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wsalda1998@gmail.com

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A Fuzzy Inventory Model for Imperfect Process, Obsolescence and its Greenhouse Gas Emissions under Vendor Managed Inventory – Consignment Stock (VMI-CS) Policy

W S Alda[1]*, S Rexlin Jeyakumari[2]

1 Research Scholar, PG and Research Department of Mathematics, Holy Cross College (Autonomous), Affiliated to Bharathidasan University, Tiruchirappalli, 620002, Tamil Nadu, Inia

2 Assistant Professor, PG and Research Department of Mathematics, Holy Cross College (Autonomous), Affiliated to Bharathidasan University,, Tiruchirappalli, 620002, Tamil Nadu, India

Abstract

Objectives: To discuss a green inventory management system with greenhouse gas emissions, obsolescence, imperfect process, and Vendor Managed Inventory – Consignment Stock (VMI – CS) policy in a fuzzy nature by employing hexagonal fuzzy numbers to tackle uncertain situations. **Methods:** To find the optimum solution, we use two methods such as centroid method and the graded mean integration representation method for hexagonal fuzzy numbers. We use different sets of data for Marchi et al's model[(1)]and for the proposed model with obsolescence to verify the results. Here two supply chain models are proposed, where the crisp model is first discussed with total cost and optimum order quantity. As a second part, the fuzzy model is formulated with fuzzy total cost and fuzzy optimal order quantity. **Findings:** The results of our discussion represent that the solution obtained from fuzzy by using two methods is slightly the same as the solution found from the crisp model. But since five parameters with ambiguous nature are solved under fuzzy by considering them in the form of a hexagonal fuzzy number, uncertainty is faced here. To show the applicability of the proposed model, numerical examples are given. **Novelty:** Implementation of obsolescence cost and its emissions in a green inventory system and the process of considering and solving parameters under fuzzy is a unique and important feature of our study. It helps sectors to know the effects of obsolete inventory, the need to include it, and also to rectify vagueness in their specified data. Hence this study is novel in the sense it discusses the above criteria under two fuzzy techniques.

2020 Mathematics Subject Classification: 90B05

Keywords: Hexagonal fuzzy numbers; Centroid method; Graded mean integration representation method; Obsolescence cost; Greenhouse gas emissions

1 Introduction

Awareness of environmental sustainability among manufacturers has increased considerably in the last decade. Reducing environmental effects, such as energy usage and greenhouse gas (GHG) emissions, is the main focus of environmental sustainability. Energy usage rises mainly due to the imperfect process of items. Because faster production of products reduces energy usage but affects the quality of products, resulting in an imperfect process. Hence considering quality as well as environmental issues together makes the production process in a better way. In 2019, a supply chain model was examined by Marchi et al with energy consumption, greenhouse gas emissions, and imperfect processes under various coordination decisions like traditional agreement and Vendor Managed Inventory – Consignment Stock (VMI – CS) policies[1].In 2022, a sustainable inventory management model with energy consumption, imperfect production, and green investment in a closed-loop supply chain[2] is discussed by Wakhid Ahmad Jauhari. With regulations for returning defective items, a sustainable supply chain model was developed under the Vendor Managed Inventory – Consignment Stock (VMI-CS) agreement policy in 2022 by Javad Asadkhani et al[3].

As obsolescence costs constitute an essential part of carrying costs and obsolete inventory may result in substantial financial losses, there is a research scope to discuss this in green inventory models. Obsolescence costs are the charges incurred when a product or inventory loses value or becomes obsolete due to changing tastes among customers, advances in technology, or market trends. And if obsolescence costs occur in a supply chain, then subsequently it contributes to carbon emissions and waste production. Hence, considering emissions of obsolescence cost along with other emissions in a supply chain model helps to study about controllable carbon emissions in various criteria. An economic order quantity and economic production quantity model for a two-level supply chain that takes social, ecological, and economic concerns is discussed by Vahid Reza Soleymanfar et al in 2021 with the consideration of obsolescence cost function[4] and its emissions. A sustainable inventory management model[5] under controlled carbon emissions, including backorders and deterioration, is studied by Umakanta Mishra et al in 2021, where they have discussed obsolete inventory along with their cost and emission parameters.

In the discussed cases of green inventory systems by Marchi et al[1], based on the literature on obsolescence in sustainable inventory systems, we have added obsolescence cost and its emissions from Umakanta Mishra's[5] work. In addition to this, identifying the unpredictable state of each and every parameter in a particular model is essential, as the absence of accurate or reliable information regarding the inventory levels, costs, supply, or demand may result in understocking or overstocking, as well as in lost opportunities and resource waste. So to face this crucial situation, implementing fuzzy in green inventory is worthwhile. The conceptualization of including fuzzy into green inventory is performed by many researchers. Jiseong Noh and et al proposed a green supply chain model in 2019[6] with greenhouse gas emissions, considering demand as a fuzzy parameter. Combined work of green products, imperfect production, and fuzzy in an inventory model[7] was carried out by Subrata Panja et al in 2019 with the retailer's credit period as a fuzzy parameter. In 2023, an environmentally friendly supply chain model[8] is discussed for imperfect quality items by Riju Chaudhary et al with demand as a fuzzy parameter. A Fuzzy Environment-Based Sustainable Green Supply Chain Model was proposed by Osama et al in 2023 with Carbon Emissions for Defective Items under Learning[9] where they considered demand as a fuzzy parameter. Using a genetic algorithm, a sustainable supply chain model with variable production is discussed by Karthick et al[10] in 2024, where they have taken the carbon emission factor as a triangular intuitionistic fuzzy number.

1.1 Research Gap:

On analyzing all the above literature reviews, it is found that many researchers have studied imperfect processes, Vendor Managed Inventory – Consignment Stock (VMI – CS) Policy, energy usage, and obsolescence in different models under fuzzy green supply chain management for single parameters alone. But there is a research gap for models with the combination of the above different cases under fuzzy perspective for various parameters. Because of the unique need to observe and analyzing about obsolete items in this modern era, including obsolescence cost and its emission, studying about it in a model is essential.

Hence the present model works by adding obsolescence cost and its emissions from Umakanta Mishra's work [5] in a combination of supply chain model with greenhouse gas emissions, imperfect process, Vendor Managed Inventory – Consignment Stock (VMI – CS) agreement policy [1] and solves it by using two fuzzy methods such as Centroid method and Graded Mean Integration Representation method to remove uncertainty in various parameters and to find optimum fuzzy total cost and optimum fuzzy order quantity. Hence this model works in a way to fill the research gap of finding the solution for uncertainty under the above different cases. Also, the results are compared.

2 Methodology

The approach of the model comprises the definitions, its assumptions, and notations utilized in the creation of both crisp and fuzzy models. Our present work describes a green inventory management system with a specific idea of obsolescence and the introduction of fuzziness into the parameters for demand, cost of maintaining items to restore a machine at the breakdown stage, setup cost, ordering cost, and emission tax as hexagonal fuzzy numbers. We solve this model under two fuzzy methodologies and analyze how the considered fuzzy parameters in the green inventory model create results with different fuzzy approaches.

The prevalent methodology for defuzzification is the centroid method. This approach utilizes the center of gravity to transform fuzzy numbers into precise values. And also, the graded mean integration representation method is an efficient and beneficial methodology as it enables the use of fuzzy more conveniently and acceptably while also reducing the difficulty and monotony of the original membership function's huge operation. Hence, in our model, while solving the crisp model in terms of fuzzy, we use the centroid method for hexagonal fuzzy numbers, which is derived from the Maheswari et al [11] model, and the graded mean integration representation method for hexagonal fuzzy numbers, which is referred to from the Theo et al [12] model.

2.1 Definitions

2.1.1 Fuzzy Set:

A fuzzy set $\widetilde{A}$ defined on a Universe of discourse X may be written as a collection of ordered pairs, $\widetilde{A} = \left\{(x, \mu_{\widetilde{A}(x)): \mathrm{x} \in X, \mu_{\widetilde{A} \in (0,1]}}\right\}$, where each pair $(x, \mu_{\widetilde{A}}(x))$ is called a singleton and the element $\mu_{\widetilde{A}}(x)$ belongs to the interval$(0,1]$.The function $\mu_{\widetilde{A}}(x)$ is called a membership function.

2.1.2 Hexagonal Fuzzy Number:

A fuzzy number $\widetilde{A} = (a,b,c,d,e,f)$ where $a < b < c < d < e < f$are defined on R is called hexagonal fuzzy number if its membership function is

$$\mu_{\hat{\lambda}}(x) = \begin{cases} \frac{(x-a)}{2(b-a)}, & a \le x \le b \\ \frac{1}{2} + \frac{(x-b)}{2(c-d)}, & b \le x \le c \\ 1, & \mathrm{c} \le x \le d \\ 1 - \frac{(x-d)}{2(e-d)}, & d \le x \le e \\ \frac{(f-x)}{2(f-c)}, & \mathrm{e} \le x \le f \\ 0 & \text{, Otherwise} \end{cases}$$

2.1.3 Fuzzy Arithmetical Operations:

Some of the fuzzy arithmetical operations for hexagonal fuzzy numbers under function principle are as follows,

Let us assume $\widetilde{A} = (a_1, a_2, a_3, a_4, a_5, a_6)$ and $\widetilde{B} = (b_1, b_2, b_3, b_4, b_5, b_6)$ as two hexagonal fuzzy numbers. Then

(i) The addition of $\widetilde{A}$ and $\widetilde{B}$ is $\widetilde{A} + \widetilde{B} = (a_1+b_1, a_2+b_2, a_3+b_3, a_4+b_4, a_5+b_5, a_6+b_6)$, where a_1, a_2, a_3, a_4, a_5, a_6, b_1, b_2, b_3, b_4, b_5 $and\, b_6$ are any real numbers.

(ii) The multiplication of $\widetilde{A}$ and $\widetilde{B}$ is $\widetilde{A} \times \widetilde{B} = (a_1 b_1,\, a_2 b_2,\, a_3 b_3,\, a_4 b_4,\, a_5 b_5,\, a_6 b_6)$,
where $a_1, a_2, a_3, a_4, a_5, a_6, b_1, b_2, b_3, b_4, b_5\, and\, b_6$ are any real numbers.

(iii) The subtraction of $\widetilde{A}$ and $\widetilde{B}$ is $\widetilde{A} - \widetilde{B} = (a_1 - b_6,\, a_2 - b_5,\, a_3 - b_4,\, a_4 - b_3,\, a_5 - b_2,\, a_6 - b_1)$,where $-\widetilde{B} = (-b_6, -b_5, -b_4, -b_3, -b_2, -b_1)$,also$a_1, a_2, a_3, a_4, a_5, a_6, b_1, b_2, b_3, b_4, b_5\, and\, b_6$ are any real numbers.

(iv) The division of $\widetilde{A}$ and $\widetilde{B}$ is $\frac{\widetilde{A}}{\widetilde{B}} = \left(\frac{a_1}{b_6}, \frac{a_2}{b_5}, \frac{a_3}{b_4}, \frac{a_4}{b_3}, \frac{a_5}{b_2}, \frac{a_6}{b_1}\right)$, where $\frac{1}{\widetilde{B}} = \widetilde{B}^{-1} = \left(\frac{1}{b_6}, \frac{1}{b_5}, \frac{1}{b_4}, \frac{1}{b_3}, \frac{1}{b_2}, \frac{1}{b_1}\right)$, $b_1, b_2, b_3, b_4, b_5\, and\, b_6$are positive real numbers. Also $a_1, a_2, a_3, a_4, a_5, a_6, b_1, b_2, b_3, b_4, b_5\, and\, b_6$ are any real numbers.

(v) For any$\alpha \in R$,

a) If $\alpha \geq 0$, then$\alpha \times \widetilde{A} = (\alpha a_1, \alpha a_2, \alpha a_3, \alpha a_4, \alpha a_5, \alpha a_6)$.

b) If $\alpha < 0$, then $\alpha \times \widetilde{A} = (\alpha a_6, \alpha a_5, \alpha a_4, \alpha a_3, \alpha a_2, \alpha a_1)$.

2.1.4 Obsolescence Cost:

It is the costs that represent the expenses for inventory items becoming expired, losing their value because of customer choice of preferred items and because of trending market behaviour.

2.2 Two Fuzzy Methods:

2.2.1 Centroid Method:

If $\widetilde{B} = (b_1, b_2, b_3, b_4, b_5, b_6)$ is a hexagonal fuzzy number, then the centroid method from $\widetilde{B}$ to $\tilde{0}$ is defined as,

$$d\left(\widetilde{B},\tilde{0}\right) = \int_0^1 d\left(\left(B_L(\alpha)\right], \left(B_R(\alpha)\right], \tilde{0}\right) d\alpha$$

$$= \frac{(b_1 + b_2 + b_3 + b_4 + b_5 + b_6)}{6}.$$

2.2.2 Graded Mean Integration Representation Method:

If $\widetilde{C} = (c_1,\, c_2,\, c_3,\, c_4,\, c_5,\, c_6)$ is a hexagonal fuzzy number, then the formula for graded mean integration representation method is given by,

$$d_G\left(\widetilde{C}\right) = \frac{c_1 + 3c_2 + 2c_3 + 2c_4 + 3c_5 + c_6}{12}.$$

2.3 Assumptions:

- Time horizon for a single product is infinite.
- The failure rate of defective products and equipment depends on the production rate.
- Demand is not a constant value and higher than the rate of production.
- Obsolete inventory is added to the model.

2.4 Notations:

2.4.1 Crisp Parameters

- α_p— Percentage of damaged items at the out of control state of machine.
- O—Cost of ordering.
- f_a—first parameter in the emission function for manufacturing process.
- f_{ar}—first parameter in the emission function for reworking process.
- β_p—Quality function first non-negative parameter for its estimation.
- f_b— Second parameter in the emission function for manufacturing process.
- f_{br}— Second parameter in the emission function for reworking process.
- f_c— Third parameter in the emission function for manufacturing process.
- f_{cr}— Third parameter in the emission function for reworking process.
- t_{ec}— Tax of emission.
- S_c— Cost of screening items.
- D—Demand.
- G—Emission of greenhouse gases from the manufacturing process.
- G_r—Emission of greenhouse gases from the reworking process.
- G_{tr}—Emissions generated by transporting products.
- e_g—Amount of CO_2 emissions from truck for diesel.

- ν_r—Parameter that relates the rate of production and reworking process.
- g_t—Number of gallons per travelled distance per truck.
- η_t—Count of trucks per trip.
- P_b—Physical component for the cost of holding items from buyer's approach.
- f_v—Financial component for the cost of holding items from vendor's side.
- P_v—Physical component for the cost of holding items from vendor's side.
- λ_r—Reliability function parameter.
- C_m—Cost of maintaining items to restore a machine at the breakdown stage.
- V_r—Cost of vendor's reworking process.
- C_s—Setup cost.
- T_c—Capacity of truck.
- w_q—Quality function second non-negative parameter for its estimation.
- $E(N)$—Expected number of damaged products.
- n_s—Number of shipments.
- O_W—Weight of obsolete inventory.
- O_r— Obsolescence rate of inventory in percentage.
- O_e— average carbon emission of obsolescence inventory.
- α— Period length of positive inventory level.
- t—Cycle time of Inventory.
- OCE—Emissions generated from obsolescence.
- OCT—Obsolescence total cost.
- l_W—Obsolete items weight in warehouse in kg per unit.
- S_P—Selling price per unit.
- S_e—Scrap estimated value.
- $S_P - S_e$— Expense of disposing and collection of products in storage those are obsolete or discarded.
- P—Rate of production.
- Q^*—Order Quantity.
- TC—Total cost.

2.4.2 Fuzzy Parameters:

- $\widetilde{D}$—Fuzzy Demand
- $\widetilde{C}_m$—Fuzzy Cost of maintaining items to restore a machine at the breakdown stage.
- $\widetilde{C}_s$—Fuzzy Setup cost.
- $\widetilde{O}$— Fuzzy Cost of ordering.
- $\tilde{t}_{ec}$— Tax of emission in fuzzy.
- $\widetilde{Q}^*$—Fuzzy Order Quantity.
- $d\left(T\widetilde{C}\right)$—Fuzzy Total cost using Centroid Method.
- $d_G\left(T\widetilde{C}\right)$—Fuzzy Total cost using Graded Mean Integration Representation Method.

2.5 Supply chain model with carbon emissions:

The considered model is a supply chain model under energy usage, GHG emissions, and imperfect process production under the VMI-CS agreement policy.

Due to faster production, at certain instant production process results in producing defective items. Hence the expected number of defective items is given by,

$E(N) = \alpha_p\left(\beta_p + \omega_q P^2\right) \times \frac{(n_s Q)^2}{2P}$.

Under energy requirements, the rate of reworking of products is given by,

$$V_r = \nu_r P.$$

Apart from the above cases, on observing the effects of obsolete items obsolescence cost is included in the model. Hence Obsolescence total cost is given by,

$OCT = \frac{(S_P - S_e) l_W \alpha D}{2}$.

The greenhouse gas emissions from the production, and reworking process, because of the movement of products through transport and from obsolescence cost is given by,

$G = f_a P^2 - f_b P + f_c$

$G_r = f_{ar} V_r^2 - f_{br} V_r + f_{cr}$

$G_{tr} = n_s \eta_t \times \frac{D}{n_s Q} \times g_t e_g$

–.

$OCE = \frac{O_W O_r O_e t_{ec} \alpha^2 Dt}{2}$.

Including the above factors, the total cost under VMI - CS agreement policy with the addition of Obsolescence emissions is given by,

$$TC = \frac{C_s D}{n_s Q} + \frac{OD}{Q} + (f_v + P_v) . \frac{Q}{2} . \frac{D}{P} + (f_v + P_b) \times \frac{Q}{2} \times \left(\left(1 - \frac{D}{P} \right) n_s + \frac{D}{P} \right] + S_c D +$$

$$t_{ec} \left(DG + G_{tr} + E(N) G_r \times \frac{D}{n_s Q} \right) + C_m \lambda_r . \frac{D}{P} + \frac{O_W O_r O_e t_{ec} \alpha^2 Dt}{2}.$$

After substituting the parameters G, G_{tr}, $E(N)$ and G_r with their expressions, we have the total cost as,

$$TC = \frac{C_s D}{n_s Q} + \frac{OD}{Q} + (f_v + P_v) . \frac{Q}{2} . \frac{D}{P} + (f_v + P_b) \times \frac{Q}{2} \times \left(\left(1 - \frac{D}{P} \right) n_s + \frac{D}{P} \right] + S_c D +$$

$$t_{ec} \left(D \left(f_a P^2 - f_b P + f_c \right) + \frac{D g_t e_g}{T_c} + \left(\frac{\alpha_p \left(\beta_p + \omega_q P^2 \right) \times n_s Q}{2P} \right) \times D \left(f_{ar} (\nu_r P)^2 - f_{br} (\nu_r P) + f_{cr} \right) \right)$$

$$+ C_m \lambda_r . \frac{D}{P} + \frac{O_W O_r O_e t_{ec} \alpha^2 Dt}{2}. \tag{2.5a}$$

Differentiating (2.5a) with respect to Q and equating to zero gives the Optimum Order Quantity and it is given by,

$$\varrho^* = \sqrt{\frac{2 \left[\frac{C_s}{n_s} + O \right] D}{\left[\begin{array}{l} \frac{(f_v + P_v) \cdot D}{P} + (f_v + P_b) \times \left[\left(1 - \frac{D}{P} \right) n_s + \frac{D}{P} \right] + \\ t_{ec} \left[\frac{\alpha_p (\beta_p + \omega_q P^2) \times \left(f_{ar} (v_r P)^2 - f_{br} (v_r P) + f_{cr} \right) \times n_s D}{p} \right] \end{array} \right]}}$$

2.6 Fuzzy Green Inventory Model:

In the considered supply chain model with carbon emissions, several cost parameters and emission parameters are taken as hexagonal fuzzy numbers to remove vagueness by applying two fuzzy techniques and comparing them.

Suppose $\widetilde{D} = (d_1, d_2, d_3, d_4, d_5, d_6)$

$\widetilde{C}_m = \left(C_{m_1}, C_{m_2}, C_{m_3}, C_{m_4}, C_{m_5}, C_{m_6} \right)$

$\widetilde{C}_s = \left(C_{s_1}, C_{s_2}, C_{s_3}, C_{s_4}, C_{s_5}, C_{s_6} \right)$

$\widetilde{O} = (O_1, O_2, O_3, O_4, O_5, O_6)$

$\tilde{t}_{ec} = \left(t_{ec_1}, t_{ec_2}, t_{ec_3}, t_{ec_4}, t_{ec_5}, t_{ec_6} \right)$

are taken as non – negative hexagonal fuzzy numbers, then the fuzzy total cost is given by,

$$T\widetilde{C} = \frac{\widetilde{C}_s \widetilde{D}}{n_s Q} + \frac{\widetilde{O}\widetilde{D}}{Q} + (f_v + P_v) . \frac{Q}{2} . \frac{\widetilde{D}}{P} + (f_v + P_b) \times \frac{Q}{2} \times \left(\left(1 - \frac{\widetilde{D}}{P} \right) n_s + \frac{\widetilde{D}}{P} \right] + S_c \widetilde{D} +$$

$$\tilde{t}_{ec} \left(\widetilde{D} \left(f_a P^2 - f_b P + f_c \right) + \frac{\widetilde{D} g_t e_g}{T_c} + \left(\frac{\alpha_p \left(\beta_p + \omega_q P^2 \right) \times n_s Q}{2P} \right) \times \widetilde{D} \left(f_{ar} (\nu_r P)^2 - f_{br} (\nu_r P) + f_{cr} \right) \right)$$

$$+\widetilde{C}_m\lambda_r\cdot\frac{\widetilde{D}}{P}+\frac{O_WO_rO_e\tilde{t}_{ec}\alpha^2\widetilde{D}t}{2}.$$

$$\left[\frac{C_{s_1}d_1}{n_sQ}+\frac{O_1d_1}{Q}+(f_v+P_v)\cdot\frac{Q}{2}\cdot\frac{d_1}{P}+(f_v+P_b)\times\frac{Q}{2}\times\left[\left(1-\frac{d_1}{P}\right)n_s+\frac{d_1}{P}\right]+S_cd_1+\right.$$
$$t_{cc_1}\left(d_1\left(f_aP^2-f_bP+f_c\right)+\frac{d_1g_te_g}{T_c}+\left(\frac{\alpha_p\left(\beta_p+\omega_qP^2\right)\times n_sQ}{2P}\right)\times d_1\left(f_{ar}\left(v_rP\right)^2-f_{br}\left(v_rP\right)+f_{cr}\right)\right)$$
$$+C_{m_1}\lambda_r\cdot\frac{d_1}{P}+\frac{O_WO_rO_et_{c_1}\alpha^2d_1t}{2},$$
$$\frac{C_{s_2}d_2}{n_sQ}+\frac{O_2d_2}{Q}+(f_v+P_v)\cdot\frac{Q}{2}\cdot\frac{d_2}{P}+(f_v+P_b)\times\frac{Q}{2}\times\left[\left(1-\frac{d_2}{P}\right)n_s+\frac{d_2}{P}\right]+S_cd_2+$$
$$t_{ec_2}\left(d_2\left(f_aP^2-f_bP+f_c\right)+\frac{d_2g_te_g}{T_c}+\left(\frac{\alpha_p\left(\beta_p+\omega_qP^2\right)\times n_sQ}{2P}\right)\times d_2\left(f_{ar}(vP)^2-f_{br}(vP)+f_{cr}\right)\right)$$
$$+C_{m_2}\lambda_r\cdot\frac{d_2}{P}+\frac{O_WO_rO_et_{ec_x}\alpha^2d_2t}{2},$$
$$\frac{C_{s_s}d_3}{n_sQ}+\frac{O_3d_3}{Q}+(f_v+P_v)\cdot\frac{Q}{2}\cdot\frac{d_3}{P}+(f_v+P_b)\times\frac{Q}{2}\times\left[\left(1-\frac{d_3}{P}\right)n_s+\frac{d_3}{P}\right]+S_cd_3+$$
$$t_{\text{ecs}}\left(d_3\left(f_aP^2-f_bP+f_c\right)+\frac{d_3g_te_g}{T_c}+\left(\frac{\alpha_p\left(\beta_p+\omega_qP^2\right)\times n_sQ}{2P}\right)\times d_3\left(f_{ar}(v,P)^2-f_{br}(vP)+f_{cr}\right)\right)$$
$$+C_{m_2}\lambda_r\cdot\tfrac{d_3}{P}+\tfrac{O_WO_rO_et_{ec_3}\alpha^2d_3t}{2},$$
$$+\tfrac{O_4d_4}{Q}+(f_v+P_v)\cdot\tfrac{Q}{2}\cdot\tfrac{d_4}{P}+(f_v+P_b)\times\tfrac{Q}{2}\times\left[\left(1-\tfrac{d_4}{P}\right)n_s+\tfrac{d_4}{P}\right]+S_cd_4+$$
$$t_{ec_4}\left(d_4\left(f_aP^2-f_bP+f_c\right)+\frac{d_4g_te_g}{T_c}+\left(\frac{\alpha_p\left(\beta_p+\omega_qP^2\right)\times n_sQ}{2P}\right)\times d_4\left(f_{ar}(vP)^2-f_{br}(vP)+f_{cr}\right)\right)$$
$$+C_{m_4}\lambda_r\cdot\frac{d_4}{P}+\frac{O_WO_rO_et_{c_t}\alpha^2d_4t}{2},$$
$$\frac{C_{s_s}d_5}{n_sQ}+\frac{O_sd_5}{Q}+(f_v+P_v)\cdot\frac{Q}{2}\cdot\frac{d_5}{P}+(f_v+P_b)\times\frac{Q}{2}\times\left[\left(1-\frac{d_5}{P}\right)n_s+\frac{d_5}{P}\right]+S_cd_5+$$
$$t_{ec_s}\left(d_5\left(f_aP^2-f_bP+f_c\right)+\frac{d_sg_te_g}{T_c}+\left(\frac{\alpha_p\left(\beta_p+\omega_qP^2\right)\times n_sQ}{2P}\right)\times d_5\left(f_{ar}\left(v_rP\right)^2-f_{br}\left(v_rP\right)+f_{cr}\right)\right)$$
$$+C_{m_s}\lambda_r\cdot\frac{d_s}{P}+\frac{O_WO_rO_et_{ecs}\alpha^2d_st}{2},$$
$$\frac{C_{s_6}d_6}{n_sQ}+\frac{O_6d_6}{Q}+(f_v+P_v)\cdot\frac{Q}{2}\cdot\frac{d_6}{P}+(f_v+P_b)\times\frac{Q}{2}\times\left[\left(1-\frac{d_6}{P}\right)n_s+\frac{d_6}{P}\right]+S_cd_6+$$
$$t_{ec_c}\left(d_6\left(f_aP^2-f_bP+f_c\right)+\frac{d_6g_te_g}{T_c}+\left(\frac{\alpha_p\left(\beta_p+\omega_qP^2\right)\times n_sQ}{2P}\right)\times d_6\left(f_{ar}\left(v_rP\right)^2-f_{br}\left(v_tP\right)+f_{cr}\right)\right)$$
$$+C_{m_6}\lambda_r\cdot\frac{d_6}{P}+\frac{O_WO_rO_et_{cc_6}\alpha^2d_6t}{2}.$$

2.6.1 By using Centroid Method:

By applying Centroid Method we arrive at the fuzzy total cost,

$$d\left(T\tilde{C}\right)=\frac{1}{6}\begin{bmatrix}
\begin{bmatrix}\frac{C_{s_1}d_1}{n_sQ}+\frac{O_1d_1}{Q}+\left(f_v+P_v\right).\frac{Q}{2}.\frac{d_1}{P}+\left(f_v+P_b\right)\times\frac{Q}{2}\times\left[\left(1-\frac{d_1}{P}\right)n_s+\frac{d_1}{P}\right]+S_cd_1+\\
t_{ec_1}\left(d_1\left(f_aP^2-f_bP+f_c\right)+\frac{d_1g_te_g}{T_c}+\left(\frac{\alpha_p\left(\beta_p+\omega_qP^2\right)\times n_sQ}{2P}\right)\times d_1\left(f_{ar}\left(\nu_rP\right)^2-f_{br}\left(\nu_rP\right)+f_{cr}\right)\right)\\
+C_{m_1}\lambda_r.\frac{d_1}{P}+\frac{O_WO_rO_et_{ec_1}\alpha^2d_1t}{2}\end{bmatrix}+\\
\begin{bmatrix}\frac{C_{s_2}d_2}{n_sQ}+\frac{O_2d_2}{Q}+\left(f_v+P_v\right).\frac{Q}{2}.\frac{d_2}{P}+\left(f_v+P_b\right)\times\frac{Q}{2}\times\left[\left(1-\frac{d_2}{P}\right)n_s+\frac{d_2}{P}\right]+S_cd_2+\\
t_{ec_2}\left(d_2\left(f_aP^2-f_bP+f_c\right)+\frac{d_2g_te_g}{T_c}+\left(\frac{\alpha_p\left(\beta_p+\omega_qP^2\right)\times n_sQ}{2P}\right)\times d_2\left(f_{ar}\left(\nu_rP\right)^2-f_{br}\left(\nu_rP\right)+f_{cr}\right)\right)\\
+C_{m_2}\lambda_r.\frac{d_2}{P}+\frac{O_WO_rO_et_{ec_2}\alpha^2d_2t}{2}\end{bmatrix}+\\
\begin{bmatrix}\frac{C_{s_3}d_3}{n_sQ}+\frac{O_3d_3}{Q}+\left(f_v+P_v\right).\frac{Q}{2}.\frac{d_3}{P}+\left(f_v+P_b\right)\times\frac{Q}{2}\times\left[\left(1-\frac{d_3}{P}\right)n_s+\frac{d_3}{P}\right]+S_cd_3+\\
t_{ec_3}\left(d_3\left(f_aP^2-f_bP+f_c\right)+\frac{d_3g_te_g}{T_c}+\left(\frac{\alpha_p\left(\beta_p+\omega_qP^2\right)\times n_sQ}{2P}\right)\times d_3\left(f_{ar}\left(\nu_rP\right)^2-f_{br}\left(\nu_rP\right)+f_{cr}\right)\right)\\
+C_{m_3}\lambda_r.\frac{d_3}{P}+\frac{O_WO_rO_et_{ec_3}\alpha^2d_3t}{2}\end{bmatrix}+\\
\begin{bmatrix}\frac{C_{s_4}d_4}{n_sQ}+\frac{O_4d_4}{Q}+\left(f_v+P_v\right).\frac{Q}{2}.\frac{d_4}{P}+\left(f_v+P_b\right)\times\frac{Q}{2}\times\left[\left(1-\frac{d_4}{P}\right)n_s+\frac{d_4}{P}\right]+S_cd_4+\\
t_{ec_4}\left(d_4\left(f_aP^2-f_bP+f_c\right)+\frac{d_4g_te_g}{T_c}+\left(\frac{\alpha_p\left(\beta_p+\omega_qP^2\right)\times n_sQ}{2P}\right)\times d_4\left(f_{ar}\left(\nu_rP\right)^2-f_{br}\left(\nu_rP\right)+f_{cr}\right)\right)\\
+C_{m_4}\lambda_r.\frac{d_4}{P}+\frac{O_WO_rO_et_{ec_4}\alpha^2d_4t}{2}\end{bmatrix}+\\
\begin{bmatrix}\frac{C_{s_5}d_5}{n_sQ}+\frac{O_5d_5}{Q}+\left(f_v+P_v\right).\frac{Q}{2}.\frac{d_5}{P}+\left(f_v+P_b\right)\times\frac{Q}{2}\times\left[\left(1-\frac{d_5}{P}\right)n_s+\frac{d_5}{P}\right]+S_cd_5+\\
t_{ec_5}\left(d_5\left(f_aP^2-f_bP+f_c\right)+\frac{d_5g_te_g}{T_c}+\left(\frac{\alpha_p\left(\beta_p+\omega_qP^2\right)\times n_sQ}{2P}\right)\times d_5\left(f_{ar}\left(\nu_rP\right)^2-f_{br}\left(\nu_rP\right)+f_{cr}\right)\right)\\
+C_{m_5}\lambda_r.\frac{d_5}{P}+\frac{O_WO_rO_et_{ec_5}\alpha^2d_5t}{2}\end{bmatrix}+\\
\begin{bmatrix}\frac{C_{s_6}d_6}{n_sQ}+\frac{O_6d_6}{Q}+\left(f_v+P_v\right).\frac{Q}{2}.\frac{d_6}{P}+\left(f_v+P_b\right)\times\frac{Q}{2}\times\left[\left(1-\frac{d_6}{P}\right)n_s+\frac{d_6}{P}\right]+S_cd_6+\\
t_{ec_6}\left(d_6\left(f_aP^2-f_bP+f_c\right)+\frac{d_6g_te_g}{T_c}+\left(\frac{\alpha_p\left(\beta_p+\omega_qP^2\right)\times n_sQ}{2P}\right)\times d_6\left(f_{ar}\left(\nu_rP\right)^2-f_{br}\left(\nu_rP\right)+f_{cr}\right)\right)\\
+C_{m_6}\lambda_r.\frac{d_6}{P}+\frac{O_WO_rO_et_{ec_6}\alpha^2d_6t}{2}.\end{bmatrix}
\end{bmatrix}$$

———————————————-(2.6.1a)

Differentiating (2.6.1a) with respect to Q and equating to zero we obtain the required fuzzy optimum order quantity by using centroid method and it is given by,

$$\tilde{Q}^{*}=\sqrt{\frac{\left[2\left(\left(\frac{C_{s_1}}{n_s}+O_1\right)d_1+\left(\frac{C_{s_2}}{n_s}+O_2\right)d_2+\left(\frac{C_{s_3}}{n_s}+O_3\right)d_3+\left(\frac{C_{s_4}}{n_s}+O_4\right)d_4+\left(\frac{C_{s_5}}{n_s}+O_5\right)d_5+\left(\frac{C_{s_6}}{n_s}+O_6\right)d_6\right)\right]}{\left[\begin{array}{l}\frac{(f_v+P_v)}{P}(d_1+d_2+d_3+d_4+d_5+d_6)+(f_v+P_b)\left[\begin{array}{l}\left(\left(1-\frac{d_1}{P}\right)n_s+\frac{d_1}{P}\right)+\left(\left(1-\frac{d_2}{P}\right)n_s+\frac{d_2}{P}\right)+\left(\left(1-\frac{d_3}{P}\right)n_s+\frac{d_3}{P}\right)+\\ \left(\left(1-\frac{d_4}{P}\right)n_s+\frac{d_4}{P}\right)+\left(\left(1-\frac{d_5}{P}\right)n_s+\frac{d_5}{P}\right)+\left(\left(1-\frac{d_6}{P}\right)n_s+\frac{d_6}{P}\right)\end{array}\right]\\ +\left(\frac{\alpha_p\left(\beta_p+\omega_q P^2\right)\times n_s\times\left(f_{ar}\left(v_r P\right)^2-f_{br}\left(v_r P\right)+f_{cr}\right)}{P}\right)\times\left(t_{ec_1}d_1+t_{ec_2}d_2+t_{ec_3}d_3+t_{ec_4}d_4+t_{ec_5}d_5+t_{ec_6}d_6\right)\end{array}\right]}}$$

2.6.2 By using Graded Mean Integration Representation Method:

By applying graded mean integration representation method, we arrive at the fuzzy total cost,

$$d_G\left(T\tilde{C}\right)=\frac{1}{12}\left[\begin{array}{l}
\left[\begin{array}{l}\frac{C_{s_1}d_1}{n_sQ}+\frac{O_1d_1}{Q}+(f_v+P_v).\frac{Q}{2}.\frac{d_1}{P}+(f_v+P_b)\times\frac{Q}{2}\times\left[\left(1-\frac{d_1}{P}\right)n_s+\frac{d_1}{P}\right]+S_cd_1+\\
t_{ec_1}\left(d_1\left(f_aP^2-f_bP+f_c\right)+\frac{d_1g_te_g}{T_c}+\left(\frac{\alpha_p\left(\beta_p+\omega_qP^2\right)\times n_sQ}{2P}\right)\times d_1\left(f_{ar}\left(v_rP\right)^2-f_{br}\left(v_rP\right)+f_{cr}\right)\right)\\
+C_{m_1}\lambda_r.\frac{d_1}{P}+\frac{O_WO_rO_et_{ec_1}\alpha^2d_1t}{2}\end{array}\right]+\\
3\left[\begin{array}{l}\frac{C_{s_2}d_2}{n_sQ}+\frac{O_2d_2}{Q}+(f_v+P_v).\frac{Q}{2}.\frac{d_2}{P}+(f_v+P_b)\times\frac{Q}{2}\times\left[\left(1-\frac{d_2}{P}\right)n_s+\frac{d_2}{P}\right]+S_cd_2+\\
t_{ec_2}\left(d_2\left(f_aP^2-f_bP+f_c\right)+\frac{d_2g_te_g}{T_c}+\left(\frac{\alpha_p\left(\beta_p+\omega_qP^2\right)\times n_sQ}{2P}\right)\times d_2\left(f_{ar}\left(v_rP\right)^2-f_{br}\left(v_rP\right)+f_{cr}\right)\right)\\
+C_{m_2}\lambda_r.\frac{d_2}{P}+\frac{O_WO_rO_et_{ec_2}\alpha^2d_2t}{2}\end{array}\right]+\\
2\left[\begin{array}{l}\frac{C_{s_3}d_3}{n_sQ}+\frac{O_3d_3}{Q}+(f_v+P_v).\frac{Q}{2}.\frac{d_3}{P}+(f_v+P_b)\times\frac{Q}{2}\times\left[\left(1-\frac{d_3}{P}\right)n_s+\frac{d_3}{P}\right]+S_cd_3+\\
t_{ec_3}\left(d_3\left(f_aP^2-f_bP+f_c\right)+\frac{d_3g_te_g}{T_c}+\left(\frac{\alpha_p\left(\beta_p+\omega_qP^2\right)\times n_sQ}{2P}\right)\times d_3\left(f_{ar}\left(v_rP\right)^2-f_{br}\left(v_rP\right)+f_{cr}\right)\right)\\
+C_{m_3}\lambda_r.\frac{d_3}{P}+\frac{O_WO_rO_et_{ec_3}\alpha^2d_3t}{2}\end{array}\right]+\\
2\left[\begin{array}{l}\frac{C_{s_4}d_4}{n_sQ}+\frac{O_4d_4}{Q}+(f_v+P_v).\frac{Q}{2}.\frac{d_4}{P}+(f_v+P_b)\times\frac{Q}{2}\times\left[\left(1-\frac{d_4}{P}\right)n_s+\frac{d_4}{P}\right]+S_cd_4+\\
t_{ec_4}\left(d_4\left(f_aP^2-f_bP+f_c\right)+\frac{d_4g_te_g}{T_c}+\left(\frac{\alpha_p\left(\beta_p+\omega_qP^2\right)\times n_sQ}{2P}\right)\times d_4\left(f_{ar}\left(v_rP\right)^2-f_{br}\left(v_rP\right)+f_{cr}\right)\right)\\
+C_{m_4}\lambda_r.\frac{d_4}{P}+\frac{O_WO_rO_et_{ec_4}\alpha^2d_4t}{2}\end{array}\right]+\\
3\left[\begin{array}{l}\frac{C_{s_5}d_5}{n_sQ}+\frac{O_5d_5}{Q}+(f_v+P_v).\frac{Q}{2}.\frac{d_5}{P}+(f_v+P_b)\times\frac{Q}{2}\times\left[\left(1-\frac{d_5}{P}\right)n_s+\frac{d_5}{P}\right]+S_cd_5+\\
t_{ec_5}\left(d_5\left(f_aP^2-f_bP+f_c\right)+\frac{d_5g_te_g}{T_c}+\left(\frac{\alpha_p\left(\beta_p+\omega_qP^2\right)\times n_sQ}{2P}\right)\times d_5\left(f_{ar}\left(v_rP\right)^2-f_{br}\left(v_rP\right)+f_{cr}\right)\right)\\
+C_{m_5}\lambda_r.\frac{d_5}{P}+\frac{O_WO_rO_et_{ec_5}\alpha^2d_5t}{2}\end{array}\right]+\\
\left[\begin{array}{l}\frac{C_{s_6}d_6}{n_sQ}+\frac{O_6d_6}{Q}+(f_v+P_v).\frac{Q}{2}.\frac{d_6}{P}+(f_v+P_b)\times\frac{Q}{2}\times\left[\left(1-\frac{d_6}{P}\right)n_s+\frac{d_6}{P}\right]+S_cd_6+\\
t_{ec_6}\left(d_6\left(f_aP^2-f_bP+f_c\right)+\frac{d_6g_te_g}{T_c}+\left(\frac{\alpha_p\left(\beta_p+\omega_qP^2\right)\times n_sQ}{2P}\right)\times d_6\left(f_{ar}\left(v_rP\right)^2-f_{br}\left(v_rP\right)+f_{cr}\right)\right)\\
+C_{m_6}\lambda_r.\frac{d_6}{P}+\frac{O_WO_rO_et_{ec_6}\alpha^2d_6t}{2}.\end{array}\right]
\end{array}\right]$$

———————————————-(2.6.2a)

Differentiating (2.6.2a) with respect to Q and equating to zero we obtain the required fuzzy optimum order quantity by using graded mean integration representation method and it is given by,

$$\tilde{Q}^{*}=\sqrt{\frac{\left[2\left(\left(\frac{C_{s_1}}{n_s}+O_1\right)d_1+3\left(\frac{C_{s_2}}{n_s}+O_2\right)d_2+2\left(\frac{C_{s_3}}{n_s}+O_3\right)d_3+2\left(\frac{C_{s_4}}{n_s}+O_4\right)d_4+3\left(\frac{C_{s_5}}{n_s}+O_5\right)d_5+\left(\frac{C_{s_6}}{n_s}+O_6\right)d_6\right)\right]}{\left[\begin{array}{l}\frac{(f_v+P_v)}{P}\left(d_1+3d_2+2d_3+2d_4+3d_5+d_6\right)+\left(f_v+P_b\right)\left[\begin{array}{l}\left(\left(1-\frac{d_1}{P}\right)n_s+\frac{d_1}{P}\right)+3\left(\left(1-\frac{d_2}{P}\right)n_s+\frac{d_2}{P}\right)+2\left(\left(1-\frac{d_3}{P}\right)n_s+\frac{d_3}{P}\right)+\\ 2\left(\left(1-\frac{d_4}{P}\right)n_s+\frac{d_4}{P}\right)+3\left(\left(1-\frac{d_5}{P}\right)n_s+\frac{d_5}{P}\right)+\left(\left(1-\frac{d_6}{P}\right)n_s+\frac{d_6}{P}\right)\end{array}\right]\\ +\left(\frac{\alpha_p\left(\beta_p+\omega_q P^2\right)\times n_s\times\left(f_{ar}\left(v_r P\right)^2-f_{br}\left(v_r P\right)+f_{cr}\right)}{P}\right)\times\left(t_{ec_1}d_1+3t_{ec_2}d_2+2t_{ec_3}d_3+2t_{ec_4}d_4+3t_{ec_5}d_5+t_{ec_6}d_6\right)\end{array}\right]}}$$

3 Results and Discussion

3.1 Numerical Analysis

To illustrate our work, we have given numerical analysis for both crisp green inventory system and fuzzy green inventory system. To solve this model and to verify the results, we use different values apart from Marchi et al [1] data and Umakanta Mishra et al [5] data for obsolescence and its emissions. The assumed values are given below.

3.1.1 Crisp Sense:

The corresponding assumed values for the parameters in the considered green supply chain model are as follows:

$C_s=1100$, $n_s=2$, $O=300$, $D=1000$, $P_v=54$, $P=2000$, $f_v=4$,$t=1.68$. $P_b=10$, $t_{ec}=17$, $\alpha_p=20\%=0.2$, $\beta_p=0.25$, $\omega_q=10^{-6}$, $f_{ar}=8.33\times10^{-7}$, $\nu_r=1.1$, $f_{br}=0.001$, $f_{cr}=1.3$, $S_c=0.4$,$f_a=3\times10^{-7}$, $f_b=0.0011$, $f_c=1.3$, $g_t=275$, $e_g=0.01008413$,$T_c=90$, $C_m=900$, $\lambda_r=0.65$, $O_W=0.4$, $O_r=0.6$, $O_e=0.4$, $\alpha=1$,

By using these values Order Quantity and Total cost for the crisp green inventory system is calculated and it is given by,

Order Quantity:

$$Q^{*}=133.5931906.$$

Total Cost:

$$TC=20412.39401.$$

3.1.2 Fuzzy Sense:

Depending on the set of data taken, as in the crisp green inventory system, the values of fuzzy parameters, which are taken as hexagonal fuzzy numbers, are given below.

$$\widetilde{D}=(999.97,999.98,999.99,1000.01,1000.02,1000.03)$$

$$\widetilde{C}_s=(1099.97,1099.98,1099.99,1100.01,1100.02,1100.03)$$

$$\widetilde{O}=(299.97,299.98,299.99,300.01,300.02,300.03)$$

$$\tilde{t}_{ec}=(16.97,16.98,16.99,17.01,17.02,17.03)$$

$$\widetilde{C}_m = (899.97, 899.98, 899.99, 900.01, 900.02, 900.03)$$

Using these values, the fuzzy order quantity and fuzzy total cost is calculated.

Fuzzy Order Quantity by using Centroid Method:

$$\widetilde{Q}^* = 133.5931898.$$

Fuzzy Total Cost by using Centroid Method:

$$d\left(T\widetilde{C}\right) = 20412.39429.$$

Fuzzy Order Quantity by using Graded Mean Integration Representation Method:

$$\widetilde{Q}^* = 133.59319.$$

Fuzzy Total Cost by using Graded Mean Integration Representation Method:

$$d_G(T\widetilde{C}) = 20412.39424.$$

Table 1. Comparison of Two Fuzzy Methods with the C risp Model and Marchi et al Model

Optimal values	Crisp		Fuzzy	
	Marchi et al model	Model extended from Marchi et al with obsolescence	Centroid Method	Graded Mean Integration Representation Method
EOQ	133.5931906	133.5931906	133.5931898	133.59319
Total Cost	19041.51401	20412.39401	20412.39429	20412.39424

3.2 Comparison

An examined study of the developed model under a crisp and fuzzy system is shown in Table 1. In the Marchi et al [1] model, authors have determined the optimum order quantity and total cost for the green inventory system with an imperfect process and VMI – CS Policy. While in our model we have provided the optimum order quantity and total cost with the additional specification of obsolescence cost, whose value is 1370.88. Though the result of adding obsolescence increases expenses, this model provides a better outcome of analyzing an idea of obsolete inventory in a green supply chain system with its emission parameter O_e.

Our results in fuzzy perspective specify that economic order quantity and total cost obtained from two fuzzy methods, centroid and graded mean integration representation method, give slightly the same results, indicating the similarity of the two methods. However, the uncertainty in the parameters D, C_m, C_s, O and t_{ec} are tackled in a fuzzy approach by considering hexagonal fuzzy numbers with 0.01difference for their coordinates. Hence, in terms of uncertainty, both fuzzy methods provide an efficient result. Also, our work is unique in the sense that several researchers analyzed green inventory systems with fuzzy by considering only one parameter as a fuzzy parameter [6–10]but here w[e] discussed fuzziness for five parameters.

4 Conclusion

Mostly industries prompting to develop sustainability in their respective firms that are facing obstacles in the form of imperfect processes and obsolescence seek better results and ideas to face this issue. And usually in the real world, the demand and cost of maintaining, setting, and ordering products cannot be specifically determined in a day–to– day life. Hence, indeterminacy exists in the green inventory system even for tax, which is allocated for the emission of carbon. So, in this model, we have provided a way to remove such indeterminacy in certain parameters under a fuzzy perspective and have represented fuzziness in those parameters by using hexagonal fuzzy numbers. A fuzzy total cost and fuzzy optimum order quantity are obtained by using two fuzzy methods that provide slightly the same results. But an extension is made in our work with a comparison of two fuzzy approaches compared to Marchi et al's model[1] and would help sectors implement this in their respective fields to remove vagueness.

The present study can also be extended by including some other additional criteria that are mostly needed in the current scenario of green product production. Also, this study can be extended by applying several fuzzy methodologies and discussing uncertainty in parameters with various fuzzy numbers.

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