INDIAN JOURNAL OF SCIENCE AND TECHNOLOGY

RESEARCH ARTICLE

OPEN ACCESS

Received: 29-08-2023

Accepted: 05-03-2024

Published: 25-05-2024

Editor: ICDMMMDE-2023 Special Issue Editors: Dr. G .Mahadevan and Prof. Dr. P. Balasubramaniam

Citation: Uma G, Amutha S, Anbazhagan N, Shanthi P (2024) Fractional Domination Number of the Fractal Graphs. Indian Journal of Science and Technology 17(SP1): 34-39. https://doi.org/10.17485/IJST/v17sp1.110

* **Corresponding author**.

amuthas@alagappauniversity.ac.in

Funding: RUSA Phase 2.0(F24 – 51/2014 – U), DST-FIST(SR/FIST/MS-I/2018/17), DST-PURSE 2nd Phase programme (SR/PURSE Phase 2/38), Govt. of India

Competing Interests: None

Published By Indian Society for Education and Environment (iSee)

ISSN
Print: 0974-6846
Electronic: 0974-5645

Fractional Domination Number of the Fractal Graphs

G Uma[1], S Amutha[2]*, N Anbazhagan[1], P Shanthi[1]

1 Department of Mathematics, Alagappa University, Karaikudi, Karaikudi, 630 003, Tamil Nadu
2 Ramanujan Centre for Higher Mathematics, Alagappa University, Karaikudi, 630 003, Tamil Nadu, India

Abstract

Background/Objectives: A fractal is a collection of clearly defined structures that are uneven or irregular and can be divided into smaller pieces. Each of these smaller pieces has a relationship to the overall structure at random ranges that are analogous or identical. Fractal graphs are a highly effective way to construct well-defined patterns. The fractional domination number in a graph refers to the weight of a minimum fractional dominating function. The study is to find the bounds of the fractional domination and their related parameters in the fractal graphs. **Methods:** The sharp bounds and the constant ratio of fractional domination and their related parameters were calculated using an iterative process in all iterations of a self-similarity fractal graph. **Findings:** The study established the relation between the fractional domination number and other relevant parameters as fractional independence domination numbers for each iteration in the fractal graphs and explained how the concept of fractional domination works on fractal graphs such as the Koch snowflake and the Sierpinski triangle. **Novelty:** Previous researchers have studied bounds on maximum matching and dominating sets in fractal graphs such as the Koch snowflake and the Sierpinski triangle. The present study enhances the idea of finding the bounds of fractional domination parameters in a fractal graph. The construction of new beautiful ornaments, locating damaged cells for drug injection in medical science, placing stones in jewels, plumping work, and designing fractal antennas all can be made possible by this work in self-similarity fractal graphs.

Keywords: Fractals; Fractional Dominating Functions; Fractional Domination Number; Fractional Independence Domination Number; Fractional Domination Chain

1 Introduction

Let G be a fractal graph with vertices and edges. The special characteristics of graphs, such as self-similarity, recursive function, and detailed structure at arbitrary tiny scales, are present in fractal graphs. Fractals are beautiful visuals that have people in awe. Fractals are regarded as things with well-defined self-similarity. Each of these items, regardless of how they were split or divided, retains the same characteristics and a mathematically determined structure. The Koch snowflake is one of the better-known early fractals. Koch Island or Koch Star are other names for Koch snowflake. It has been developed iteratively at various phases. Each subsequent step is created by adding outward folds to every side of the initial, equilateral triangle, resulting in a smaller triangle. In each repetition, the curve's overall length increased by 4/3. The line segment from the previous iteration is multiplied by four in each subsequent iteration. The Koch snowflake has 3×4^mvertices in its mth iteration. In classical geometry, the Sierpinski triangle is the most well-known fractal graph. Inside, it is made entirely of triangles. It consists of an overall equilateral triangle with equal lengths on each of its three sides. The inside of the structure is entirely composed of triangles. It is applied in building construction and jewelry design. An equilateral triangle can be used as a starting point for a Sierpinski triangle. Attach the center of the triangle to each side, forming another equilateral triangle. This process can be repeated to cover 3/4 of the original triangle. The remaining portion can be removed. Each side of the triangle is reduced by 1/3 of its initial length. Continuously follow these steps to create new structures at each iteration. The Sierpinski triangle has an infinite area and diameter. It consists of 3^s edges in each iteration. A function $f : V \rightarrow [0, 1]$ is called a "Fractional Dominating Functions" if, for each vertex v in the vertex set V,$\sum_{u\varepsilon N[v]} f(u) \geq 1$.The minimum weight of all other fractional dominating functions is called fractional domination number denoted by $\gamma_f(G)$ [1]. An independent function $f : V \rightarrow [0, 1]$ is called a maximal independent function (MIF) if for every $v \in V$ with f(v)=0, we have $\sum_{u\varepsilon N[v]} f(u) \geq 1$. The fractional independent domination numbers are denoted as i_f(G), which is defined as the minimum cardinality of the set {|f|: f is an MIF of G}[2]. In [3], which is based on fractal geometry and deduces some of the properties of fractal graphs, [4] investigated the effects of deviations composing the fractal of the Sierpinski triangle, and [5] presented the design and simulation of a Koch curve fractal antenna. In [6], complex networks use combinatorial methods and algorithms for studying fractal properties. The results of the effectiveness confirmation and feasibility studies of using a fractal algorithm for making stronger structures are covered in [7]. Some bounds and effects of removing a vertex on the fractional domination number are discussed in [8]. In [9], the results of bounds or the exact value of the total domination number of new graphs are obtained. Extend the result to the range on incidence graphs using the fractional total domatic numbers given in [10]. [11] proposed the idea of finding the dominating sets in the linear benzenoid graph as fractals. The maximal matching and dominating sets on fractal graphs were found in recent studies [12,13]. We assessed the bounds of fractional domination number and related parameters in the fractal graph, motivated by the development of the fractional concept.

2 Methodology

Fractional domination number of a Koch snowflake

Let K_m be the Koch snowflake consisting of 3×4^m vertices and edges, where m is the iteration. i.e., each iteration of the Koch Snowflake appears to have a self-similarity fractal structure, as shown in Figure 1 [12].

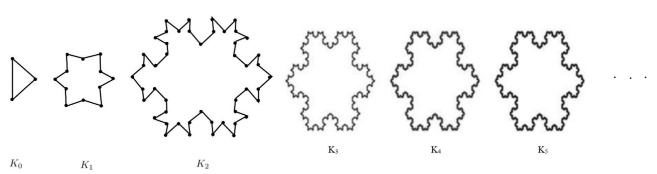

Fig 1. Koch Snowflake

Theorem2.1

The Koch snowflake consists of 3×4^m vertices and edges of each iteration. Then, it has the common ratio between the number of vertices and the cardinality number of the fractional domination number. i.e., for every iteration, the fractional domination number of the Koch snowflake, $\gamma_f(K_m) = \frac{3\times4^m}{3}$ where m≥0.

Proof

Let K_m be the Koch snowflake, which consists of 3×4^mvertices in each iteration. When $m = 0$, the initial iteration of Koch snowflake is an equilateral triangle. The value of the vertices is assigned by the definition of the fractional domination number, as shown in Figure 2.

Either way, the fractional domination number is the same for adding the weight of minimum dominating functions. i.e, $\gamma_f(K_0) = 1+0+0 = \frac{1}{3}+\frac{1}{3}+\frac{1}{3} = 1$. When m = 1, the first iteration of a Koch snowflake has $3 \times 4^1 = 12$ vertices, and the fractional domination number $\gamma_f(K_1) = \frac{3\times4^1}{3} = 4$ is calculated as shown in Figure 3 .

Hence, either way for the first iteration,

Fig 2. K_0

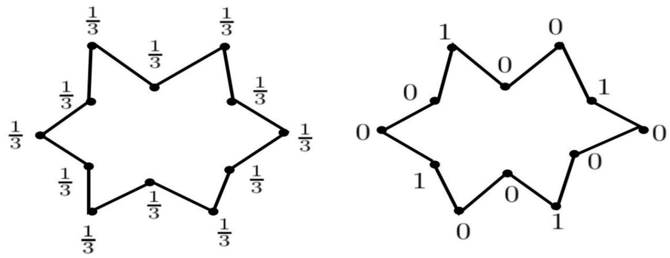

Fig 3. K_1

$\gamma_f(K_1) = 1+1+1+1+0+0+0+0+0+0+0+0 = \frac{1}{3}+\frac{1}{3}+\frac{1}{3}+\frac{1}{3}+\frac{1}{3}+\frac{1}{3}+\frac{1}{3}+\frac{1}{3}+\frac{1}{3}+\frac{1}{3}+\frac{1}{3}+\frac{1}{3} = \frac{3\times 4^1}{3} = 4.$
When m=2, the second iteration of a Koch snowflake has $3\times 4^2 = 48$ vertices and the fractal is shown in Figure 4 .

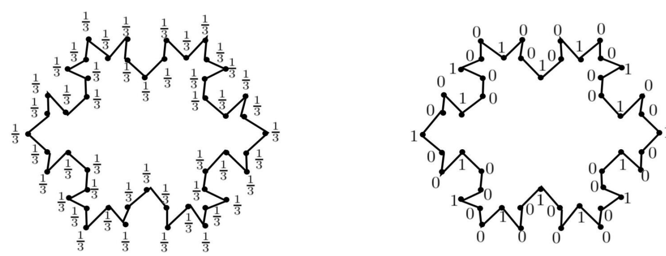

Fig 4. K_2

Therefore for the second iteration,
$\gamma_f(K_2) = \frac{1}{3}+\frac{1}{3} = 1+1+1+1+1+1+1+1+1+1+1+1+1+1+1+1+0+0+0+0+0+0+0+0+0+0+0+0+0+0+0+0+0+0+0+0+0+0+0+0+0+0+0+0+0+0+0+0 = \frac{3\times 4^2}{3} = 16.$

For every m, it can be proved by using mathematical induction method.

Step(i): The initial iteration holds for a fractional domination number of a Koch snowflake.

Step(ii): Let us assume that this theorem is true for all m=t. Assume that in the m=t^th^ iteration, the fractional domination number of Koch snowflake has $\frac{3\times 4^t}{3}$.

Step(iii): To prove this theorem for m = t+ 1. In the first component vertices of the Koch snowflake have of 3×4^t fractional domination number and the second integrant is connected with the first integrant by the weight of 4 multiple of fractional domination number by permutation. Hence, fractional domination number of m = t + 1 hexagon is $\frac{3\times 4^t\times 4}{3} = \frac{3\times 4^{t+1}}{3}$. The theorem is proved for all m. The Induction method is proved.

Remark 2.2 Similar to the above theorem, Koch snowflake has a fractional independent domination number that is the same as the fractional domination number. Hence, $\gamma_f(K_m) = i_f(K_m) = \frac{3\times 4^m}{3}$ for every m iteration.

3 Results and Discussion

Fractional Domination Number of a Sierpinski Triangle

Let T_s be the Sierpinski triangle consisting of 3^s edges where $s \geq 1$ is the iteration. i.e., each iteration of the Sierpinski triangle appears to have a self-similarity fractal structure, as shown in Figure 5 [12].

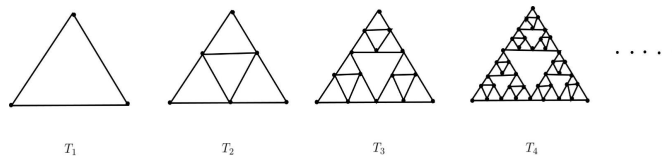

Fig 5. Sierpinski Triangle

Theorem 3.1

The Sierpinski triangle has 3^s edges where $s \geq 1$ is the iteration. Then, in the first iteration, the Sierpinski triangle of 3 edges has 1 fractional domination number; in the second iteration, 9 edges have $\frac{3}{2}$ of fractional domination number; and the next level of iterations has $3^{s-2}$where ($s \geq 3$) of fractional domination number.

i. e. $\gamma_f(T_s) = \left\{ \begin{array}{c} 1\ s = 1 \\ \frac{3}{2} s = 2 \\ 3^{s-2} s \geq 3 \end{array} \right\}$

Proof

The Sierpinski triangle has 3^s edges where s is the iteration. When s = 1 in the first iteration of the Sierpinski triangle is an equilateral triangle and it has 3 edges and 1 of fractional domination number is shown in Figure 6.

Fig 6. T_1

Hence, in both ways, $\gamma_f(T_1) = \frac{1}{3} + \frac{1}{3} + \frac{1}{3} = 1 + 0 + 0 = 1$. When s = 2, Sierpinski triangle has 3^2= 9 edges and the $\frac{3}{2}$ of fractional domination number is shown in Figure 7.

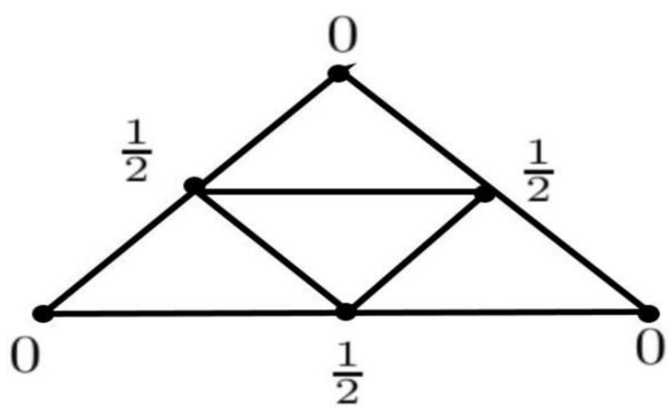

Fig 7. T_2

Hence, for the second iteration, $\gamma_f(T_2) = \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + 0 + 0 + 0 = \frac{3}{2}$.

After two iterations, the common ratio between the number of edges and the fractional domination number has generalised for every s $\geq$ 3 iterations. Hereafter, the Sierpinski triangle implies 3^s edges of the fractal graph have 3^{s-2} of fractional domination number where s $\geq$ 3. When the iteration increases and the fractional domination number increases by 3^{s-2} for every s $\geq$ 3 iterations.

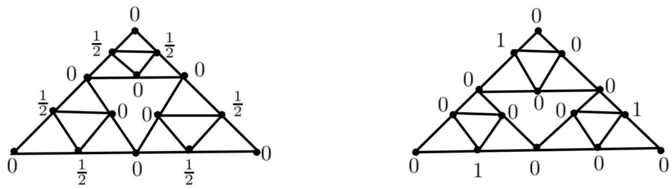

Fig 8. T_3

i.e.

$\gamma_f(T_3) = \frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+0+0+0+0+0+0+0+0+0 = 1+1+1+0+0+0+0+0+0+0+0+0+0+0+0 = 3.$

Therefore, it is true for s = 3. When s = 4, Sierpinski triangle is shown in Figure 9 has 3^4= 81 edges and $\gamma_f(T_4) = 3^{s-2} = 3^{4-2} = 3^2 = 9$.

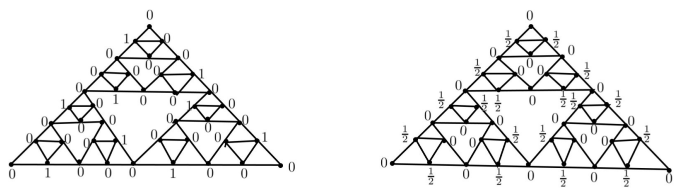

Fig 9. T_4

Hence, from the Figure 9,

$\gamma_f(T_4) = \frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+0+0+0+0+0+0+0+0+0+0+0+0+0+0+0+0+0+0+0+0+0+0+0+0 = 1+1+1+1+1+1+1+1+1+0+0+0+0+0+0+0+0+0+0+0+0+0+0+0+0+0+0+0+0+0+0+0+0+0+0+0+0+0+0+0+0+0+0+0+0+0+0 = 9.$

It is easily proved by using the mathematical induction method.

Step(i): When s = 3 iteration, the fractional domination number of $3^{3-2} = 3$ is true.

Step(ii): Let us assume that the theorem is true for all s = k. Assume that in the k^{th} iteration, the Sierpinski triangle has 3^k edges and $3^{k-2}$of fractional domination number.

Step(iii): To prove this theorem is true for s = k + 1. i.e., to prove that this Sierpinski triangle has 3^{k+1} edges and 3^{k-1} of fractional domination number. Let the Sierpinski triangle have 3^{k+1} edges. It can be divided into two integrants where one has 3^k and the other has 3 edges. Assumption of Step(ii) implies the first integrant of 3^k edges has 3^{k-2} of fractional domination and by the way of the permutation method, the second integrant is connected with the first integrant by the multiple of 3 of fractional domination number for every iteration of assigning the value to the vertices according to the definition of fractional domination number. In this way, permutation implies $3^{k-2} \times 3 = 3^{k-1}$ of fractional domination cardinality. This theorem is proved for all s.

4 Conclusion

The structural analysis of the fractal graphs using fractional domination numbers and fractional independent domination numbers was detailed in this study. It calculated generalizations and bounds for the fractional domination number in the Sierpinski triangle and Koch snow flake. Future work will involve in determining the parameter values in the fractional domination chain$ir_f(G) \leq \gamma_f(G) \leq i_f(G) \leq \beta_f(G) \leq \Gamma_f(G) \leq IR_f(G)$and generalizing it to how fractal graphs such as Koch snowflake and Sierpinski triangle work. Also to characterize the fractal graphs where the fractional domination related parameters are equal, such as$\gamma_f(G) = i_f(G)$.

Acknowledgement

This article has been written with the joint financial support of the RUSA Phase 2.0(F24 − 51/2014 − U), DST-FIST(SR/FIST/MS-I/2018/17), DST-PURSE 2nd Phase programme (SR/PURSE Phase 2/38), Govt. of India.

Declaration

Declaration Presented in 9th INTERNATIONAL CONFERENCE ON DISCRETE MATHEMATICS AND MATHEMATICAL MOD- ELLING IN DIGITAL ERA (ICDMMMDE-2023) during March 23-25, 2023, Organized by the Department of Mathematics, The Gandhigram Rural Institute (Deemed to be University), Gandhigram - 624302, Dindigul, Tamil Nadu, India. ICDMMMDE-23 was supported by GRI-DTBU, CSIR.

References

1) Shanthi P, Amutha S, Anbazhagan N, Uma G, Joshi GP, Cho W. Characterization of Fractional Mixed Domination Number of Paths and Cycles. *Journal of Mathematics*. 2024;2024:1–11. Available from: https://dx.doi.org/10.1155/2024/6619654.
2) Karunambigai MG, Sathishkumar A. Independent Function on Intuitionistic Fuzzy Graph. 2021. Available from: https://www.nveo.org/index.php/journal/article/view/4264/3511.
3) Mary CJCL, Kamali R. An Analysis of Fractals in various types of graphs. *European Chemical Bulletin*;2023(13):94–102. Available from: https://www.eurchembell.com/uploads/paper/d6c785d85ba154c3e39d87c062871526.pdf.
4) Anitas EM. Structural Properties of Molecular Sierpiński Triangle Fractals. *Nanomaterials*. 2020;10(5):925–925. Available from: https://dx.doi.org/10.3390/nano10050925.
5) Francois O, Palacios G, Erick R, Vargas D, Stakem PH, Enrique C, et al. Koch Snowflake Fractal Antenna Design in the Deep Space Bands for a Constellation of Cubesat Explorer. *Proceedings of CECNet AJ Tallón-Ballesteros*. 2021. Available from: https://ebooks.iospress.nl/pdf/doi/10.3233/FAIA210419.
6) Skums P, Bunimovich L. Graph fractal dimension and structure of fractal networks: a combinatorial perspective. *Journal of Complex Networks*;2020(4). Available from: https://arxiv.org/pdf/1912.11385.pdf.
7) Zhikharev LA. A Sierpiński triangle geometric algorithm for generating stronger structures. *Journal of Physics: Conference Series*. 2021;1901(1):012066–012066. Available from: https://dx.doi.org/10.1088/1742-6596/1901/1/012066.
8) Shanthi P, Amutha S, Anbazhagan N, Prabu SB. Effects on fractional domination in graphs. *Journal of Intelligent Fuzzy Systems*. 2023;44(5):7855–7864. Available from: https://dx.doi.org/10.3233/jifs-222999.
9) Sigarreta JM. Total Domination on Some Graph Operators. *Mathematics*. 2021;9(3):241–241. Available from: https://dx.doi.org/10.3390/math9030241.
10) Zhang Y, Zhang X. On the fractional total domatic numbers of incidence graphs. *Mathematical Modelling and Control*. 2023;3(1):73–79. Available from: https://dx.doi.org/10.3934/mmc.2023007.
11) Raji M, Jayalalitha G. Molecular Graph of Linear Benzenoid Graph as Fractals using Domination Sequence. 2021. Available from: https://www.mililink.com/upload/article/1805460295aams_vol_212_december_2021_a9_p615-619_m._raji_and_g._jayalalitha.pdf.
12) Tharaniya P, Jayalalitha G. Advances in intelligent systems and computing, Sheng- Lung peng- Rong Xia Hao, Souvik pal. *Proceedings of first international conference on mathematical modeling and computational sciences*;p. 978–981. Available from: https://doi.org/10.1007/978-981-33-4389-4_1.
13) Sathyamoorthy RD, Tharaniya P, Senthil S, Jayalalitha G. Computation of Domination Sets in Koch Snowflake. *Annals of Romanian Society for Cell Biology*. 2021;2021(6):1583–6258. Available from: https://www.annalsofrscb.ro/index.php/journal/article/view/8089/5959.