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Studying the Absorption and Fluorescence Parameters of Ho: YAP Laser Using the Judd-Oflet Model

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Abstract

Objectives: Study and find the absorption and fluorescence parameters for the Ho: YAP laser crystal, such as branching ratios, intensity parameters, oscillation strength, fluorescence line strength, absorption line strength, the spontaneous emission factor, the time life of the upper irritated level, the emission cross-section, and the absorption cross-section and prove that this laser is Ho: YAP laser. **Method:** Absorption and fluorescence parameters are calculated by employing the equations of the Judd-Oflet model and using the absorption coefficient to calculate the area under the curve. **Findings:** The properties of the compelling medium were calculated theoretically using the Judd-Oflet Model. These include branching ratios β_2 = 0.27, β_3 = 0.72, intensity parameters Ω_2=1.41*10^{-20} cm^2 Ω_4=2.91*10^{-20} cm^2, Ω_6= 1.72*10^{-20} cm^2, oscillation strength 1.48*10^{-6}, fluorescence line strength S_{f1} =0, S_{f2} = 3.24*10^{-23}, S_{f3} = 4.9*10-23, absorption line strength 3.2439*10^{-23}, the spontaneous emission factor 167 s^{-1}, and the time life of the upper irritated level 5.99 ms, the emission cross-2.239*10^{-18} cm^2, the absorption cross-section 0.92*10^{-20} cm^2, these values are evidence that the active medium to be studied is Ho: YAP. Also, the ratio of the probability of spontaneous emission to stimulated emission was calculated, which is much less than one, which indicates that the laser system operates in this medium, and a laser is emitted through it. **Novelty:** In this article, we succeeded in using a mathematical model to calculate the properties of absorption and fluorescence, which are of great importance in laser work, instead of experimental work that could expose the crystal to damage and also require money, effort, and time.

Keywords: Judd-Oflet Model; Ho: YAP laser; Absorption Parameters; Fluorescence Parameters; absorption coefficient; intensity parameters

1 Introduction

Recently, laser science has attracted more attention from researchers due to its wide usage in different scientific fields such as medical, biological, industrial, chemical applications, and others [1–3]. The laser wave has different penetration abilities depending upon the types of active medium materials [4]. As a result of the distinctive properties of the laser beam, it is used for the treatment of eyebrow microblading and cosmetic tattoo pigment. [5]. Since the crystal's unique properties are used as the active medium, we can calculate it through the Judd-Offlet Model. For example, the Judd-Ofelt parameters, the spontaneous transition rates, branching ratios and the radiative lifetime were calculated for Nd: $Ca_{0.7}La_{0.3}Mg_{0.3}Al_{11.7}O_{19}$ crystal [6]. The spectroscopic parameters and Judd - Oflet parameters have been computed using a statistical method like partial regression method [7]. The Judd -Ofelt intensity parameters are calculated to realize the ligand environment among Tb^{3+} ions in host glass [8]. Judd-Oflet theory was presented in 1962 independently by (Brian R. Judd) at the University of California (George S. Ofelt) and Johns Hopkins University. It became a basic model for calculating absorption and fluorescence parameters. It successfully calculates the physical parameters of rare earth ions in solid materials [9]. In this article, we will use the Judd-Oflet model to estimate the absorption and fluorescence parameters of the Ho: YAP laser. Ho: YAP crystal is one of the commonly used crystals in solid-state laser active medium, which belongs to rare earth ions that have good spectral properties that allow it to act as an effective medium for solid-state lasers. These properties include Einstein coefficients, upper excited plane life, pumping schemes and other parameters. A theory in physical chemistry describes the intensity of electron transfer within the 4f shell of rare earth ions in solids and solutions.

2 Theory

2.1 Calculating the strength of the absorption line:

According to the (J-O) model, the transmission force of the dipole line between the initial level (J) is given, which is written in the formula $|(s,l)J>$ and the final grade (J'), which is written in the form $|\left(s',l'\right)J'>$with the following Equation [10–12]:

$$S_{ED} = \sum_{t=2,4,6} \Omega_t \; | < (s,l)\, J||\, U^{(t)}\, ||(s',\, l'\,)J > |^2 \tag{1}$$

Where: SED: the strength of the absorption line, Ω_t: Intensity Parameters.

$\left|U^{(t)}\right|$: The numerical value of the elements of the reduced matrix was calculated by (Canoll) [13].

Note:

$$\left|U^{(t)}\right|^2_{(t=2,4,6)} = |< (s',l')J'| \left|U^{(t)}\|(s,l)J >\right|^2 \tag{2}$$

The three values of Ω_t (Ω_2, Ω4, Ω_6) represent the intensity parameters of the J-O model and reflect the symmetry of the crystal field applied to the sites of the active ions. These parameters determine the strength of the transition between any two energy levels in these active ions. The numerical values of these parameters are calculated by empirical measurements of the transition force under the influence of (E.D.). Experimentally, the absorption spectrum is measured by testing a sample of the active medium with a laser according to the J-O model. Then, each absorption beam is numerically integrated to determine the area under the curve $\int K(\lambda)d\lambda$ for each beam in the calibration of the absorption spectrum by replacing this value with the following Equation to be able to determine the value of (S_{ED}) [10]:

$$\int K(\lambda)d\lambda = \frac{8\pi^2 e^2 \lambda_p}{3\,\mathrm{ch}(2J+1)n} \frac{\left(n_r^2+2\right)^2}{9} S_{ED} \tag{3}$$

Where:

$\int K(\lambda)d\lambda$: is the area under the absorption curve and represents the absorption coefficient, λ_p: the pumping wavelength, n_r: the refractive index of the compelling medium crystal, C: the speed of light, h: Planck's constant, e: the charge of the electron.

When we replace (S_{ED}) as well as the values of the scalar array $|U^{(t)}|$, It is possible to obtain the best match with the importance of the density parameters of the model (J-O). Using these parameters' values, the laser media's spectral properties and transmission possibilities can be determined.

2.2 Spectral parameters of the Ho^{3+} ion

We can use special programs developed by (Caird) for Calculating the values of the Spectral parameters as follows [14,15]:

2.2.1 The probability of spontaneous emission A_{spo}:

The probability of moving from the initial level $|\left(s',l'\right)J'>$ to the final level $|(s,l)J>$ is given by the following relationship[8]:

$$A_{spo}\left[\left(s',l'\right)J';(s,l)J\right]=\frac{64\pi^2e^2}{3h(2J+1)\lambda^3}\frac{n_r(n^2{}_r+2)}{9}\sum_{t=2,4,6}\Omega_t\,|<(s,l)J||U^{(t)}||\left(s',l'\right)J>|^2 \tag{4}$$

$$A_{spo}\left[\left(s',l'\right)J';(s,l)J\right]=\left[\frac{n_r\left(n^2{}_r+2\right)^2}{9}\right]f_{ED} \tag{5}$$

As the term $\left[\frac{n_r\left(n^2{}_r+2\right)^2}{9}\right]$ in Equation (5) represents the correction factor for the medium hosting the Holmium ion in the initial level (J').

A_{spo}: is the Einstein spontaneous emission factor.

2.2.2 Radiation Lifetime T_f:

When (A_{spo}) represents the probability that spontaneous emission will occur; as a result of the effect of the electrode dipole compound, it is[16]:

$$T_f=\sum_{s',l',J'}A_{spo}\left[\left(s',l'\right)J';(s,l)J\right]^{-1}=A_{spo}^{-1} \tag{6}$$

Where (T_f) represents the calculated value of the radiation lifetime or the decay time of the radiation.

When we substitute the numerical values of the constants in Equation (4), we get the following Equation[4]:

$$f_{ED}*T_f=1.5*10^{14}\lambda^4{}_0\left[\frac{n_r\left(n^2{}_r+2\right)^2}{9}\right]^{-1} \tag{7}$$

Where λ_0 is the wavelength transmitted in the vacuum.

2.2.3 Stimulated Emission Probability (B_{st}):

The catalytic transition probability (B_{st}) per unit time between the two points (a,b) related to the probability of spontaneous emission (A_{spo}) for the same level through the following relationship[17]:

$$\frac{A_{spo}}{B_{st}}=\frac{(hw^3)n^3{}_r}{\pi^2c^3} \tag{8}$$

$$\frac{A_{spo}}{B_{st}}=\frac{8\pi n^3{}_r(hv^3)}{c^3} \tag{9}$$

Calculating this ratio is essential in knowing whether to generate a laser. When this ratio is more significant than one, it means that the probability of spontaneous emission occurring is greater than the probability of stimulating emission, and this is not useful in which the laser cannot be generated. However, if this percentage is less than one, the probability of the stimulated emission occurring is greater than the possibility of spontaneous emission, which is very useful and indicates the generation of the laser. It is a prerequisite for the occurrence of the laser process and must at least be less than or equal to one.

2.2.4 Stimulated Emission Cross-Section (σ_{st}):

The maximum value of the cross-sectional value of the stimulated emission of Ho^{3+} ion and the transitions $\left({}^5I_8-{}^5I_7\right)$ can be explained by the following relationship[10]:

$$\sigma_{st}(\lambda_p)=\frac{\lambda_p^4}{8\pi cn^2\triangle\lambda_{eff}}A_{spo}\,[5I_8\,;4I_7] \tag{10}$$

Since: λ_p: the peak value of the wavelength in the emission beam of the fluorescence spectrum. And $\Delta\lambda_{eff}$: the adequate bandwidth of the fluorescence. These values are determined by integrating the fluorescence beam divided by the intensity value at λ_p [18]. Also, the cross-section of the stimulated emission can be calculated using the following Equation [18]:

$$\sigma_{st} = \frac{hv}{T_f I_s} \tag{11}$$

Where: I.S. is the intensity in the saturated state.

3 Methodology

In this section, we will classify the theoretical steps used to calculate the spectral parameters of the absorption and fluorescence transitions between two levels starting from $^5I_8$. When we take the received data from any spectrophotometer and determine the values of absorption (Y-axis in cm^{-1}) and values of wavelength (X-axis in nm), several steps were applied in this model to calculate the different functions.

- **The first step:** Using the absorption spectrum shown in Figure 1 [19], we calculate the absorption coefficient of the selected beam.

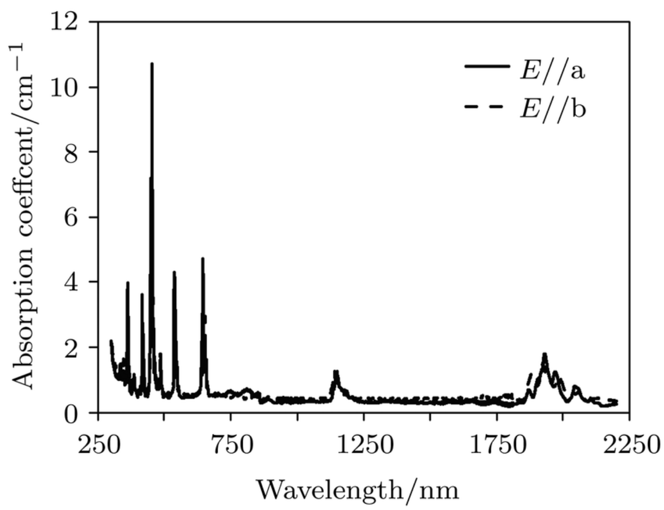

Fig 1. The characteristic absorption spectrum of the Ho: YAP laser [19]

- **The second step:** The numerical integration of the 1.91μm absorption beam was taken from the absorption spectrophotometer. Using (Simpson's rule) [14]. The area under the absorption curve $\int K(\lambda)d\lambda$ for that beam or band can be calculated by integrating the absorption spectrum. In this method, the area under the curve is divided into geometric shapes (square shapes), and the locations of these are calculated. Forms, and then adding the total rooms, we get the area under the curve.
- **The third step** is calculating the probabilities of the peak absorption cross-section and the radiative transmission from the absorption measurements. It requires knowing the density (ρ (ion/cm^3)) of the Holmium ion of the sample under study. The density can be calculated through the following Equation [16]:

$$\rho = \frac{2Dp\,NoD_d}{M} \tag{12}$$

So, N_0: Avogadro's number = (6.023*10^{23} mol^{-1}), M: molecular weight Ho = (164.93), D_p: Doping ratio = 0.01, D_d: the density of the compelling medium (gm/cm^3).

- **The fourth step**: The value of the line strength (S_{ED}) can be determined by substituting the numerical integral into the essential absorption beam $\int K(\lambda)d\lambda$ (obtained from the second step) in Equation (3).
- **The fifth step**: According to the (J-O) model, the line strength between the initial level $(s,l)\,J$ and the final level $\left(s^{'},l^{'}\right)$ can be written through Equation (1). The parameters (Ω_2, Ω_4, Ω_6) represent the intensity parameters through which the strength of the transition between any two energy levels of the active ions can be determined. by substituting the values of (S_{ED}) obtained from Equation (1) into Equation (3) as well as the elements of $|U^{(t)}|$. We compute the three values of the parameters (Ω_t), each corresponding to one value of the matrix element in the (J-O) model. These values will also be used to calculate step eight's fluorescence line strength S_f.
- **The sixth step**: Calculating the cross-section of the integrated absorption by taking the absorption between two levels under (E.D.) conditions, the line strength (S_{ED}) is related to $\int \sigma\,(v)\,dv$ as in the following equation[20–22]:

$$\int \sigma_{(v)} dv = \frac{\int K_{(v)} dv}{\rho} = \frac{8\pi^2 v}{3hcg_a}\left[\frac{\left(n^2{}_r+2\right)^2}{9} * \frac{1}{n}\right] S_{ED(ab)} \tag{13}$$

By substituting the value of S_{ED} into this equation, we have calculated this function.

- **The seventh step** is calculating the strength of oscillation (F_E.D.) by substituting the value of $\int \sigma((v)\,dv)$ in Equation (4), we can calculate the oscillation force.

$$\int \sigma_{(v)} dv = \frac{\int K_{(v)} dv}{\rho} = \frac{\pi e^2}{mc}\left[\frac{\left(n^2{}_r+2\right)^2}{9}\right] F_{ED} \tag{14}$$

- **The eighth step:** The fluorescence line strength (S_f) for the emitted wavelength (2 μm) is calculated by replacing the values of the density parameters (Ω_t) obtained from the fifth step with the numerical values of $|U^{(t)}|$ In the following equation[23–25]:

$$S_f = \Omega_t * \left[U^{(t)}\right]^2 \tag{15}$$

Note :

$$\left|U^{(t)}\right|^2 = \left| < (^5I_8 \left\| U^{(t)} \right\| (s,l) > \right|^2 \tag{16}$$

- **The ninth step:** When we replace the value of (S_f) calculated in the eighth step, as well as the refractive index of the studied sample in the fluorescence spectrum range, in Equation (4), the spontaneous emission probabilities (A_{spo}) for fluorescence transitions ($^5I_8 \rightarrow {}^5I_7$) are calculated.
- **The tenth step:** The radiation life (T_f) for the $^5I_7$ level was calculated after finding the total values of spontaneous emission probabilities by summing the inverse probabilities of all fluorescence transitions in the $^5I_7$ level and substituting them into Equation (6).
- **The eleventh step**: The branching ratio for each fluorescence transition arising from the $^5I_8$ level was calculated by substituting the spontaneous emission probabilities of the fluorescence transitions and summing the values of (A_{spo}) in the following equation:

$$\beta_r\left[\left(s^{'},l^{'}\right)J^{'};(s,l)\,J\right] = \frac{A_{spo}\left[\left(s^{'},l^{'}\right)J^{'};(s,l)\,J\right]}{\sum_{S,L,J} A_{spo}\left[\left(s^{'},l^{'}\right)J^{'};(s,l)\,J\right]} \tag{17}$$

- **The twelfth step:** Calculate the ratio (A_{spo})/B_{st}) where this value represents the ratio between the probability of spontaneous emission of any transmission and the possibility of stimulated emission, and this value was calculated by replacing the matter mentioned above for (A_{spo}) in Equation (8).

- **The thirteenth step**: In the first step, we drew the emission spectrum of the studied sample, and I/I_0 was calculated for the wavelength (1.91μm). Using (Beer-Lambert's law), the absorption coefficients of the model at these wavelengths were also calculated. The absorption cross-section can be calculated by substituting K(v) (calculated from the emission spectrum of the model) into the following equation[16,26]:

$$\sigma(v) = \frac{K(v)}{\rho} \tag{18}$$

- **The fourteenth step:** By measuring $\Delta\lambda_{eff}$ from the fluorescence spectrum and replacing this value in Equation (10) after calculating the value of (A_{spo}) from the (J-O) model, we calculate the stimulated emission cross section (σ_{st}) or using Equation (11).

4 Results and Discussion

We will list all the values we used in the above equations in Table 1 and all the values and results we obtained in Table 2.

Table 1. Input parameters through the (J –O) model

Input parameters	
n	1.5485
m_e	$9.1*10^{-31}$ kg
J_1	8
J_2	7
e	$1.6*10^{-19}$ C
C	$3*10^{10}$ cm/sec
π	3.14
h	$6.625*10^{-34}$ J sec
ρ	$64*10^{19}$ gm/cm^3
λ_p	1.91μm
λ	2 μm
N	$6.023*\ 10^{23}$ mol^{-1}
M	164.93
D.P.	0.01
D_d	8.8 gm/cm^3

Table 2. Output Parameters obtained from (J – O) model

Output parameters	
S_{ED}	$3.2439*10^{-23}$
Ω_2	$1.41*10^{-20}$ cm^2
Ω_4	$2.91*10^{-20}$ cm^2
Ω_6	$1.72*10^{-20}$ cm^2
$\int\sigma$ (v)dv	$0.12*10^{-20}$ cm^2
σ_{ab}	$0.92*10^{-20}$ cm^2
f_{ED}	$1.48*10^{-6}$
S_{f1}	0
S_{f2}	$3.24*10^{-23}$
S_{f3}	$4.9*10^{-23}$
A_{spo1}	0
A_{spo2}	46.2 s^{-1}
A_{spo3}	120.8 s^{-1}
A_{spotot}	167 s^{-1}

Continued on next page

Table 2 continued

T_f	5.99 ms
β_2	0.27
β_3	0.72
A_{spo}/B_{st}	$63.09*10^{-16}$
B_{st}	$2.64*10^{11}$
σ_{st}	$2.239*10^{-18}$ cm^2

The values and results that we obtained theoretically through the use of the (J-O) model, which is shown in Table 2, show that the laser under study is the Ho: YAP laser and this is considered a success for the Judd-Oflet model Compared to other previous studies[27,28].

5 Conclusion

In this study, we have successfully applied the Judd-Oflet model to study a rare earth laser. We calculated the fluorescence and absorption properties by applying the aforementioned theoretical model. The values calculated using the theoretical model indicated that the crystal of the active medium that is supposed to be studied is Holmium YAP. The values that were calculated are: branching ratios β_2 = 0.27, β_3 = 0.72, intensity parameters Ω_2=$1.41*10^{-20}$ cm^2 Ω_4=$2.91*10^{-20}$ cm^2, Ω_6= $1.72*10^{-20}$ cm^2, oscillation strength $1.48*10^{-6}$, fluorescence line strength S_{f1} = 0, S_{f2} = $3.24*10^{-23}$, S_{f3} = 4.9*10-23, absorption line strength $3.2439*10^{-23}$, the spontaneous emission factor 167 s^{-1}, the time life of the upper irritated level 5.99 ms, the emission cross section $2.239*10^{-18}$ cm^2, and the absorption cross section $0.92*10^{-20}$ cm^2, Also the ratio of the probability of spontaneous emission to stimulated emission was calculated, which is much less than one, which indicates that the laser system operates in this medium and a laser is emitted through it. This is considered a success for the Judd-Oflet Model, as we calculated all the optical properties theoretically without any financial cost, and this saves a lot of effort and time for researchers in calculating the properties of the Active medium or verifying the type of medium.

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