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RESEARCH ARTICLE

Upper g- Eccentric Domination in Fuzzy Graphs

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Abstract

Objectives: To introduce a contemporary aspect notorious as upper g-eccentric point set, upper g-eccentric number, upper g-eccentric dominating set, upper g-eccentric domination number in fuzzy graphs and related concepts. **Methods:** Strong arc in fuzzy graphs concepts are used to find upper g-eccentric number and upper g-eccentric domination number. **Findings:** The new idea upper g-eccentric domination in fuzzy graphs will leads to numerous application such as cell phone tower formation in remote areas. **Novelty:** Using the concepts g-distance, geodesics and strong arc in fuzzy graphs, the upper g-eccentric dominating set and upper g-eccentric domination number in fuzzy graphs are obtained. Some points are observed in this concept and some theorem related to the upper g-eccentric dominant set, its numbers are proved.

Keywords: g-Eccentric Dominating Set; g-Eccentric Domination Number; Upper g-Eccentric Domination Number

1 Introduction

FGs are used in a variety of sectors, including the internet, finance, medicine, cell phone towers, and more. Bhanumathi and Meenal Abirami [1] introduced the concept of upper eccentric domination on graphs in 2019. A.M. Ismayil and S. Muthupandiyan [2] initiated the complementary nil g-eccentric domination in fuzzy graph(FG) on the year 2020. Mariappan [3] et.al discussed domination integrity and efficient FGs in 2020. S. Muthupandiyan and A.M. Ismayil [4] initiated the concept of perfect g-eccentric domination in FGs on 2021. Enriques E [5] et.al., stated about domination in fuzzy directed graphs in 2021. Titus P [6] et. al discussed about the connected monophonic eccentric domination number of a graph in 2022. Surya SS, Mathew L. [7] introduced secure domination cover pebbling number of join of graphs in 2022. Muthupandiyan S, Ismayil AM [8] initiated connected g-eccentric domination in FG in 2022. Ganesan B [9] initiated Strong domination integrity in graphs and FGs. Manjusha OT [10] discussed about global domination in fuzzy graphs using strong arcs in 2023. Muthupandiyan S and Ismayil AM [11] initiated isolate g-eccentric domination in FG in 2023. Mehra S [12] initiated the arrow domination of some generalized graphs in 2023. Ismayil AM [13] initiated transversal eccentric domination in graphs in 2023.

Research gap

K.R Bhutani and A Rosenfeld initiated the perception strong arc and geodesics in FG in 2003. Janakiraman et al, initiated the perception of eccentric domination in graphs in 2010. Linda J.P and Sunitha M.S initiated On g-eccentric nodes g-boundary nodes and g-interior nodes of a FG in 2012. Mini Tom, M.S.Sunitha and Sunil Mathew introduced Notes on Types of arcs in fuzzy graphs in 2014. Sameeha and M.S Sunitha initiated connected geodesic number of a fuzzy graph in 2018. A Mohamed Ismayil and S Muthupandiyan [14] introduced g-eccentric domination in FG utilize strong arc, in 2020.

New idea

In this article, the upper g-eccentric point set, upper g-eccentric dominating set and their numbers in FG are introduced. The bounds on the upper g-eccentric domination number for some standard FG are obtained and some theorems related to upper g-eccentric domination in FG are stated and proved.

Overcome of existing limit

This upper g-eccentric domination in FG idea will overcome the existing idea eccentric domination in graph by fuzzy value. One can use this idea to access high frequency cell phone network coverage in remote areas with minimum number of tower.

In this paper, G is a connected FG unless otherwise stated.

The upper g-eccentric domination in fuzzy graphs concepts are currently very important to locate and to minimize the cell phone tower in remote area. There are lot of unsolved concepts are available in geodesic concepts using strong arcs like monophonic g-eccentric domination, connected monophonic g-eccentric domination, in FGs and etc.,

2 Methodology

The concept strong arc in FG were used by Sunitha and Sunil Mathew S. Eccentric domination in graphs introduced by Janakiraman et al. These ideas motivated to us to develop the concept upper g-eccentric domination in FG.

Definitions and Examples

The following definitions are in this section will help the readers to comprehend the core concepts.

Definition 2.1. [2] A FG $G=(\sigma,\mu)$ is characterized with two functions σ on V and μ on $E\subseteq V\times V$, where $\sigma: V\to[0,1]$ and $\mu: E\to[0,1]$ such that $\mu(x,y)\leq\sigma(x)\wedge\sigma(y),\forall x,y\in V$. We indicate the crisp grpah $G^*=(\sigma^*,\mu^*)$ of the FG $G=(\sigma,\mu)$ where $\sigma^*=\{x\in V:\sigma(x)>0\}$ and $\mu^*=\{(x,y)\in E:\mu(x,y)>0\}$. A FG G's order and size are determined by $p=\sum_{u\in V}\sigma(u)$ and $q=\sum_{uv\in E}\mu(u,v)$ respectively.

Definition 2.2. [2] "Let G be a FG. A way P of distance n is a chain of different vertex $u_0,u_1,\ldots,u_n$ such that $\mu(u_{i-1},u_i)>0, i=1,2,..,n$ and strength of the path P is $s(P)=min(\mu(u_{i-1},u_i), i=1,2,\ldots n\}$. The strength of connectedness among nodes u and v is defined as the highest strengths of all paths among u and v and is mentioned by $CONN_G(u,v)$. An $u-v$ path P is called a Strong path if it contains only strong arcs. An $u-v$ path is called strongest $u-v$ path if its strength is equals $CONN_G(u,v)$. An arc is strong iff its weight is equal to the strength of connectedness of its end nodes. i.e., $\mu(u,v)\geq CONN_{G-(u,v)}(u,v)$. If an $arc(u,v)$ is strong, then u dominates v or v dominates u".

Definition 2.3. [4] "A strong path P from u to v is referred to as a geodesics if there is no shortest strong path from u to v and a length of a $u-v$ geodesic is the geodesic distance(g-distance) from u to v addressed by $d_g(u,v)$. The geodesic eccentricity (g-eccentricity)(gE) $e_g(u)$ of a node u in a connected FG G is given by $e_g(u)=max\{\{d_g(u,v),v\in V\}\}$. The g-eccentric(gE) set of a vertex u is defined and mentioned by $E_g(u)=\{v:v\in d_g(u,v)=e_g(u)\}$. The lowest g-eccentricity among the nodes of G is called g-radius and mentioned by $r_g(G)=min\{e_g(u),u\in V\}$. The highest g-eccentricity among the nodes of G is called g-diameter and mentioned by $d_g(G)=max\{e_g(u),u\in V\}$. A vertex v is said to be a g-central node if $e_g(v)=r_g(G)$. Also, a vertex v in G is said to be a g-peripheral node if $e_g(v)=d_g(G)$. A FG G is said to be self centered if $r_g(G)=d_g(G)$.

Definition 2.4. [2] A subset D of V is referred to as a dominating $set(D$-set) of a FG G if for each $v\in V-D\exists$ at least one node $u\in D$ such that u dominates v.

Definition 2.5 [2]**.** The set $S\subseteq V$ in a FG $G(\sigma,\mu)$ on $G^*(V,E)$ is referred to as a g-eccentric point set(gEP-set) if for every $v\in V-S,\exists$ at least one g-eccentric point u of v in S. The gEP-set S of G is a minimal gEP-set if there is no appropriate subset (SS) S" of S is a gEP-set of G. The highest value of a minimal gEP-set is referred to as a upper g-eccentric number (UgEN) and mentioned by $E_g(G)$ and simply mentioned by E_g. Similarly the lowest value of a minimal gEP-set is referred to as a g-eccentric number and is mentioned by $e_g(G)$ and simply mentioned by e_g.

Note 2.1. The minimum gEP-set is denoted by e_g-set and the upper gEP-set($UgEP$-set) is denoted by E_g-set

Example 2.1.

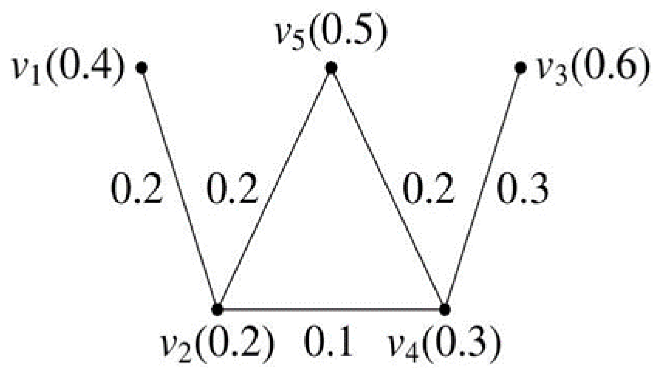

Fig 1. Fuzzy Graph$G(\sigma,\mu)$

In the FG given in Figure 1 , the g-eccentricity and the gEP-set of the vertices v_1, v_2, v_3, v_4 and v_5 are $e_g(v_1) = e_g(v_3) = 4, e_g(v_2) = e_g(v_4) = 3, e_g(v_5) = 2$ and $E_g(v_1) = E_g(v_2) = \{v_3\}, E_g(v_4) = \{v_1\}, E_g(v_5) = \{v_1, v_3\}$ respectively. Therefore, the minimal gEP-sets of G are S_1, S_2, S_3, where $E_g(v_1) = E_g(v_2) = \{v_3\}, E_g(v_4) = \{v_1\}, E_g(v_5) = \{v_1, v_3\}$ and $S_3 = \{v_1, v_3\}$. Hence, $e_g(G) = 0.6$ and $E_g(G) = 1.0$.

Definition 2.6 [2]. A dominating set $D \subseteq V(G)$ is a g-eccentric dominating $set(gED$-set) in a FG $G(\sigma,\mu)$ if for each $v \in V - D, \exists$ minimum of one gE node $u \in D$ of v. A gED-set is a minimal gED-set if no adequate SS $D" \subset D$ is a gED-set. The lowest value among all minimal gED-set is referred to as the g-eccentric domination number $(gEDN)$ and is mentioned by $\gamma_{\text{ged}}(G)$. The highest value among all minimal gED-set is referred to as a upper gEDN(UgEDN) and is mentioned by $\Gamma_{\text{ged}}(G)$. That is, $\gamma_{\text{ged}}(G) = min\{|D|\}$ and $\Gamma_{\text{ged}}(G) = max\{|D|\}$.

Note 2.2. The minimum gED-set is denoted by γ_{ged}-set and the upper gED-set(UgED-set) is denoted by Γ_{ged} -set

Example 2.2.

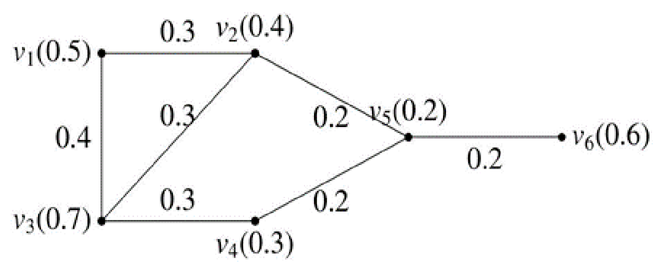

Fig 2. Fuzzy Graph$G(\sigma,\mu)$

In the FG given in Figure 2 , the g-eccentricity and the gEP-set of the vertices v_1, v_2, v_3, v_4, v_5 and v_6 are $e_g(v_1) = e_g(v_3) = e_g(v_6) = 3, e_g(v_2) = e_g(v_4) = e_g(v_5) = 2$ and $E_g(v_1) = E_g(v_3) = \{v_6\}, E_g(v_2) = \{v_4, v_6\}, E_g(v_4) = \{v_1, v_2, v_6\}, E_g(v_5) = E_g(v_6) = \{v_1, v_3\}$ respectively. Therefore, the minimal gED-sets of G are $D' = \{D_1, D_2, D_3, D_4\}$, where $D_1 = \{v_3, v_6\}, D_2 = \{v_2, v_4, v_6\}, D_3 = \{v_1, v_5, v_6\}$ and $D_4 = (v_1, v_4, v_6\}$. Hence, $(\gamma_{\text{ged}} G) = 1.3$ and $\Gamma_{\text{ged}}(G) = 1.4$.

3 Results and Discussion

This section sate about main results such as upper g-eccentric point set, upper g-eccentric number, upper g-eccentric dominating set and upper g-eccentric domination number in fuzzy graphs.

Upper g-Eccentric Point set in Fuzzy Graphs

In this section, the upper g-eccentric point set, upper g-eccentric numbers and some observations, theorem and propositions are given in a FG with applicable examples.

Proposition 3.1. Let K_σ be any complete FG. Then $e_g(K_\sigma) = \sigma_n, |\sigma^*| = n$, where $\sigma_n = max\{\sigma(u), u \in V\}$. Proof. Let K_σ be any complete FG. Then $\mu(u,v) = \sigma(u) \wedge \sigma(v), \forall u, v \in V$. Therefore, each vertex has exactly $(n-1)$ g-eccentric points. Hence, every singleton set is a minimal gEP-set and in which the maximum cardinality is an Upper gEPset. Therefore, the upper g-eccentric number of K_σ is $E_g(K_\sigma) = \sigma_n$, where $\sigma_n = max\{\sigma(u), u \in V\}$.

Observation 3.1. For any $FG\ G, \sigma_0 \leq e_g(G) \leq E_g(G) \leq p$, where $\sigma_0 = min\{\sigma(u), u \in V\}$.

Observation 3.2.

1. Let P_σ be any path FG, then

a) $E_g(P_\sigma) = \sigma_e, |\sigma^*| = n = 2,3$, where $\sigma_e = max\{\sigma(u)/u$ is an end vertex $\}$

b) $E_g(P_\sigma) \leq 2, |\sigma^*| = n \geq 4$.

2. Let S_σ be a star FG, then $E_g(S_\sigma) = \sigma_e, |\sigma^*| = n \geq 3$.

3. Let K_{σ_1,σ_2} be a complete bipartite FG, where $|\sigma_1^*| = m$ and $E_g(S_\sigma) = \sigma_e, |\sigma^*| = n \geq 3$. Then $E_g(K_{\sigma_1,\sigma_2}) = \sigma_{11} + \sigma_{22}$, where $\sigma_{11} = max_{u \in V_1}\ \sigma(u)$ and $\sigma_{22} = max_{v \in V_2}\ \sigma(v)$.

4. Let $W_\sigma, |\sigma^*| = n$ be a wheel FG, then $E_{ge}(W_\sigma) \leq 2, |\sigma^*| = n \geq 4$.

Upper g-Eccentric Domination in Fuzzy Graphs

In this part, the upper g-eccentric dominating set and its numbers are defined in FG. The relations between domination numbers, upper domination number, g-eccentric number, g-eccentric domination number and upper g-eccentric domination number are obtained. The upper g-eccentric domination numbers of some well known FG are found and theorems related to upper g-eccentric domination numbers are given and demonstrated.

Observation 3.3.

1. For any FG $G, \sigma_0 \leq \gamma(G)\gamma_{\text{ged}}(G) \leq \Gamma_{\text{ged}}(G) \leq p-1, \sigma_0 = \min\{\sigma(u), u \in V\}$
2. For a FG$G, \beta_0(G) \leq \Gamma(G)$
3. If G contains a dangling node v, then we can discover a minimal gED-set it entails v and not the assist of v.

Theorem 3.1. Let K_σ be any complete FG. Then $\Gamma_{\text{ged}}(K_\sigma) = \sigma_n, |\sigma^*| = n$, where $\sigma_n = max\{\sigma(u), u \in V\}$

Proof. Let K_σ be any complete FG. Then $\mu(u,v) = \sigma(u) \wedge \sigma(v), \forall u, v \in V$. $\therefore$ each vertex has exactly $(n-1)$ g-eccentric points. Hence, every singleton set is a minimal gED-set and in which the maximum cardinality is an Upper gEDset. Therefore, the upper g-eccentric domination number of K_σ is $\Gamma_{ged}(K_\sigma) = \sigma_n$, where $\sigma_n = max\{\sigma(u), u \in V\}$.

Theorem 3.2. Let P_σ of order $p, |\sigma^*| = n \geq 4$ be any path FG, then

$$\Gamma_{\text{ged}}(P_\sigma) \leq \begin{cases} \frac{3p-4}{4} + 1, & \text{if } n = 4k \\ \frac{3p-3}{4} + 1, & \text{if } n = 4k+1 \\ \frac{3p-2}{4} + 1, & \text{if } n = 4k+2 \\ \frac{3p-5}{4} + 1, & \text{if } n = 4k+3 \end{cases}$$

Proof. Let P_σ of order $p, |\sigma^*| = n \geq 4$ be any path FG and let $v_1, v_2, v_3, \ldots, v_n$ be the vertices of P_σ. Vertices v_1 and v_n are hanging nodes, which are again gE nodes of G. So, if any single of v_1 or v_n isn't in gED-set D then these nodes which are having this nodes as gE node should be in D.

If n is even, P_σ is bicentral, r_g of P_σ is $n/2$.

If n is odd, P_σ is unicentral, r_g of P_σ is $(n-1)/2$.

Case (i): $n = 4k$

Let, $D = \left\{v_1, v_2, v_3, \ldots, v_{\frac{n}{2}}, v_{\frac{n}{2}+2}, v_{\frac{n}{2}+4}, \ldots, v_{n-3}, v_{n-1}\right\}$ is a minimal gED-set of highest value. Hence, $\Gamma_{\text{ged}}(P_\sigma) \leq \frac{3p-4}{4} + 1$.

Case (ii) : $n = 4k+1$

Let, $D = \left\{v_1, v_2, \ldots, v_{\frac{n-1}{2}}, v_{\frac{n+3}{2}}, v_{\frac{n+7}{2}}, \ldots, v_{n-1}\right\}$ is a minimal gED-set of highest value. Hence, $\Gamma_{\text{ged}}(P_\sigma) \leq \frac{3p-3}{4} + 1$.

Case (iii) : $n = 4k+2$

Let, $D = \left\{v_1, v_2, \ldots, v_{\frac{n}{2}}, v_{\frac{n+3}{2}}, v_{\frac{n+5}{2}}, \ldots, v_{n-1}\right\}$ is a minimal gED-set of highest value.

Hence, $\Gamma_{\text{ged}}(P_\sigma) \leq \frac{3p-2}{4} + 1$.

Case (iv) : $n = 4k+3$

$D = \left\{v_1, v_2, \ldots, v_{\frac{n-1}{2}}, v_{\frac{n+5}{2}}, v_{\frac{n+9}{2}}, v_{\frac{n+13}{2}}, \ldots, v_{n-3}, v_{n-1}\right\}$ is a minimal gED-set of highest value. Hence, $\Gamma_{\text{ged}}\ (P_\sigma) \leq \frac{3p-5^2}{4} + 1$.

Observation 3.4. Let P_σ be any path FG, then $\Gamma_{\text{ged}}\ (P_\sigma) = \sigma_{00}, |\sigma^*| = 2$ and $\Gamma_{\text{ged}}\ (P_\sigma) \leq 2, |\sigma^*| = 3$

Theorem 3.3. Let C_σ of order $p, |\sigma^*| = n \geq 7$ be any cycle FG, then $\Gamma_{\text{ged}}\ (C_\sigma) \leq \frac{p}{2} + 1$.

Proof. Let C_σ of order $p, |\sigma^*| = n \geq 7$ be any cycle FG and $v_1, v_2, v_3, \ldots, v_n$ be the vertices of C_σ.

Case (i) n - even

Let $n = 2k$, k - positive number.

If n is even, r_g of C_σ $= diam_g$ of $C_\sigma = \frac{n}{2} = k$ every node v_i has an unique gE node v_{i+k}. Also, $\gamma_{egd}\ (C_\sigma) \leq p/2 + 1$, when n is even by observation 4.1 (2).

Subcase (i) : k is even

Let, $D = \{v_1, v_2, v_4, v_6 \ldots, v_k, v_{k+3}, v_{k+5}, \ldots, v_{2k-1}\}$ is a minimal gED-set of highest value. $\therefore \Gamma_{\text{ged}}\ (C_\sigma) \leq \frac{p}{2} + 1$.

Subcase (ii) : **k is odd**

Let, $D = \{v_1, v_3, v_5, \ldots, v_k, v_{k+2}, \ldots, v_{2k-3}, v_{2k-1}\}$ is a minimal gED-set of highest value. $\therefore, \Gamma_{\text{ged}}\ (C_\sigma) \leq \frac{p}{2} + 1$.

Case (ii) : n - odd

Suppose, $n = 2k+1$, k is a non-negative number. Here r_g of $C_\sigma = d_g$ of $C_\sigma = \frac{n-1}{2} = k$, every node v_i has precisely 2 gE nodes v_{i+k} and v_{i+k+1}.

Subcase (i) : k is even

Let, $D = \{v_1, v_3, v_5, \ldots, v_{k+1}, v_{k+3}, \ldots, v_{2k-1}, v_{2k+1}\}$ is a minimal gED-set of highest value. $\therefore \Gamma_{\text{ged}}\ (C_\sigma) \leq \frac{p}{2} + 1$.

Subcase (ii) : **k is odd**

Let, $D = \{v_1, v_3, v_5, \ldots, v_k, v_{k+2}, \ldots, v_{2k-3}, v_{2k-1}\}$ is a minimal gED-set of highest value. $\therefore \Gamma_{\text{ged}}\ (C_\sigma) \leq \frac{p}{2} + 1$.

Hence, $\Gamma_{\text{ged}}\ (C_\sigma) \leq \frac{p}{2} + 1$.

Observation 3.5. Let C_σ of order $p, |\sigma^*| = n$ be a cycle FG, then

$$\Gamma_{\text{ged}}\ (C_\sigma) \leq \{\} \begin{cases} 2, \text{ if } |\sigma^*| = 3 \\ 3, \text{ if } |\sigma^*| = 4 \\ 4, \text{ if } |\sigma^*| = 5 \end{cases}$$

Theorem 3.4. Let W_σ of order $p, |\sigma^*| = n \geq 6$ be a wheel FG, then

$$\Gamma_{\text{ged}}\ (W_\sigma) \leq \begin{cases} \frac{p}{2} + 1, |\sigma^*| = n, \text{ if } n \text{ is even} \\ \frac{p-1}{2} + 1, |\sigma^*| = n, \text{ if } n \text{ is odd} \end{cases}$$

Proof.$W_\sigma = C_\sigma + K_{\sigma_1}, |\sigma^*| = n \geq 6, |\sigma_1^*| = 1$. Let $v_1, v_2, v_3, \ldots, v_n$ be the nodes of C_σ. Suppose n is even, $D = \{v_1, v_3, v_5, \ldots, v_{n-1}\}$ is a minimal gED-set of highest value. Additionally, if n is odd, $D = \{v_1, v_3, v_5, \ldots, v_{n-2}\}$ is a minimal gED-set of highest value.

Observation 3.6. Let $W_\sigma, |\sigma^*| = n$ be a wheel FG, then

$$\Gamma_{ged}\ (W_\sigma) \leq \begin{cases} 2, |\sigma^*| = 4 \\ 3, |\sigma^*| = 5 \end{cases}$$

Result 3.1

1. Let S_σ be a star FG, then $\Gamma_{\text{ged}}\ (S_\sigma) \leq 2, |\sigma^*| = n \geq 3$

2. Let K_{σ_1,σ_2} be a complete bipartite FG, where $|\sigma_1^*| = m$ and $|\sigma_2^*| = n$, then $\Gamma_{\text{ged}}\ (K_{\sigma_1,\sigma_2}) = \sigma_{11} + \sigma_{22}$, where $\sigma_{11} = max_{u \in V_1}\ \sigma(u)$ and $\sigma_{22} = max_{v \in V_2}\ \sigma(v)$.

Theorem 3.5. If $F_\sigma, |\sigma^*| = n > 4$ is a fan FG of order p, then $\Gamma_{\text{ged}}\ (F_\sigma) \leq \frac{p}{2} + 1$.

Proof. Let, $F_\sigma = P_\sigma + K_{\sigma_1}, |\sigma_1^* = 1|$. Let $v_1, v_2, \ldots, v_n$ be the nodes of the path FG P_σ and let v_0 be the g-central node of F_σ.

Case (i): n **- even**

let, $D = \{v_1, v_4, v_6, \ldots, v_n\}$ is a minimal gED-set of highest value. $\therefore \Gamma_{\text{ged}}\ (F_\sigma) \leq \frac{p}{2} + 1$

Case (ii) : n - odd

Let, $D = \{v_1, v_3, v_5, \ldots, v_n\}$ is a minimal gED-set of highest value. $\therefore \Gamma_{\text{ged}}\ (F_\sigma) \leq \frac{p}{2} + 1$.

Theorem 3.6. Let G be a connected FG. For the corona $G \bullet K_\sigma, |\sigma^*| = 1, \Gamma_{\text{ged}}\ (G \bullet K_\sigma) \leq p$.

Proof. $G \bullet K_\sigma, |\sigma^*| = 1$ has $2n$ nodes. The minimal gED-set with the highest value is formed by the set of all dangling nodes D. A minimal D-set is formed by the set of all assist nodes S, however it is not gE. If we replace S with any other node, the resulting set is not minimal. Hence, $\Gamma_{\text{ged}}\ (G \bullet K_\sigma) \leq p, |\sigma^*| = 1$.

Corollary 3.6.1.
1. $\Gamma_{\text{ged}}\ (C_{\sigma_1} \bullet K_{\sigma_2}) \leq p, |\sigma_1^*| = n, |\sigma_2^*| = 1$.
2. $\Gamma_{\text{ged}}\ (P_{\sigma_1} \bullet K_{\sigma_2}) \leq p, |\sigma_1^*| = n, |\sigma_2^*| = 1$.

Theorem 3.7. If T_σ is a tree FG with s hanging nodes, $s \leq \Gamma(T_\sigma) = \Gamma_{\text{ged}}\ (T_\sigma)$, where s is the value of hanging nodes.

Proof. Let T_σ be a tree FG and the set of all end nodes form a SS of maximal independent set of G. Hence, $s \leq \Gamma(T)$, by observation 4.1(2). If D is a Γ - set of T_σ, D has all the hanging nodes of T_σ. The g-peripheral nodes of T_σ are the gE nodes of T_σ, they are also hanging nodes. $\therefore D$ is again a Γ_{ged} -set. $\therefore \Gamma(T_\sigma) = \Gamma_{\text{ged}}\ (T_\sigma)$.

Theorem 3.8. If T_σ is a uni central tree FG with g-radius 2 , then $\Gamma_{\text{ged}}\ (T_\sigma) \leq s+1$, s is the set of all end (hanging) nodes of T_σ.

Proof. Let T_σ be a tree FG with $r_g(G) = 2$ and $d_g(G) = 4$, s be the value(number) of hanging nodes and v is a g-central node. $\exists 2$ cases.

Case (i): The node v is not a assist node. Then $\{v\} \cup D, D$ is the family of all terminal nodes form a minimal gED-set with highest value. $\therefore \Gamma_{\text{ged}}\ (T_\sigma) = s+1$.

Case (ii) : The node v is a assist node. The family of hanging nodes forms minimal gED-set with highest value. $\therefore \Gamma_{\text{ged}}\ (T_\sigma) \leq s$.

Theorem 3.9. If T_σ is a bi central tree FG with $r_g(G) = 2$, then $\Gamma_{\text{ged}}\ (T_\sigma) \leq s$, where s is the set of all terminal nodes of T_σ.

Proof.T_σ is a tree FG with $r_g(G) = 2$ and $d_g(G) = 3$, where s is the number of hanging nodes. Then $T_\sigma = \bar{K}_{\sigma_1}, + K_{\sigma_2} + K_{\sigma_3} + \bar{K}_{\sigma_4}, |\sigma_1^*| = m, |\sigma_2^*| = 1 = \left|\sigma_3^*\right|, |\sigma_4^*| = n$, and $s = m+n$. $\therefore \Gamma_{ged}\ (T_\sigma) \leq m+n \leq s$.

Theorem 3.10. Let G be a FG of $r_g(G) = 1$ and $d_g(G) = 2$ with t-central nodes, then $k - t \leq \Gamma_{\text{ged}}\ (G)$, k be the clique value of G.

Proof. Let k is a clique value of G. Then $k \geq t$, because all the g-central nodes are in close proximity to every remaining.

Let $v_1, v_2, \ldots, v_k$ be the nodes of G, which form a clique in G. Among this let $v_1, v_2, \ldots, v_t$ be the g-central nodes of G. Let $D = \{v_{t+1}, v_{t+2}, \ldots, v_{t+k}\} \subseteq E_2 = \{v \in V(G)/e(v) = 2\}$.$D$ or $D \cup \{v_i\}, 1 \leq i \leq t$ is a minimal gED-set. $\therefore, k - t \leq \Gamma_{\text{ged}}\ (G)$.

Remark 3.1. Let $D = \{v_{t+1}, v_{t+2}, \ldots, v_{t+k}\}$. Any node of E_2 not in D is gE to few node of D. Hence, $\Gamma_{ed}(G) = k - t$ if a node of E_2 is in D or in close proximity to minimum of one node of D. Otherwise $S = D \cup \{v_i\}, (v_i\ )$ is a g-central node) is a gED-set and hence, $\Gamma_{\text{ged}}\ (G) \geq k - t + 1$.

Remark 3.2. If G is a self-centered FG of $d_g(G) = 2$ then $\gamma_{\text{ged}}\ (G) \leq 2 \Leftrightarrow G$ contains a D edge, is not in a triangle.

Remark 3.3. If G is a FG with $r_g(G) = 2, d_g(G) = 3$ three then $\gamma_{\text{ged}}\ (G) \leq 2 \Leftrightarrow G$ contains a γ-set $D = \{u, v\}, d_g(u, v) = 3$ in G and if P_σ is a path $uxyv$ joining u and v then $e_g(x) = 2 = e_g(y), e_g(u) = e_g(v) = 3$.

Theorem 3.11 Let G be a FG with $r_g(G) = 2, d_p(G) = 3$. Then $\Gamma_{ged}(G) \geq 3$.

Proof:

If $\gamma_{\text{ged}}\ (G) \geq 3$, then $\Gamma_{\text{ged}}\ (G) \geq 3$.Imagine $\left(\gamma_{gel}(G)\right) \leq 2$. Iet $D = \{u, v\}$ be a $\gamma_{g \text{ and}}$ -set Then by remark 4.4 has an independent(unique) y-set $D = \{u, v\}, d(u, v) = 3$ in G and if P_a is a path ieryp correcting u and v then $\varepsilon_g(x) = 2 = e_q(y), e_g(u) = e_q(v) = 3$. Since $\{w, v\}$ is an unique D-set of $G, \exists$ no $w \in V(G)$, which is in close proximity to both u and v. $\therefore$ an element of $N_1(x)$ is in close proximity to exactly one of u and xImagine $D_1 = |w, x| \cup N_1(x) \cup S, S$ is the st of nodes pruximity to u with $c_g(u) = 3$ - Because of $e_g(x) = 2, N_1(x)$ dominates all nods of G including v. If there are nodes in $N_1(x)$ with gE then they are in D_1.The node $v \notin D_1$ and if $\exists\ w$ in $N_2(x)$ proximity to v, it is dominated by few node of $N_1(x)$ and is gF to u and $\mu \in N_1(x)$. $\therefore D_1 = \{x\} \cup N_1(x) \cup S$. S and $N_1(x)$ may have common nodes and $N_1(x)$ also has nodes of gE 2 , which ax proximity to v Also, the set $[u, x, y\}$ is a SS of D_1. So, $\exists$ a petites gED-set W of G with lowest 3 nodes and $W \subseteq D_1$. $\therefore \Gamma_{ged}(G) \geq 3$.

4 Conclusion

In this study, using strong arc in fuzzy graphs, the following points were discussed.

i) The upper gEP-set, upper gED-set and their numbers in FG are obtained.

ii) Bounds on upper g-eccentric number and upper g-eccentric domination number for few standard FGs ate obtained.

iii) Theorems related to upper g-ecoentric point number and upper gecoentric domination number were stated and proved.

iv) In addition, we have labelled FG with $r_g(G) > 1$ for which $\Gamma_{ged}(G) \leq 2$.

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