳ_2) * ȷ_2), q\right)\right\}, (\breve{\varepsilon} \times \tilde{\eta})((ɳ_1, ɳ_2), q)\right)\right\}$$

Therefore (i) and (ii), $(\breve{\varepsilon} \times \tilde{\eta})$ be a $QFTI$ $Ŧ(\hat{G} \times \tilde{I})$.

Proposition: 3.9

Let, $\breve{\varepsilon} \in [0,1]^{\hat{G}}$, $\tilde{\eta} \in [0,1]^{\tilde{I}}$ and $q \in Q$. If, $(\breve{\varepsilon} \times \tilde{\eta})$ be a $QFTI$ $Ŧ(\hat{G} \times \tilde{I})$ thus preferably any one of the two following proposals must be valid. |

(i) $\breve{\varepsilon}(0,q) \geq \breve{\varepsilon}(ɭ,q)$ then either, $\tilde{\eta}(0,q) \geq \breve{\varepsilon}(ɭ,q)$ or $\tilde{\eta}(0,q) \geq \tilde{\eta}(ɳ,q)$, $\forall\, ɭ \in \hat{G}$, $ɳ \in \tilde{I}$ and $q \in Q$.

(ii) $\tilde{\eta}(0,q) \geq \tilde{\eta}(ɳ,q)$ then either, $\breve{\varepsilon}(0,q) \geq \tilde{\eta}(ɳ,q)$ or $\breve{\varepsilon}(0,q) \geq \breve{\varepsilon}(ɭ,q)$, $\forall\, ɭ \in \hat{G}$, $ɳ \in \tilde{I}$ and $q \in Q$.

Proof:

(i) Let neither of the proposals (i) & (ii) holds, then $(ɭ, ɳ) \in \hat{G} \times \tilde{I}$ and $q \in Q$ st, $\breve{\varepsilon}(0,q) < \breve{\varepsilon}(ɭ,q)$ and $\tilde{\eta}(0,q) < \tilde{\eta}(ɳ,q)$.

Thus

$$(\breve{\varepsilon} \times \tilde{\eta})((ɭ, ɳ), q) = Ŧ\left\{min\left(\breve{\varepsilon}(ɭ,q), \tilde{\eta}(ɳ,q)\right)\right\}$$
$$> Ŧ\left\{min\left(\breve{\varepsilon}(0,q), \tilde{\eta}(0,q)\right)\right\}$$
$$= (\breve{\varepsilon} \times \tilde{\eta})((0,0), q)$$

and its contradiction with, $(\breve{\varepsilon} \times \tilde{\eta})$ be a $QFTI$ $Ŧ(\hat{G} \times \tilde{I})$.

(ii) Let, $\tilde{\eta}(0,q) < \tilde{\eta}(ɳ,q)$ such that, $(ɭ, ɳ) \in \hat{G} \times \tilde{I}$ and $q \in Q$

We have, $\breve{\varepsilon}(0,q) < \tilde{\eta}(ɳ,q)$ & $\breve{\varepsilon}(0,q) < \tilde{\eta}(ɭ,q)$.

So, $\tilde{\eta}(0,q) \geq \tilde{\eta}(ɳ,q) > \breve{\varepsilon}(0,q)$ & $\breve{\varepsilon}(0,q) = Ŧ\left\{min\left(ɟ(ɭ,q), \tilde{\eta}(ɳ,q)\right)\right\}$

Thus

$$(\breve{\varepsilon} \times \tilde{\eta})((ɭ, ɳ), q) = Ŧ\left\{min\left(\breve{\varepsilon}(ɭ,q), \tilde{\eta}(ɳ,q)\right)\right\}$$
$$> Ŧ\left\{min\left(\breve{\varepsilon}(0,q), \tilde{\eta}(0,q)\right)\right\}$$
$$= ɟ(0,q)$$
$$= Ŧ\left\{min\left(\breve{\varepsilon}(0,q), \tilde{\eta}(0,q)\right)\right\}$$
$$= (\breve{\varepsilon} \times \tilde{\eta})((0,0), q)$$

and it is contradiction with, $(\breve{\varepsilon} \times \tilde{\eta})$ be a $QFTI$ $Ŧ(\hat{G} \times \tilde{I})$.

4 Conclusion

This study has achieved some of the properties of the t-norm of the Q-QFTSA and QFTI of the BP algebra. It extended on the features of Cartesian products of QFTSAs and QFTIs of BP algebras, establishing some key concepts. This approach can be extended to IFS, intervals weighted FSs, and BFs to yield novel findings in further studies.

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Declaration

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