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Impact of Learning on Ordering Policy for Deteriorating Imperfect Quality Items with Promotion Dependent Demand, Partial Trade-Credit and Shortages

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Abstract

Objective: The objective of the developed inventory model is to find the optimal solution for the retailer, while performing inspection for the defectives, and also to offer partial credit periods, where shortages are allowed. **Methods:** To derive the required solution which benefits the retailer, the partial derivatives method for the decision variables where used and to solve the complicated differential equations the necessary and sufficient conditions has been derived and the theorem and its proofs are given. **Findings:** In this research findings, the optimal credit period policy and promotional effort dependent demand pattern that offers more profit to the retailer, provided with inspection policy for finding defective items in the lot. Shortages are allowed and completely backordered. Numerical examples and sensitivity analysis are performed to verify the reliability of the developed model. **Novelty:** This study presents a new model wherein the retailer offers credit period and with shortages that will apt the real life situation as the model developed here is for deteriorating items, which is inevitable to maintain the perfect system.

Keywords: MultiEchelon Inventory Model; Sustainability; Shortages; Credit Period Offers; Learning In Inspection

1 Introduction

In a competitive market, the supplier or even retailer must sell his products as soon as possible; to do so, he must employ strategies that allow him to sell the product sooner or find a way to increase his profit [(1)]. Firms typically implement the discount strategy through price cuts, credit period offers, and product discounts [(2)]. Some of the firms, buys a lot of damaged products to inspect it [(3)], and found out the items which can be used or not. If the inspection process is performed by humans, there can be a lack of classification, there might be errors [(4)]. This can be cured by reducing the production of imperfect items[5–7], and increasing the rate of production. In some of the models, the purchasing cost is charged at the time of placing orders, but it affects the supplier-retailer relationship. Therefore, it is then transformed into delayed payments system, where the supplier offers some credit period[8] to the retailer to pay the purchase cost[9]. By giving this credit period offer, the supplier or retailer may classify their customers as good and bad in list for paying the purchase cost in time. From this classification, the old customers offered the full credit payment and new customers are allowed to pay the partial delayed payment.

From the known details, in the literature, the model for imperfect quality deteriorating items with two levels of inspection and learning with partial credit payment[10] classified the new customer as good and bad debt is rarely investigated[11]. The ideology and motivation of this paper[12,13] is to find out the maximum profit for the retailer and to reduce the imperfect quality items[14], and through the learning in inspection[15], reduce the misclassification of products is obtained[16]. In some models, the researchers analysed the results for the non-instantaneous deterioating items with inspection policy and other offers has been considered[17,18]. The main reason for adapting the two level trade credit policy is because of the real life situations, that happens for the payment delays.

The limitations of the developed model is where there would be a possibility of finding defective items after the learning in inspection from the retailer's side. So, that could be the possible drawback of the model. Similarly, the credit period offers may create some goodwill loss to the retailer among the customer's side. The promotional dependent demand may cause some loss to the retailer, if the promotion is not utilised properly.

With all this previous literature works, we have analyzed and developed a model for imperfect quality deteriorating items, with promotional dependent demand, learning in in section and offering the partial credit policy offers for old and new customers is not developed anywhere and here we present our model through the following section, in section 1, we have discussed the possibilities of the inventory management strategies over years and current idea and motivation explained, in section 2, in section 3, problem description is given, in section 4, the notations and assumptions of the developed mathematical model is given, in section 5 the inventory level of the model in various time period is given and the profit function is derived in this section. In section 7 and 6, the numerical examples to prove the reliability of the developed model and sensitivity analysis in section 9 to verify the result obtained in numerical example is given. Finally in section 8 , 10 result analysis, conclusion and future research directions are given.

2 Methodology

In this article, the three echelon model for imperfect quality deteriorating items, with shortages are developed, as in previous literature they have worked for imperfect production inventory model[10] with no shortages and for model with non-deteriorating items. The supplier here offers the full credit period offer to the retailer. The retailer classifies his customers into two types as new and old[14], and the full credit period offer is given to the old customer and partial credit period offered to the new ones and they are classified as good debt and bad debt by the paying behavior, the good customers are the ones who pays the amount during the credit period or before it ends. The other ones are the customers who pays always after the end of the credit period which makes the retailer to loss certain amount of money in his profit. The retailer also performs the inspection process to classify the imperfect quality products from the lot. There is a chance of error in inspection as the good product can be omitted as the imperfect quality product and similarly, the imperfect ones as the good product. After learning, the retailer makes the inspector the correct inspection process at all. Shortages are allowed and completely backlogged.

2.1 Notations and Assumptions

Parameters	Description
A	Ordering cost per unit item
θ	Deteriorating rate of the product
C_p	Purchasing cost per unit

Parameters	Description
Q	Lot size
I_c	Screening cost for the retailer
δ	Level of imperfect quality items
y_0	Retailer's rate of inspection before learning
f	Frequency of promotions
Δ	Learning in inspection and increased inspection rate
y	Number of units inspected
p_1	Probability of type 1 error by inspector
p_2	Probability of type 2 error by inspector
s, s_{ip}	Selling price of perfect & imperfect item
C_{rp}	Cost for rejecting perfect item
C_{ai}	Cost for accepting imperfect item
t_s	Total inspection time
C_h	Holding cost for the retailer
M	Payment delay allowed (supplier to retailer)
R	Payment delay allowed (retailer to customer)
T	Cycle length in total
I_p, I_e	Interest paid and earned per year
A_c	Cost of Promotion
a_1	Rate of old customers
β_p	New customers who pay immediately
β_w	Customers who pay during the credit period
z_1, z_2	Learning percentage in type 1 and 2 errors
C_s	Shortage cost per unit
ρ	Backordering quantity

2.2 Assumptions

1. Single item inventory model for supplier-retailer-customer is developed

2. The items deteriorates at a constant rate. The retailer performs an inspection to separate the imperfect quality products from the lot to avoid any inconvenience. There are possible chances for any misclassifcation of products while inspection.

3. To avoid the misclassification, learning from the previous inspection process, in this model inspection rate is to be presented as, $y = y_0 F(T)$, where $F(T) = \left(1 + \frac{\Delta \beta f^{\pi}}{T}\right)$. For calculation, π- promotional elasticity, is taken as 3 here, β, scaling factor.

4. Supplier offers the retailer a full credit period of M. Here, the new customers receives partial trade credit and the old ones get the full credit period offer.

5. The retailer classifies the clients as good and bad ones who pays the amount while buying as a single payment and the customer who doesn't paid well at the correct time he offered.

6. Shortages are allowed and completely backordered

7. Demand is dependent on the promotional frequency, $D = \alpha + \beta f^4$, where α, scale demand in the market, β, promotional elasticity.

2.3 Inspection and Learning Process

In the inspection, there might be a possibility of errors, that can happen in two ways, there are

1. Classifying the perfect product as imperfect
2. Classifying the imperfect product as perfect

The classification can be represented in mathematical form as,

$$F_1 = \left(p_1 - \tfrac{1}{T} z_1 p_1\right); \quad F_2 = \left(p_2 - \tfrac{1}{T} z_2 p_2\right), \ where\ F_1,\ F_2 \geq 0$$

The demand is the sum of perfect items and mistakenly classified as perfect items, by learning process, the lot size can be derived by,

$Q(1-\delta) \geq (\alpha+\beta f^4)T + Q(1-\delta)\left(p_1 - \frac{1}{T}z_1 p_1\right)$

$$Q = \frac{(\alpha+\beta f^4)T}{(1-\delta)\left(1-p_1+\frac{1}{T}z_1 p_1\right)} \tag{1}$$

2.4 Model Formulation

Let $I_1(t)$ be the inventory level at time $(0 \leq t \leq t_s)$ the equation is

$\frac{dI_1(t)}{dt} + \theta I_1(t) = -D \quad 0 \leq t \leq t_s$

with boundary condition $I_1(t)$, when $t=0$,is Q, the resulting differential equation is,

$I_1(t) = \frac{D}{\theta}\left(e^{-\theta t}-1\right) + Qe^{-\theta t}$

Before learning, the number of defective items found out in the screening process at time $t_s = Q/y$ is δQ. After learning, the inventory level at the time period t_s and the back order is

$$I(t_s) = Qe^{-\theta t_s} + \frac{D}{\theta}\left(e^{-\theta t_s}-1\right) - (1-\delta)Q\left(p_1 - \frac{1}{T}z_1 p_1\right) - Q\delta\left(1-\left(p_2-\frac{1}{T}z_2 p_2\right)\right) - \rho$$

The inventory level at $t = t_s$, after removing the imperfect items and backorder is at $t_s \leq t \leq t_1$,

$\frac{dI_2(t)}{dt} + \theta I_2(t) = -D \quad t_s \leq t \leq t_1$

By applying b.c's, $I_2(t_s) = I_1(t_s)$,

$$I_2(t) = \frac{D}{\theta}\left(e^{-\theta t}-1\right) + Qe^{-\theta t} - Qe^{-\theta(t_s-t)}\left((1-\delta)\left(p_1-\frac{z_1p_1}{T}\right)+\delta\left(1-\left(p_2-\frac{z_2p_2}{T}\right)\right)\right] - \rho e^{-\theta(t_s-t)}$$

Then, for $t = t_1$, $I_2(t_1) = 0$,

$$I_2(t_1) = \frac{D}{\theta}\left(e^{-\theta t_1}-1\right) + Qe^{-\theta t_1} - Qe^{\theta(t_s-t_1)}\left((1-\delta)\left(p_1-\frac{z_1p_1}{T}\right)+\delta\left(1-\left(p_2-\frac{z_2p_2}{T}\right)\right)\right] - \rho e^{-\theta(t_s-t_1)}$$

$$\therefore t_1 = \frac{1}{\theta}\left[\log\left(\frac{D}{\theta}+Q-Qe^{\theta t_s}\left[(1-\delta)\left(p_1-\frac{z_1p_1}{T}\right)+\delta\left(1-\left(p_2-\frac{z_2p_2}{T}\right)\right)\right]-\rho e^{\theta t_s}\right)-\log\left(\frac{D}{\theta}\right)\right]$$
$$\log\left(\frac{D}{\theta}+Q-Qe^{\theta t_s}\left((1-\delta)\left(p_1-\frac{z_1p_1}{T}\right)+\delta\left(1-\left(p_2-\frac{z_2p_2}{T}\right)\right)\right]-\rho e^{\theta t_s}\right)$$
$$-\log\left(\frac{D}{\theta}\right) \tag{2}$$

where $T = t_1 + t_2$, $t_2 = \frac{\rho}{D}$.

2.5 Cost Components

Profit can be calculated by finding the difference between sales revenue and other costs. The total cost components are given below,

1. Sales Revenue: In this model, sales revenue, is not just a single part, we have to calculate the revenue by calculating the income through perfect goods, presumed income through misclassified item, retailer's lost income due to bad debt, and finally the revenue obtained through selling of scrap items are given below respectively.

$R_a = \frac{1}{T}\left[s(1-\delta)\left(1-\left(p_1-\frac{z_1p_1}{T}\right)\right)Q + s\delta Q\left(p_2-\frac{z_2p_2}{T}\right)\right]$

$R_b = -\frac{s\delta}{T}\left(p_2-\frac{z_2p_2}{T}\right)Q$

$R_c = \frac{1}{T}\left[-s(1-\beta_p)(1-\delta)\left(1-\left(p_1-\frac{z_1p_1}{T}\right)\right)(1-\beta_w)(1-a_1)Q\right]$

$R_d = \frac{1}{T}\left[s_{ip}Q\left[\delta\left(1-\left(p_2-\frac{z_2p_2}{T}\right)\right)+(1-\delta)\left(p_1-\frac{z_1p_1}{T}\right)+\delta\left(p_2-\frac{z_2p_2}{T}\right)\right]\right]$

$SR = R_a + R_b + R_c + R_d$

2. Ordering cost: $OC = \frac{A}{T}$

3. Purchasing cost: $PC = \frac{C_p Q}{T}$
4. Screening cost: $SCR = \frac{I_c Q}{T}$
5. Inspection errors cost: $I\,EC = \frac{1}{T}\left[C_{rp}Q(1-\delta)\left(p_1 - \frac{z_1 p_1}{T}\right) + C_{ai}\delta Q\left(p_2 - \frac{z_2 p_2}{T}\right)\right]$
6. Promotion investment cost: $PI = \frac{A_c f^4}{T}$
7. Shortage cost: $SC = \frac{C_S \rho (t_2 + 2t_S)}{T}$
8. Holding cost and Deterioration cost: The holding and deterioration cost for the retailer's inventory cycle during, $[0, T]$ is given here,

$$HC + DC =$$
$$\frac{C_h + C_p\theta}{T}\left[\begin{array}{l} \frac{Q}{\theta}\left(1 - e^{-\theta t_s}\right) - \frac{D}{\theta^2}\left[\theta t_s + e^{-\theta t_s} - 1\right] + \frac{Q}{\theta}\left(e^{-\theta t_s} - e^{-\theta t_1}\right) + \frac{D}{\theta^2}\left(e^{-\theta t_s} - e^{-\theta t_1}\right) \\ + \frac{D}{\theta}(t_s - t_1) + \frac{Q}{\theta}\left[(1-\delta)\left(p_1 - \frac{z_1 p_1}{T}\right) + \delta\left(1 - \left(p_2 - \frac{z_2 p_2}{T}\right)\right)\right]\left(e^{\theta(t_s - t_1)} - 1\right) \\ + \frac{\beta}{\theta}\left(e^{\theta(t_s - t_1)} - 1\right) \end{array}\right] \quad (3)$$

2.6 Delay in Payments

According to credit period offers M and R, there are two possible sub cases,

Case 1: $R \le M$, Case 2: $M \le R$.

Case (1) $(R \le M$

Here are some other possibilities regarding the length of the inventory cycle T, they are

Subcase 1.1: $(M \le t_s \le T)$

Here, WKT, $M \le t_s$, some of the items remains with the retailer. Since, the retailer has to pay the interest for the time distance between M and t_s. Since the interest have to be paid is,

$IP_{t_s} = \frac{C_p I_p}{T}(t_s - M)Q(1-\delta)\left(p_1 - \frac{z_1 p_1}{T}\right) + \delta\left(1 - \left(p_2 - \frac{z_2 p_2}{T}\right)\right)$

From the diagram, the retailer can get some revenue after R, from both customers. The revenue can be calculated by multiplying the total area of $\triangle' s$. As per the model, the new one's has to pay the interest as an instantaneous cut during $(0, M)$. Interest earned from the old customers is followed as,

$I_{e_{a_{11}}} = \frac{s I_e D a_1 (M-R)^2}{2T}$

The retailer has to pay the interest between M to $T + R$ for the payment delays of old customers a_1. Therefore,

$I_{p_{a_{11}}} = \frac{C_p I_p D a_1 (T+R-M)^2}{2T}$

Retailer's interest earned during $[0, M]$ from new customers $(1 - a_1)$ due to instant payment is

$I_{e_{a_{12}}} = \frac{s I_e D \beta_p (1-a_1) M^2}{2T}$

Retailer has to pay the interest amount between M and cycle length T for the immediate payment from the latest customers $(1 - a_1)$, it is

$I_{p_{a_{12}}} = \frac{C_p I_p D \beta_p (1-a_1)(T-M)^2}{2T}$

Retailer also earned some interest between $R\ to\ S$, from new one's, $(1 - a_1)$, whom are classified as good customers, in case, they delay in paying, it is,

$I_{e_{a_{13}}} = \frac{s I_e D (1-a_1)(1-\beta_p)(M-R)^2}{2T}$

Interest to be paid by the retailer between M to $T + R$, for delayed payment of recent customers $(1 - a_1)$. They are,

$I_{p_{a_{13}}} = \frac{C_p I_p D (1-a_1)(1-\beta_p)(T+R-M)^2}{2T}$

Retailer's profit function

$$Retailer^{'}s\ Profit\ TP_1 = SR - OC - PC - SCR - IEC - PI$$

$-SC - DC - HC - Interest\ paid + Interest\ Earned$

For Subcase (1.1), $(M \le t_s \le T)$

TP1

$$\left[\begin{array}{l} \frac{1}{T}\left[s(1-\delta)\left(1-\left(p_1-\frac{z_1p_1}{T}\right)\right)Q+s\delta Q\left(p_2-\frac{z_2p_2}{T}\right)\right]-\frac{s\delta}{T}\left(p_2-\frac{z_2p_2}{T}\right)Q \\ \frac{1}{T}\left[-s\left(1-\beta_p\right)(1-\delta)\left(1-\left(p_1-\frac{z_1p_1}{T}\right)\right)\left(1-\beta_w\right)\left(1-a_1\right)Q\right] \\ +\frac{1}{T}\left[s_{ip}Q\left[\delta\left(1-\left(p_2-\frac{z_2p_2}{T}\right)\right)+(1-\delta)\left(p_1-\frac{z_1p_1}{T}\right)+\delta\left(p_2-\frac{z_2p_2}{T}\right)\right]\right] \\ -\frac{A}{T}-\frac{C_pQ}{T}-\frac{I_cQ}{T}-\frac{1}{T}\left[C_{rp}Q(1-\delta)\left(p_1-\frac{z_1p_1}{T}\right)+C_{ai}\delta Q\left(p_2-\frac{z_2p_2}{T}\right)\right]-\frac{A_cf^4}{T}-\frac{C_s\rho(t_2+2t_S)}{T} \\ \frac{C_h+C_p\theta}{T}\left[\begin{array}{l} \frac{Q}{\theta}\left(1-e^{-\theta t_s}\right)-\frac{D}{\theta^2}\left[\theta t_s+e^{-\theta t_s}-1\right]+\frac{Q}{\theta}\left(e^{-\theta t_s}-e^{-\theta t_1}\right)+\frac{D}{\theta^2}\left(e^{-\theta t_s}-e^{-\theta t_1}\right) \\ +\frac{D}{\theta}\left(t_s-t_1\right)+\frac{Q}{\theta}\left[(1-\delta)\left(p_1-\frac{z_1p_1}{T}\right)+\delta\left(1-\left(p_2-\frac{z_2p_2}{T}\right)\right)\right]\left(e^{\theta(t_s-t_1)}-1\right) \\ +\frac{\rho}{\theta}\left(e^{\theta(t_s-t_1)}-1\right) \end{array}\right] \\ -\frac{C_pI_p}{T}\left(t_s-M\right)Q(1-\delta)\left(p_1-\frac{z_1p_1}{T}\right)+\delta\left(1-\left(p_2-\frac{z_2p_2}{T}\right)\right) \\ -\frac{C_pI_pDa_1(T+R-M)^2}{2T}-\frac{sI_pD\beta_p(1-a_1)(T-M)^2}{2T}-\frac{sI_pD(1-a_1)\left(1-\beta_p\right)(T+R-M)^2}{2T} \\ +\frac{sI_eDa_1(M-R)^2}{2T}+\frac{sI_eD\beta_p(1-a_1)M^2}{2T}+\frac{sI_eD(1-a_1)\left(1-\beta_p\right)(M-R)^2}{2T} \end{array}\right]$$

Subcase 1.2: $(t_s \leq T \leq M \leq R+T)$

Retailer's interest, between t_s and M for the damaged items combined with the one's that are misclassified as damaged.

$$I_{e_t}=\frac{S_{ip}I_e}{T}(M-t_s)Q\left((1-\delta)\left(p_1-\frac{z_1p_1}{T}\right)+\delta\left(1-\left(p_2-\frac{z_2p_2}{T}\right)\right)\right]$$

The interest retailer could earn by sales, and since $M \geq T$, the retailer doesn't need to pay any interest to the supplier, where $I_{p_{a_{22}}}=0$. Therefore,

$$I_{e_{a_{21}}}=\frac{sI_eD\beta_p(1-a_1)(M-R)^2}{2T}$$
$$I_{e_{a_{22}}}=\frac{sI_eD\beta_p(1-a_1)T^2}{2T}+\frac{sI_eD\beta_p(1-a_1)(M-T)}{T}$$
$$I_{e_{a_{23}}}=\frac{sI_eD(1-a_1)(1-\beta_p)\beta_w(M-R)^2}{2T}$$
$$I_{p_{a_{21}}}=\frac{C_pI_pDa_1(T+R-M)^2}{2T}$$
$$I_{p_{a_{23}}}=\frac{C_pI_pD(1-a_1)(1-\beta_p))(T+R-M)^2}{2T}$$

For $(t_s \leq T \leq M \leq R+T)$

TP$_2$=

$$\left[\begin{array}{l} \frac{1}{T}\left[s(1-\delta)\left(1-\left(p_1-\frac{z_1p_1}{T}\right)\right)Q+s\delta Q\left(p_2-\frac{z_2p_2}{T}\right)\right]-\frac{s\delta}{T}\left(p_2-\frac{z_2p_2}{T}\right)Q \\ \frac{1}{T}\left[-s\left(1-\beta_p\right)(1-\delta)\left(1-\left(p_1-\frac{z_1p_1}{T}\right)\right)\left(1-\beta_w\right)\left(1-a_1\right)Q\right] \\ +\frac{1}{T}\left[s_{ip}Q\left[\delta\left(1-\left(p_2-\frac{z_2p_2}{T}\right)\right)+(1-\delta)\left(p_1-\frac{z_1p_1}{T}\right)+\delta\left(p_2-\frac{z_2p_2}{T}\right)\right]\right] \\ -\frac{A}{T}-\frac{C_pQ}{T}-\frac{I_cQ}{T}-\frac{1}{T}\left[C_{rp}Q(1-\delta)\left(p_1-\frac{z_1p_1}{T}\right)+C_{ai}\delta Q\left(p_2-\frac{z_2p_2}{T}\right)\right]-\frac{A_cf^4}{T}-\frac{C_s\rho(t_2+2t_s)}{T} \\ \frac{C_h+C_p\theta}{T}\left[\begin{array}{l} \frac{Q}{\theta}\left(1-e^{-\theta t_s}\right)-\frac{D}{\theta^2}\left[\theta t_s+e^{-\theta t_s}-1\right]+\frac{Q}{\theta}\left(e^{-\theta t_s}-e^{-\theta t_1}\right)+\frac{D}{\theta^2}\left(e^{-\theta t_s}-e^{-\theta t_1}\right) \\ +\frac{D}{\theta}\left(t_s-t_1\right)+\frac{Q}{\theta}\left[(1-\delta)\left(p_1-\frac{z_1p_1}{T}\right)+\delta\left(1-\left(p_2-\frac{z_2p_2}{T}\right)\right)\right]\left(e^{\theta(t_s-t_1)}-1\right) \\ +\frac{\rho}{\theta}\left(e^{\theta(t_s-t_1)}-1\right) \end{array}\right] \\ +\frac{s_{ip}I_e}{T}\left(M-t_s\right)Q\left[(1-\delta)\left(p_1-\frac{z_1p_1}{T}\right)+\delta\left(1-\left(p_2-\frac{z_2p_2}{T}\right)\right)\right] \\ +\frac{sI_eD\beta_p(1-a_1)(M-R)^2}{2T}+\frac{sI_eD\beta_p(1-a_1)T^2}{2T}+\frac{sI_eD\beta_p(1-a_1)(M-T)}{T} \\ +\frac{sI_eD(1-a_1)\left(1-\beta_p\right)\beta_w(M-R)^2}{2T}-\frac{C_pI_pDa_1(T+R-M)^2}{2T}-\frac{C_pI_pD(1-a_1)\left(1-\beta_p\right))(T+R-M)^2}{2T} \end{array}\right]$$

Subcase 1.3: $(T \leq T+R \leq T+t_s \leq M)$

Retailer's interest for the damaged items, miscalculated as perfect items is,

$$I_{e_{t_3}}=\frac{s_{ip}I_eQ}{T}\left(M-t_s\right)\left[(1-\delta)\left(p_1-\frac{z_1p_1}{T}\right)+\delta\left(1-\left(p_2-\frac{z_2p_2}{T}\right)\right)+(M-T-t_s)\,\delta\left(p_2-\frac{z_2p_2}{T}\right)\right]$$

where, $I_{p_{a_{31}}}=I_{p_{a_{32}}}=I_{p_{a_{33}}}=0$.

$$I_{e_{a_{31}}}=\frac{sI_eDa_1T^2}{2T}+\frac{sI_eDa_1T(M-T_R)}{T}$$
$$I_{e_{a_{32}}}=\frac{sI_eD\beta_p(1-a_1)T^2}{2T}+\frac{sI_eD\beta_p(1-a_1)T(M-T)}{T}$$

$I_{e_{a_{33}}}=\frac{sI_eD(1-a_1)(1-\beta_p)\beta_wT^2}{2T}+\frac{sI_eD(1-a_1)(1-\beta_p)\beta_wT(M-T-R)}{T}$

For $(T\leq T+R\leq T+t_s\leq M)$

$TP_3=$

$$\left[\begin{array}{l}\frac{1}{T}\left[s(1-\delta)\left(1-\left(p_1-\frac{z_1p_1}{T}\right)\right)Q+s\delta Q\left(p_2-\frac{z_2p_2}{T}\right)\right]-\frac{s\delta}{T}\left(p_2-\frac{z_2p_2}{T}\right)Q\\ \frac{1}{T}\left[-s\left(1-\beta_p\right)(1-\delta)\left(1-\left(p_1-\frac{z_1p_1}{T}\right)\right)(1-\beta_w)(1-a_1)Q\right]\\ +\frac{1}{T}\left[s_{ip}Q\left[\delta\left(1-\left(p_2-\frac{z_2p_2}{T}\right)\right)+(1-\delta)\left(p_1-\frac{z_1p_1}{T}\right)+\delta\left(p_2-\frac{z_2p_2}{T}\right)\right]\right]\\ -\frac{A}{T}-\frac{C_pQ}{T}-\frac{I_cQ}{T}-\frac{1}{T}\left[C_{rp}Q(1-\delta)\left(p_1-\frac{z_1p_1}{T}\right)+C_{ai}\delta Q\left(p_2-\frac{z_2p_2}{T}\right)\right]-\frac{A_cf^4}{T}-\frac{C_s\rho(t_2+2t_s)}{T}\\ \frac{C_h+C_p\theta}{T}\left[\begin{array}{l}\frac{Q}{\theta}\left(1-e^{-\theta t_s}\right)-\frac{D}{\theta^2}\left[\theta t_s+e^{-\theta t_s}-1\right]+\frac{Q}{\theta}\left(e^{-\theta t_s}-e^{-\theta t_1}\right)+\frac{D}{\theta^2}\left(e^{-\theta t_s}-e^{-\theta t_1}\right)\\ +\frac{D}{\theta}(t_s-t_1)+\frac{Q}{\theta}\left[(1-\delta)\left(p_1-\frac{z_1p_1}{T}\right)+\delta\left(1-\left(p_2-\frac{z_2p_2}{T}\right)\right)\right]\left(e^{\theta(t_s-t_1)}-1\right)\\ +\frac{\rho}{\theta}\left(e^{\theta(t_s-t_1)}-1\right)\end{array}\right]\\ +\frac{S_{ip}I_eQ}{T}(M-t_s)\left[(1-\delta)\left(p_1-\frac{z_1p_1}{T}\right)+\delta\left(1-\left(p_2-\frac{z_2p_2}{T}\right)\right)+(M-T-t_s)\delta\left(p_2-\frac{z_2p_2}{T}\right)\right]\\ +\frac{sI_eDa_1T^2}{2T}+\frac{sI_eDa_1T(M-T_R)}{T}+\frac{sI_eD\beta_p(1-a_1)T^2}{2T}+\frac{sI_eD\beta_p(1-a_1)T(M-T)}{T}\\ +\frac{sI_eD(1-a_1)\left(1-\beta_p\right)\beta_wT^2}{2T}+\frac{sI_eD(1-a_1)\left(1-\beta_p\right)\beta_wT(M-T-R)}{T}\end{array}\right]$$

Case (2) $(M\leq R)$

Subcase 2.1: $(M\leq t_s\leq T)$

The retailer's interest have to be paid during this time is given here,

$I_{p_t}=\frac{C_pI_p}{T}(t_s-M)Q\left[(1-\delta)\left(p_1-\frac{z_1p_1}{T}\right)+\theta\left(1-\left(p_2-\frac{z_2p_2}{T}\right)\right)\right]$

Here, $I_{e_{a_{41}}}=I_{e_{a_{43}}}=0.$

$I_{e_{a_{42}}}=\frac{sI_eD\beta_p(1-a_1)M^2}{2T}$

$I_{p_{a_{41}}}=\frac{C_pI_pDa_1T^2}{2T}+\frac{C_pI_pDa_1T(R-M)}{T}$

$I_{p_{a_{42}}}=\frac{C_pI_pD\beta_p(1-a_1)(T-M)^2}{2T}$

$I_{p_{a_{43}}}=\frac{C_pI_pD(1-a_1)(1-\beta_p)T^2}{2T}+\frac{C_pI_pDT(1-a_1)(1-\beta_p)(R-M)}{T}$

For $(M\leq t_s\leq T)$

$TP_4=$

$$\left[\begin{array}{l}\frac{1}{T}\left[s(1-\delta)\left(1-\left(p_1-\frac{z_1p_1}{T}\right)\right)Q+s\delta Q\left(p_2-\frac{z_2p_2}{T}\right)\right]-\frac{s\delta}{T}\left(p_2-\frac{z_2p_2}{T}\right)Q\\ \frac{1}{T}\left[-s\left(1-\beta_p\right)(1-\delta)\left(1-\left(p_1-\frac{z_1p_1}{T}\right)\right)(1-\beta_w)(1-a_1)Q\right]+\frac{C_pI_pDT(1-a_1)\left(1-\beta_p\right)(R-M)}{T}\\ +\frac{1}{T}\left[s_{ip}Q\left[\delta\left(1-\left(p_2-\frac{z_2p_2}{T}\right)\right)+(1-\delta)\left(p_1-\frac{z_1p_1}{T}\right)+\delta\left(p_2-\frac{z_2p_2}{T}\right)\right]\right]\\ -\frac{A}{T}-\frac{C_pQ}{T}-\frac{I_cQ}{T}-\frac{1}{T}\left[C_{rp}Q(1-\delta)\left(p_1-\frac{z_1p_1}{T}\right)+C_{ai}\delta Q\left(p_2-\frac{z_2p_2}{T}\right)\right]-\frac{A_cf^4}{T}-\frac{C_s\rho(t_2+2t_s)}{T}\\ \frac{C_h+C_p\theta}{T}\left[\begin{array}{l}\frac{Q}{\theta}\left(1-e^{-\theta t_s}\right)-\frac{D}{\theta^2}\left[\theta t_s+e^{-\theta t_s}-1\right]+\frac{Q}{\theta}\left(e^{-\theta t_s}-e^{-\theta t_1}\right)+\frac{D}{\theta^2}\left(e^{-\theta t_s}-e^{-\theta t_1}\right)\\ +\frac{D}{\theta}(t_s-t_1)+\frac{Q}{\theta}\left[(1-\delta)\left(p_1-\frac{z_1p_1}{T}\right)+\delta\left(1-\left(p_2-\frac{z_2p_2}{T}\right)\right)\right]\left(e^{\theta(t_s-t_1)}-1\right)\\ +\frac{\rho}{\theta}\left(e^{\theta(t_s-t_1)}-1\right)\end{array}\right]\\ -\frac{C_pI_p}{T}(t_s-M)Q\left[(1-\delta)\left(p_1-\frac{z_1p_1}{T}\right)+\theta\left(1-\left(p_2-\frac{z_2p_2}{T}\right)\right)\right]+\frac{sI_eD\beta_p(1-a_1)M^2}{2T}\\ -\frac{C_pI_pDa_1T^2}{2T}-\frac{C_pI_pDa_1T(R-M)}{T}-\frac{C_pI_pD\beta_p(1-a_1)(T-M)^2}{2T}-\frac{C_pI_pD(1-a_1)\left(1-\beta_p\right)T^2}{2T}\end{array}\right]$$

Subcase 2.2: $(t_s\leq T\leq M)$

Since the time period is less than M, the product remains in the side of the retailer. The interest earned and to be paid by the retailer is given here, Here $I_{e_{a_{51}}}=I_{e_{a_{53}}}=I_{p_{a_{51}}}=I_{p_{a_{52}}}=0.$

$I_{e_t}=\frac{S_{ip}I_e(M-t_s)Q}{T}\left[(1-\delta)\left(p_1-\frac{z_1p_1}{T}\right)+\delta\left(1-\left(p_2-\frac{z_2p_2}{T}\right)\right)\right]$

$I_{e_{a_{52}}}=\frac{sI_eD\beta_p(1-a_1)T^2}{2T}+\frac{sI_eD\beta_p(1-a_1)T(M-T)}{T}$

$I_{p_{a_{53}}}=\frac{C_pI_pDT(1-a_1)(1-\beta_p)T}{2T}+\frac{C_pI_pDT(1-a_1)(1-\beta_p)(R-M)}{T}$

For $(t_s\leq T\leq M)$

$TP_5=$

$$\left[\begin{array}{l} \frac{1}{T}\left[s(1-\delta)\left(1-\left(p_1-\frac{z_1p_1}{T}\right)\right)Q+s\delta Q\left(p_2-\frac{z_2p_2}{T}\right)\right]-\frac{s\delta}{T}\left(p_2-\frac{z_2p_2}{T}\right)Q \\ \frac{1}{T}\left[-s\left(1-\beta_p\right)(1-\delta)\left(1-\left(p_1-\frac{z_1p_1}{T}\right)\right)\left(1-\beta_w\right)\left(1-a_1\right)Q\right]+\frac{C_pI_pDT(1-a_1)\left(1-\beta_p\right)(R-M)}{T} \\ +\frac{1}{T}\left[s_{ip}Q\left[\delta\left(1-\left(p_2-\frac{z_2p_2}{T}\right)\right)+(1-\delta)\left(p_1-\frac{z_1p_1}{T}\right)+\delta\left(p_2-\frac{z_2p_2}{T}\right)\right]\right] \\ -\frac{A}{T}-\frac{C_pQ}{T}-\frac{I_cQ}{T}-\frac{1}{T}\left[C_{rp}Q(1-\delta)\left(p_1-\frac{z_1p_1}{T}\right)+C_{ai}\delta Q\left(p_2-\frac{z_2p_2}{T}\right)\right]-\frac{A_cf^4}{T}-\frac{C_s\rho(t_2+2t_s)}{T} \\ \frac{C_h+C_p\theta}{T}\left[\begin{array}{l} \frac{Q}{\theta}\left(1-e^{-\theta t_s}\right)-\frac{D}{\theta^2}\left[\theta t_s+e^{-\theta t_s}-1\right]+\frac{Q}{\theta}\left(e^{-\theta t_s}-e^{-\theta t_1}\right)+\frac{D}{\theta^2}\left(e^{-\theta t_s}-e^{-\theta t_1}\right) \\ +\frac{D}{\theta}\left(t_s-t_1\right)+\frac{Q}{\theta}\left[(1-\delta)\left(p_1-\frac{z_1p_1}{T}\right)+\delta\left(1-\left(p_2-\frac{z_2p_2}{T}\right)\right)\right]\left(e^{\theta(t_s-t_1)}-1\right) \\ +\frac{\rho}{\theta}\left(e^{\theta(t_s-t_1)}-1\right) \end{array}\right] \\ +\frac{s_{ip}I_e(M-t_s)Q}{T}\left[(1-\delta)\left(p_1-\frac{z_1p_1}{T}\right)+\delta\left(1-\left(p_2-\frac{z_2p_2}{T}\right)\right)\right]+\frac{sI_eD\beta_p(1-a_1)T^2}{2T} \\ +\frac{sI_eD\beta_p(1-a_1)T(M-T)}{T}-\frac{C_pI_pDT(1-a_1)\left(1-\beta_p\right)T}{2T}-\frac{C_pI_pDT(1-a_1)\left(1-\beta_p\right)(R-M)}{T} \end{array}\right]$$

2.7 Solution Procedure

To find the solution for the derived mathematical model, and to calculate the global optimal values, which gives the maximum profit to the retailer, with respect to the time T and advertisement paramter f. To derive the optimal value of the function, we have to show that the derived profit function is concave with respect to the decision variable, T and f. To verify the concavity, we have to find out the second derivative of the respective profit function, for all the case 1 and case 2 should be non-positive.

$$\frac{\partial^2 TP_i(f,T)}{\partial T^2}\leq 0 \; where \; i=1,2,3,4,5.$$

$$\frac{\partial^2 TP_i(f,T)}{\partial f^2}\leq 0 \; where \; i=1,2,3,4,5.$$

$$\frac{\partial^2 TP_i(f,T)}{\partial f^2}\times\frac{\partial^2 TP_i(f,T)}{\partial T^2}-\left(\frac{\partial^2 TP_i(f,T)}{\partial f\partial T}\right)^2\geq 0 \; where \; i=1,2,3,4,5.$$

The values of the above referred derivative functions of the respective equations, is concave for all the subcases and the optimal values are obtained and unique.

2.8 Numerical Example

Example : For this study, data set for the numerical example is adopted from the work of Moradi, as we have developed their model for deteriorating items, the data are given in the Table 1.

Table 1. Numerical data

Parameters	values	Parameters	values	Parameters	values
A	10001000	C_p	4001000	C_h	101000
I_c	81000	S_{ip}	3701000	C_{rp}	8001000
C_{ai}	10001000	s	5601000	y_0	1000
α	600	I_p	0.05	I_e	0.02
δ	0.1	p_1	0.05	p_2	0.05
β	0.01	A_c	0.361000	Δ	0.2
β_p	0.2	a_1	0.4	β_w	0.95
z_1	0.001	z_2	0.001	ρ	283.97
C_s	83340	θ	0.7		

Table 2. Optimal values of profit and other functions for case 1 and 2

Parameter	case 1.1	case 1.2	case 1.3	case 2.1	case 2.2
θ	0.7	0.7	0.7	0.7	0.7
T	0.9225	0.8965	0.9215	0.9202	0.9345
TP	2915082453	2921892347	2920969058	2918390864	2915082453

With the numerical values given in the table 1, the optimal values of the developed model is obtained in the table 2.

3 Results and Discussion

From the obtained results , in the numerical example 1, we have obtained the profit values of the subcases of case 1 and 2, out of this subcase 1.2 has the profit value higher than the other cases with the minimum possible time period of the inventory cycle. The deterioration rate of the inventory model is fixed and constant, for all the possibilities discussed. Here the shortages are allowed in the considerable rate and those are completely backlogged. Considerably, the subcase 1.3 has the second highest profit value and the minimum time period $T = 0.9215$. The learning in inspection may reduce the rate of the time period of the whole inventory cycle. In real life situations, there are lot of possibilities for a product to deteriorate at the rate θ given, certainly in critical situations such as disaster and pandemic there are more possibilities for a product to deteriorate fully with no sale due to demand fluctuations. Since the higher the deterioration rate, it may affect the profit of the retailer if the cycle time is higher, it makes the product deterioration increase. Similarly, for shortages, we can't avoid the shortage possibilities in real life even if there is a demand fluctuation possibilities, therefore the backlogged demand help us to overcome the shortage situation. Finally, by considering all this possibilities in our developed model, the profit function TP_2 has the highest profit value.

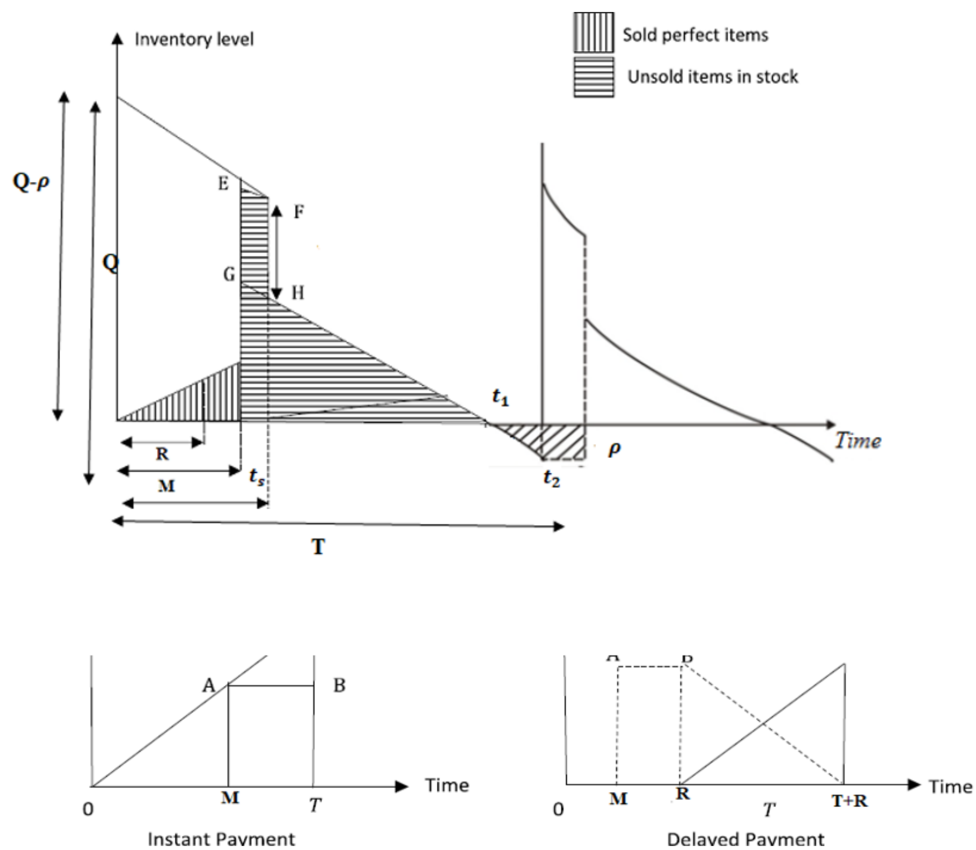

Fig 1. Inventory level for subcase 1.1

3.1 Sensitivity Analysis

1. The inspector's inspection rate p_1, where he classifies the perfect product as imperfect one's, if this happens at greater level, the profit of the firm will decrease, if he whether misclassifies the imperfect product as perfect one p_2 , the firm may lose its goodwill among the customer and also it might affect the profit.
2. After learning process, the impact of production of imperfect products may decrease, whether during this time if the inspection time period is higher it would affect the product's deterioration level θ , and then it may lead us to more product shortages, this would decrease the total profit of the retailer.
3. By changing the frequency of advertisement after learning, helps us to sell the products as soon as possible, but if the error p_2 increases, it may result in loss of customers to the firm.

4. Demand rate D, decreases with increase in type 2 error of misclassification, it creates more loss to the firm than usual. If the learning rate increases, the time period of the inventory cycle may decrease and it will give more profit to the retailer. The results by changing various parameters of the developed model and obtained results are given in the Figures 2, 3, 4 and 5.

Table 3. Changes in deterioration cost values of the profit function

θ	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**
DC	27992.87	33387.13	38715.94	43980.83	49183.49
TP_1 **Case 1.1**	2917837060	2918175605	2918510381	2918840890	2919166676
TP_2 **Case 1.2**	2921219026	2921557571	2921892347	2922222856	2922548642
TP_3 **Case 1.3**	2920295737	2920634282	2920969058	2921299567	2921625354
TP_4 **Case 2.1**	2917717543	2918056088	2918390864	2918721373	2919047159
TP_5 **Case 2.2**	2914409132	2914747677	2915082453	2915412962	2915738748

Table 4. Changes in Shortage cost values of the profit function

C_s	**81000**	**82000**	**83340**	**84000**	**85000**
TP_1 **Case 1.1**	2,91,97,64,877	2,91,85,10,381	2,91,92,28,7674	2,91,81,56,549	2,91,76,20,440
TP_2 **Case 1.2**	2,91,27,76,394	2,91,22,40,285	2,91,15,21,899	2,91,11,68,067	2,91,06,31,957
TP_3 **Case 1.3**	2,92,22,23,554	2,92,15,20,731	2,92,09,69,058	2,91,85,06,758	2,91,78,03,936
TP_4 **Case 2.1**	2,91,96,45,360	2,91,91,09,250	2,91,83,90,864	2,91,80,37,032	2,91,75,00,923
TP_5 **Case 2.2**	2,91,63,36,949	2,91,58,00,839	2,91,50,82,453	2,91,50,82,453	2,91,41,92,511

Table 5. Changes in inspection, the values of the profit function

t_s	**0.5**	**0.4**	**0.3**	**0.2**	**0.1**
Case 1.1	2860699428	2869846408	2878735162	2887383148	2895806644
Case 1.2	2864911431	2874589726	2884009797	2893189101	2902143913
Case 1.3	2863922035	2873557299	2882939535	2892086199	2901013570
Case 2.1	2860694985	2869841965	2878730719	2887378705	2895802201
Case 2.2	2858101536	2867779832	2877199903	2886379206	2895334019

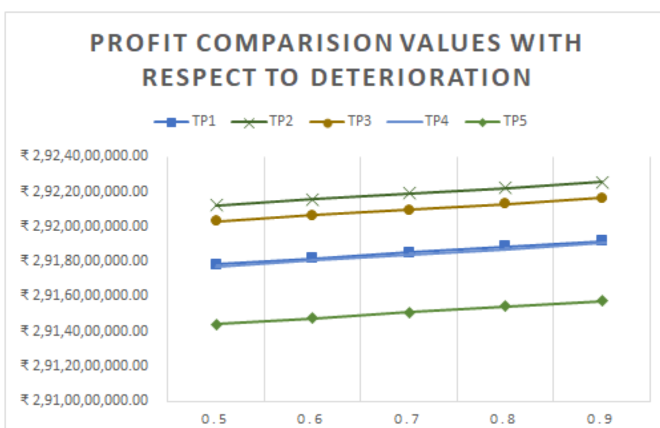

Fig 2. Change in deterioration rate

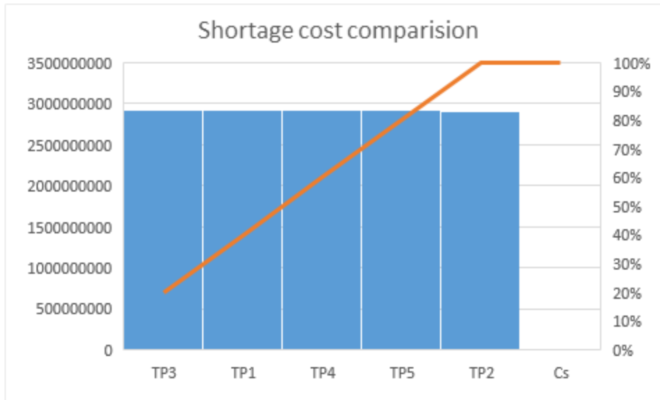

Fig 3. Change in shortage cost value w.r to profit

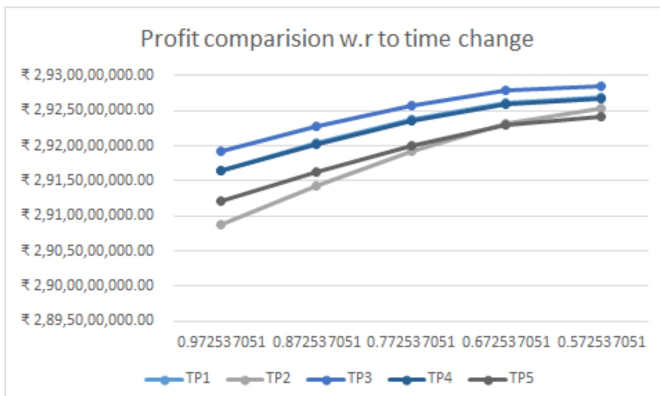

Fig 4. Profit comparison w.r to time

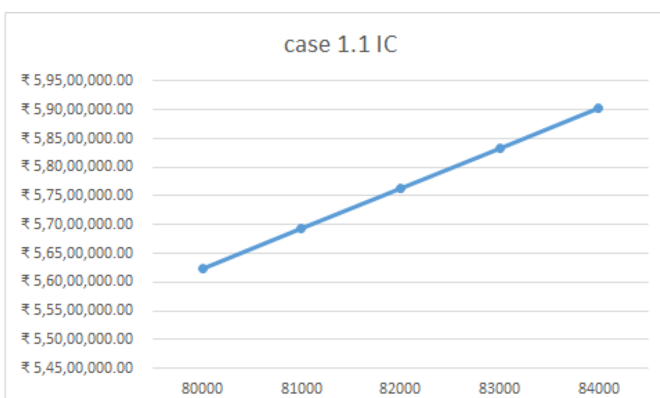

Fig 5. Inspection cost comparison

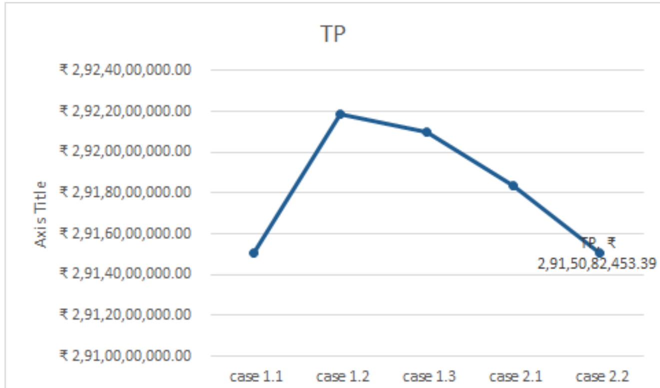

Fig 6. Profit comp for different cases

4 Conclusion

In general, firms tend to minimize the time period of production, and to reduce the ratio of production of imperfect products, to maximize the profit and to improve their standard in the market. The products must be inspected and have to separate the imperfect products from the perfect ones, in our model, we have considered the deteriorating items, with imperfect quality products present in the lot, the inspector have to inspect carefully the whole lot. But there are always the possibilities to have the error that he can classify the imperfect product as perfect one and perfect product as imperfect one, during this inspection process , if the learning of the inspector can help the firm to avoid the misclassification, it also avoid the uncomfortable situation to the retailer and it induces the customer to buy more, the lot has become null in short period. Shortages are considered in this model, and backlogged. The promotional effort of the demand pattern induces the sales and we also considered two levels of delay in payment it helps the retailer and customer to face the different uncertainties. The solution procedure given in this study is to find the global optimal solution for the firm.

For the future research, one can change the demand pattern as stochastic in nature, price dependent, green product consideration, or some other discount policies and pre payment offers to be discussed. Various shortage situations can be added to make the model more realistic.

Declaration

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References

1) Mahapatra AS, Mahapatra MS, Sarkar B, Majumder SK. Benefit of preservation technology with promotion and time-dependent deterioration under fuzzy learning. *Expert Systems with Applications*. 2022;201:117169–117169. Available from: https://dx.doi.org/10.1016/j.eswa.2022.117169.
2) Bose D, Guha A. Economic production lot sizing under imperfect quality, on-line inspection, and inspection errors: Full vs. sampling inspection. *Computers & Industrial Engineering*. 2021;160:107565–107565. Available from: https://dx.doi.org/10.1016/j.cie.2021.107565.
3) Dey O, Giri BC. A new approach to deal with learning in inspection in an integrated vendor-buyer model with imperfect production process. *Computers & Industrial Engineering*. 2019;131:515–523. Available from: https://dx.doi.org/10.1016/j.cie.2018.12.028.
4) Fu K, Chen Z, Sarker BR. An optimal decision policy for a single-vendor single-buyer production-inventory system with leaning effect, fuzzy demand and imperfect quality. *Journal of Information and Optimization Sciences*. 2019;40(3):633–658. Available from: https://dx.doi.org/10.1080/02522667.2018.1427026.
5) Jayaswal MK, Sangal I, Mittal M, Malik S. Effects of learning on retailer ordering policy for imperfect quality items with trade credit financing. *Uncertain Supply Chain Management*. 2019;7(1):49–62. Available from: https://dx.doi.org/10.5267/j.uscm.2018.5.003.
6) Jayaswal MK, Mittal M, Sangal I. Ordering policies for deteriorating imperfect quality items with trade-credit financing under learning effect. *International Journal of System Assurance Engineering and Management*. 2021;12(1):112–125. Available from: https://dx.doi.org/10.1007/s13198-020-01038-y.
7) Jayaswal MK, Mittal M, Alamri OA, Khan FA. Learning EOQ Model with Trade-Credit Financing Policy for Imperfect Quality Items under Cloudy Fuzzy Environment. *Mathematics*. 2022;10(2):246–246. Available from: https://dx.doi.org/10.3390/math10020246.
8) Khanna A, Kishore A, Sarkar B, Jaggi CK. Inventory and pricing decisions for imperfect quality items with inspection errors, sales returns, and partial backorders under inflation. *RAIRO - Operations Research*. 2020;54(1):287–306. Available from: https://dx.doi.org/10.1051/ro/2018102.

9) Khanna A, Kishore A, Sarkar B, Jaggi C. Supply Chain with Customer-Based Two-Level Credit Policies under an Imperfect Quality Environment. *Mathematics*. 2018;6(12):299–299. Available from: https://dx.doi.org/10.3390/math6120299.
10) Mahato C, Mahata GC. Optimal inventory policies for deteriorating items with expiration date and dynamic demand under two-level trade credit. *OPSEARCH*. 2021;58(4):994–1017. Available from: https://dx.doi.org/10.1007/s12597-021-00507-7.
11) Moradi S, Gholamian MR, Sepehri A. An inventory model for imperfect quality items considering learning effects and partial trade credit policy. *OPSEARCH*. 2023;60(1):276–325. Available from: https://dx.doi.org/10.1007/s12597-022-00602-3.
12) Moradi S, Gholamian MR, Sepehri A. An inventory model for imperfect quality items considering learning effects and partial trade credit policy. *OPSEARCH*. 2023;60(1):276–325. Available from: https://dx.doi.org/10.1007/s12597-022-00602-3.
13) Tiwari S, Cárdenas-Barrón LE, Shaikh AA, Goh M. Retailer's optimal ordering policy for deteriorating items under order-size dependent trade credit and complete backlogging. 2020. Available from: https://doi.org/10.1016/j.cie.2018.12.006.
14) Sepehri A, Gholamian MR. Joint pricing and lot-sizing for a production model under controllable deterioration and carbon emissions. *International Journal of Systems Science: Operations & Logistics*. 2022;9(3):324–338. Available from: https://dx.doi.org/10.1080/23302674.2021.1896049.
15) Tiwari S, Cárdenas-Barrón LE, Malik AI, Jaggi CK. Retailer's credit and inventory decisions for imperfect quality and deteriorating items under two-level trade credit. *Computers & Operations Research*. 2022;138:105617–105617. Available from: https://dx.doi.org/10.1016/j.cor.2021.105617.
16) Shah NH, Rabari K, Patel E. An Inventory Model for Deteriorating Items with Constant Demand Under Two-Level Trade-Credit Policies. In: Inventory Optimization. Springer Singapore. 2021;p. 91–103.
17) Das SC, Manna AK, Rahman MS, Shaikh AA, Bhunia AK. An inventory model for non-instantaneous deteriorating items with preservation technology and multiple credit periods-based trade credit financing via particle swarm optimization. *Soft Computing*. 2021;25(7):5365–5384. Available from: https://dx.doi.org/10.1007/s00500-020-05535-x.
18) Tripathy M, Sharma G, Sharma AK. An EOQ inventory model for non-instantaneous deteriorating item with constant demand under progressive financial trade credit facility. *OPSEARCH*. 2022;59(4):1215–1243. Available from: https://dx.doi.org/10.1007/s12597-022-00573-5.