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Adjacency Sequence and Adjacency Spectrum of Power Fuzzy Graphs

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Abstract

Objectives: To find the adjacency sequence, spectrum of the power fuzzy graphs and discuss its properties. **Methods:** The spectrum and energy of the power fuzzy graphs are derived using the adjacency matrices. Energy of the power fuzzy graph is computed by adding the absolute eigenvalues of the adjacency matrix. **Findings:** The condition for a power fuzzy graph, ${G_f}^k$ to be vertex regular in terms of adjacency sequence has been verified. The energy sequence for the power fuzzy graph with increasing k has been established. **Novelty:** The concept of minimizing the interval of the edge membership values when the powers of fuzzy graph increase makes difference in the spectrum and energy of fuzzy graphs with the same initial structure. The concept of adjacency matrix, spectrum and energy of power fuzzy graph is applied in power supply load distribution system.

Keywords: Fuzzy graph; Power fuzzy graph; Adjacency sequence; Spectrum; Energy

1 Introduction

Fuzzy sets, initially introduced by Lotfi Zadeh in 1965 [1] were the groundwork and fundamental inspiration to develop the theory of fuzzy graphs by Azriel Rosenfeld in 1975 [2]. Graphs are essential tools in network theory and mathematics because they provide an organised method of representing interactions between entities. Specifically, fuzzy graphs having variable degrees of intensity between vertices can help to find the capacity and represent systems with ambiguous connections. An expansion of the fuzzy graph model, a power fuzzy graph is designed to use the strength of connections between nodes

The adjacency sequence for fuzzy graph was defined and categorized by Nagoor Gani, A., and K. Radha. in 2008 [3]. Further, adjacency sequence is defined and discussed for extensions of fuzzy graphs such as intuitionistic fuzzy graph [4], bipolar fuzzy graph [5], etc. The adjacency matrix, spectrum and energy of fuzzy graph were defined and explored by Narayanan, A., and Mathew, S. in 2013 [6]. More results based on the adjacency matrices [7], spectral properties [8] and energy [9,10], of fuzzy graphs were analyzed. Similarly for intuitionistic fuzzy graph [11], bipolar fuzzy graph [12] and picture fuzzy graph [13] the spectrum and energy are defined and discussed. Other than adjacency sequence, Laplacian sequence, α, β, γ- sequences for fuzzy graphs are discussed by Mathew, J., and Mathew, S[14]. This motivates to define and discuss the problem of finding adjacency sequences and adjacency spectrum of power fuzzy graph. A useful tool for modelling and analysing complex systems, like power distribution networks, can be done by power fuzzy graphs, using the ideas adjacency sequences, adjacency matrices, eigenvalues, and graph energy. The stability of network links was measured using edge connectivity of fuzzy graphs[15]. While the connections between the nodes increase the power load fluctuations, damaged places can be verified and rectified, and also able to allocate the power between nodes efficiently, by these concepts. The required basic definitions are given in section 2. The results based on adjacency sequence, Laplacian matrix of power fuzzy graph and conclusion are given in section 3 and 4 respectively.

2 Methodology

The concept of finding adjacency sequence of vertices and spectrum of eigen values in power fuzzy graphs using the method of vertex adjacency of the fuzzy graph. The study follows the definitions.

2.1 Definition[3]

The adjacency sequence of a vertex v in a fuzzy graph G_f is defined as the sequence of membership values of edges adjacent to v arranged in increasing order. It is denoted as $as(v)$.

2.2 Definition[6]

Let $G_f = (V, E, \sigma, \mu)$ be a fuzzy graph with vertices $v_i \in V$ where $i = 1 \; to \; n$. The adjacency matrix $A(G_f)$ is an $n \times n$ matrix defined as $A\left(G_f\right) = [a_{ij}]$ where $a_{ij} = \begin{cases} \mu(v_i, v_j), \; if \; i \neq j \\ 0, \; if \; i = j \end{cases}$.

2.3 Example

Consider a fuzzy graph G_f with 7 vertices and 11 edges in Figure 1. The adjacency matrix is

$$A\left(G_f\right) = \begin{bmatrix} 0 & 0.6 & 0.1 & 0 & 0 & 0.4 & 0 \\ 0.6 & 0 & 0 & 0.3 & 0 & 0 & 0.4 \\ 0.1 & 0 & 0 & 0 & 0.5 & 0.3 & 0 \\ 0 & 0.3 & 0 & 0 & 0.2 & 0 & 0.1 \\ 0 & 0 & 0.5 & 0.2 & 0 & 0.6 & 0.5 \\ 0.4 & 0 & 0.3 & 0 & 0.6 & 0 & 0 \\ 0 & 0.4 & 0 & 0.1 & 0.5 & 0 & 0 \end{bmatrix}$$

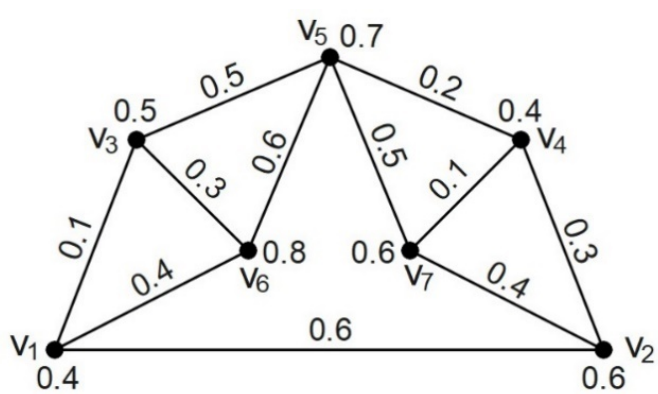


Fig 1. Fuzzy graph of a Moser spindle graph

2.4 Definition [16]

Let $G_f = (V, E, \sigma, \mu)$ be a fuzzy graph. The size of G_f is defined as $size(G_f) = \sum_{(v_1, v_2) \in V \times V} \mu(v_1, v_2)$.

2.5 Definition [6]

The set of eigenvalues $\{\lambda_1, \lambda_2, ..., \lambda_n\}$ of a fuzzy graph G_f is called the spectrum of G_f. The sum of absolute values of eigenvalues is called the energy of G_f.

2.6 Example

The spectrum and energy of the fuzzy graph in Figure 1 are $\{-0.9103, -0.8311, -0.2855, -0.08811, 0.2897, 0.5953, 1.23\}$ and 4.23 respectively.

2.7 Definition [17]

Let $G_f = (V, E, \sigma, \mu)$ be a fuzzy graph of a graph $G = (V, E)$. The power fuzzy graph of G_f is defined as ${G_f}^k = (V, E_k, \sigma, \mu_k)$ where $E_k = E \cup E^*$, for any non-adjacent vertices $u, v \in V$ in G_f there exists $uv \in E^*$ such that $l(u, v) \leq k$ where l is the length from u to v and $k > 1$ provided $min(\sigma(u)^k, \sigma(v)^k) \leq \mu_k(u, v) \leq min(\sigma(u)^{k-1}, \sigma(v)^{k-1})$.

2.8 Definition [18]

Let ${G_f}^k = (V, E_k, \sigma, \mu_k)$ be power fuzzy graph. If $deg_{{G_f}^k}(v) = p$ for all $v \in V$, that is, if each vertex has the same degree p in G_f, then ${G_f}^k$ is said to be regular. Similarly, if $tdeg_{{G_f}^k}(v) = q$ for all $v \in V$, that is, if each vertex has the same total degree q in G_f, then ${G_f}^k$ is said to be totally regular.

2.9 Note

The adjacency matrix of the connected power fuzzy graph is symmetric and the spectrum of the matrix is real.

2.10 Theorem [19]

Given an $n \times n$ complex matrix $A = [a_{ij}]$, for each i construct a Gershgorin disc $D(a_{ii}, R_i) = \{z \in C\ : |z - a_{ii}| \leq R_i\}$, where $R_i = \sum_{i \neq j} |a_{ij}|$. Then every eigenvalue of the matrix A lies within at least one of the $D(a_{ii}, R_i)$.

2.11 Theorem [20]

Let M be a symmetric matrix of size $n \times n$, and let $\lambda_1(M) \leq \lambda_2(M) \leq \cdots \leq \lambda_n(M)$ be the ordered eigenvalues of M. Then the $i-th$ eigenvalue of A is $\lambda_i(M) = \underset{dim(S_p) = i}{min} \ \underset{x \in S_p}{max} \ \frac{x^T M x}{x^T x}$, where S_p is a subspace of dimension i such that $1 \leq i \leq n$, and $x \in S_p$ is a non-zero vector.

3 Results and Discussion

3.1 Properties Based on Adjacency Sequence

3.1.1 Theorem

A power fuzzy graph ${G_f}^k = (V, E_k, \sigma, \mu_k)$ is vertex regular, if the number of elements in the adjacency sequence of all vertices is even (or odd) and $as(v_i)$ is constant but the converse is not true.

Proof: Assume ${G_f}^k$ is not a regular power fuzzy graph. Consider two vertices v_1 and v_2 with even (or odd) number of adjacent vertices such that $\sum as(v_1) = p_1 \neq \sum as(v_2) = p_2$. Then adjacency sequences $as(v_1)$ and $as(v_2)$ are different. Conversely, assume ${G_f}^k$ is a regular power fuzzy graph. Suppose there exist vertices v_1 and v_2 with even (or odd) number of adjacent vertices with different adjacency sequences $as(v_1)$ and $as(v_1)$ respectively. Then $\sum as(v_1) = p_1 \neq \sum as(v_2) = p_2$. This contradicts the assumption. Hence ${G_f}^k$ is regular.

3.1.2 Example

Consider the 8-cycle power fuzzy graph ${G_f}^1 = (V, E_1, \sigma, \mu_1)$ in Figure 2**(a)** with 2 elements in adjacency sequence such that $as(v_i) = \{0.1, 0.1\}$ for $i = 1\ to\ 8$, ${G_f}^2 = (V, E_2, \sigma, \mu_2)$Figure 2**(b)** with 4 elements in adjacency sequence such that $as(v_i) = (0.3, 0.3, 0.3, 0.3\}$ for $i = 1$ to 8, ${G_f}^3 = (V, E_3, \sigma, \mu_3)$ in the Figure 2**(c)** where $\mu_3(v_i, v_j) = 0.07, \forall (v_i, v_j) \in E_3$ with 6 elements in adjacency sequence such that $as(v_i) = (0.07, 0.07, 0.07, 0.07, 0.07, 0.07\}$ for $i = 1$ to 8, and ${G_f}^4 = (V, E_4, \sigma, \mu_4)$ in the Figure Figure 2 **(d)** where $\mu_3(v_i, v_j) = 0.026, \forall (v_i, v_j) \in E_4$ with 7 elements in adjacency sequence such that $as(v_i) = \{0.026, 0.026, 0.026, 0.026, 0.026, 0.026, 0.026\}$ for $i = 1$ to 8. Here G_f^1, G_f^2, G_f^3 and ${G_f}^4$ are vertex regular.

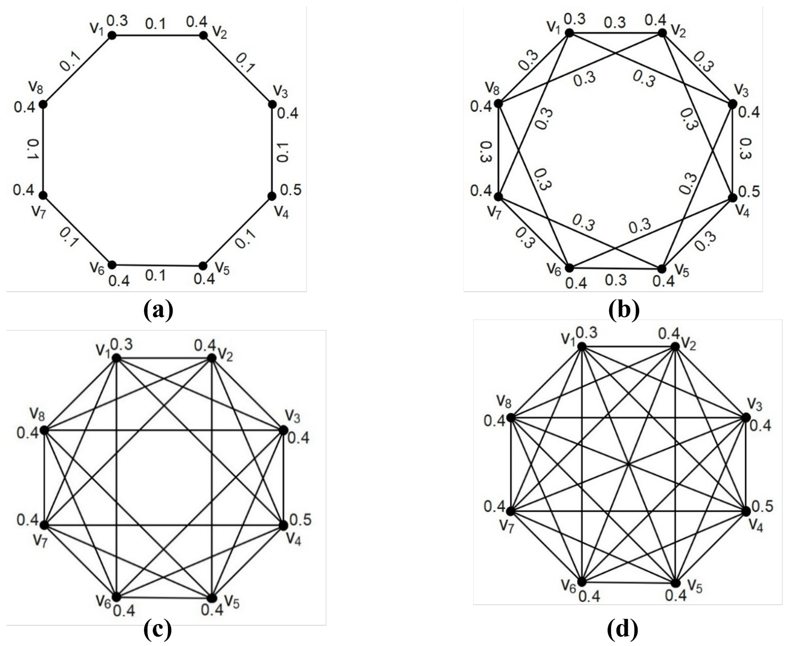


Fig 2. (a) Regular power fuzzy graph ${G_f}^1$ (b) Regular power fuzzy graph ${G_f}^2$ (c) Regular power fuzzy graph ${G_f}^3$(d) Regular power fuzzy graph ${G_f}^4$

3.1.3 Example

In the Figure 3**(a)**, $as(v_7) \neq as(v_i)\ i = 1\ to\ 6$. Consider the vertex regular power fuzzy graphs ${G_f}^1$ and ${G_f}^2$ in Figure 2**(a)** and Figure 2 **(b)**, such that $deg(v_i) = 0.15$ and $deg(v_i) = 0.24$ for $i = 1$ to 7 respectively. Here, $as(v_1) \neq as(v_7)$ in both ${G_f}^1$ and ${G_f}^2$. Hence $as(v_i), \forall v_i \in V$ is not constant.

Table 1. Adjacency Sequence of fan graph ${G_f}^1$

Vertices	Adjacency Sequence
v_1	$as(v_1) = (0.025, 0.125)$
v_2	$as(v_2) = (0.025, 0.125)$
v_3	$as(v_3) = (0.025, 0.125)$
v_4	$as(v_4) = (0.025, 0.125)$
v_5	$as(v_5) = (0.025, 0.125)$
v_6	$as(v_6) = (0.025, 0.125)$
v_7	$as(v_7) = (0.025, 0.025, 0.025, 0.025, 0.025, 0.025)$

3.1.4 Theorem

A total power fuzzy graph ${T_f}^k = (V, E_k, \sigma, \mu_k)$ is vertex regular if and only if the adjacency sequence of all vertices is constant.

Table 2. Adjacency Sequence of fan graph $G_f{}^2$

Vertices	Adjacency Sequence
v_1	$as(v_1) = (0.04, 0.1, 0.1)$
v_2	$as(v_2) = (0.04, 0.1, 0.1)$
v_3	$as(v_3) = (0.04, 0.1, 0.1)$
v_4	$as(v_4) = (0.04, 0.1, 0.1)$
v_5	$as(v_5) = (0.04, 0.1, 0.1)$
v_6	$as(v_6) = (0.04, 0.1, 0.1)$
v_7	$as(v_7) = (0.04, 0.04, 0.04, 0.04, 0.04, 0.04)$

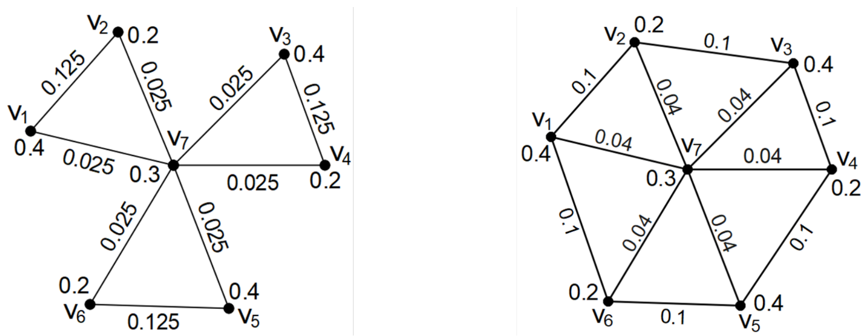
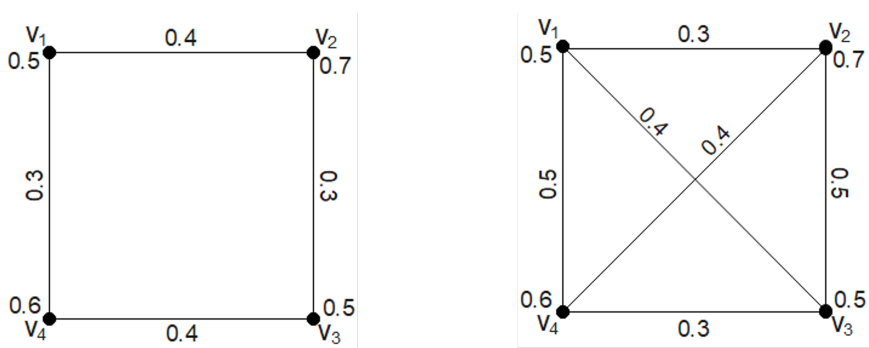


Fig 3. (a) Power fuzzy graph $G_f{}^1$ of fan graph (b) Power fuzzy graph $G_f{}^2$ of fan graph

Proof: Assume that $T_f{}^k = (V, E_k, \sigma, \mu_k)$ is regular with n vertices then for each vertex there exists $n-1$ incident edges since it is a crisp complete graph. Let $as(v) = (em_1, em_2, \ldots, em_{n-1})$ be the sequence of a vertex v. Suppose there exists a vertex w with another sequence such that an edge membership value em_{n_0} is either $em_{n_0} < em_i$ or $em_{n_0} > em_i$ for any $i = 1\ to\ n-1$, then $d(v) < d(w)$ or $d(v) > d(w)$ respectively. This leads to the contradiction of our assumption. Hence $as(v)$ is constant for all v. Conversely, let us assume that all the vertices in $T_f{}^k$ have the same adjacency sequence $as(u) \forall u \in V$. Then $\sum as(u) = p \forall u \in V$. Hence $T_f{}^k$ is regular.

3.1.5 Example

In Figure 4 **(b)**, the total power fuzzy graph has $as(v_i) = 0.3, 0.4, 0.5 \forall\ v_i$ where $i = 1, 2, 3, 4$ and it is vertex regular since $\sum as(v_i) = 1.2$.



(a) **(b)**

Fig 4. (a)Power fuzzy graph $G_f{}^1$ (b) Power fuzzy graph $G_f{}^2$

3.1.6 Note

Adjacency sequence $as(v_i)$ is not constant if the power fuzzy graph is totally regular.

3.1.7 Remark

If power fuzzy graph is edge regular then adjacency sequence $as(v_i)$ can either be constant or not.

3.2 Properties based on adjacency spectrum

3.2.1 Theorem

The sum of elements of the adjacent matrix of power fuzzy graph is twice the size of the power fuzzy graph.

Proof: Let ${G_f}^k$ be the power fuzzy graph and $A\left({G_f}^k\right)$ be the corresponding adjacency matrix of ${G_f}^k$. Let sum of elements in any adjacency matrix M be $-$ and sum of edge membership values of any power fuzzy graph be $\sum_{i<j} \mu_k(v_i, v_j),\ i \neq j$.

$$\implies Sum\left(A\left(G_f^k\right)\right) = 2\sum_{i \neq j} a_{ij}\ (\text{since}\ a_{ij} = a_{ji}\ \&\ a_{ii} = 0)$$

By the Definition 2.2, $a_{ij} = \begin{cases} \mu_k(v_i, v_j), & if\ i \neq j \\ 0, & if\ i = j \end{cases}$, we get $\sum_{i\neq j} \mu_k(v_i, v_j) = \sum_{i \neq j} a_{ij}$.

$$\implies Sum\left(A\left({G_f}^k\right)\right) = 2\sum_{i \neq j} \mu_k(v_i, v_j)$$

This implies, $Sum\left(A\left({G_f}^k\right)\right) = 2size$ (from the Definition 2.4).

3.2.2 Example

Let ${G_f}^1 = (V, E_1, \sigma, \mu_1)$, ${G_f}^2 = (V, E_2, \sigma, \mu_2)$, ${G_f}^3 = (V, E_3, \sigma, \mu_3)$ and ${G_f}^4 = (V, E_4, \sigma, \mu_4)$ be the 5-path power fuzzy graphs in Figures 5a, 5b, 5c and 5d respectively. The size of ${G_f}^1$, ${G_f}^2$, ${G_f}^3$ and ${G_f}^4$ are 1.1, 1.29, 0.789 and 0.506 respectively. Then the adjacency matrices are

$$A\left(G_f^1\right) = \begin{bmatrix} 0 & 0.3 & 0 & 0 & 0 \\ 0.3 & 0 & 0.1 & 0 & 0 \\ 0 & 0.1 & 0 & 0.2 & 0 \\ 0 & 0 & 0.2 & 0 & 0.5 \\ 0 & 0 & 0 & 0.5 & 0 \end{bmatrix},$$

$$A\left(G_f^2\right) = \begin{bmatrix} 0 & 0.3 & 0.09 & 0 & 0 \\ 0.3 & 0 & 0.15 & 0.2 & 0 \\ 0.09 & 0.15 & 0 & 0.05 & 0.1 \\ 0 & 0.2 & 0.05 & 0 & 0.4 \\ 0 & 0 & 0.1 & 0.4 & 0 \end{bmatrix},$$

$$A\left(G_f^3\right) = \begin{bmatrix} 0 & 0.08 & 0.03 & 0.3 & 0 \\ 0.08 & 0 & 0.009 & 0.1 & 0.09 \\ 0.03 & 0.009 & 0 & 0.02 & 0.01 \\ 0.3 & 0.1 & 0.02 & 0 & 0.15 \\ 0 & 0.09 & 0.01 & 0.15 & 0 \end{bmatrix},$$

$$A\left(G_f^4\right) = \begin{bmatrix} 0 & 0.05 & 0.003 & 0.15 & 0.1 \\ 0.05 & 0 & 0.006 & 0.03 & 0.06 \\ 0.003 & 0.006 & 0 & 0.005 & 0.002 \\ 0.15 & 0.03 & 0.005 & 0 & 0.1 \\ 0.1 & 0.06 & 0.002 & 0.1 & 0 \end{bmatrix}.$$

From the adjacency matrices we get, sum of elements of the matrices $A\left({G_f}^1\right)$, $A\left({G_f}^2\right)$, $A\left({G_f}^3\right)$ and $A\left({G_f}^4\right)$ as 2.2, 2.58, 1.578, and 1.012 respectively. This implies $Sum\left(A\left({G_f}^k\right)\right) = 2size$.

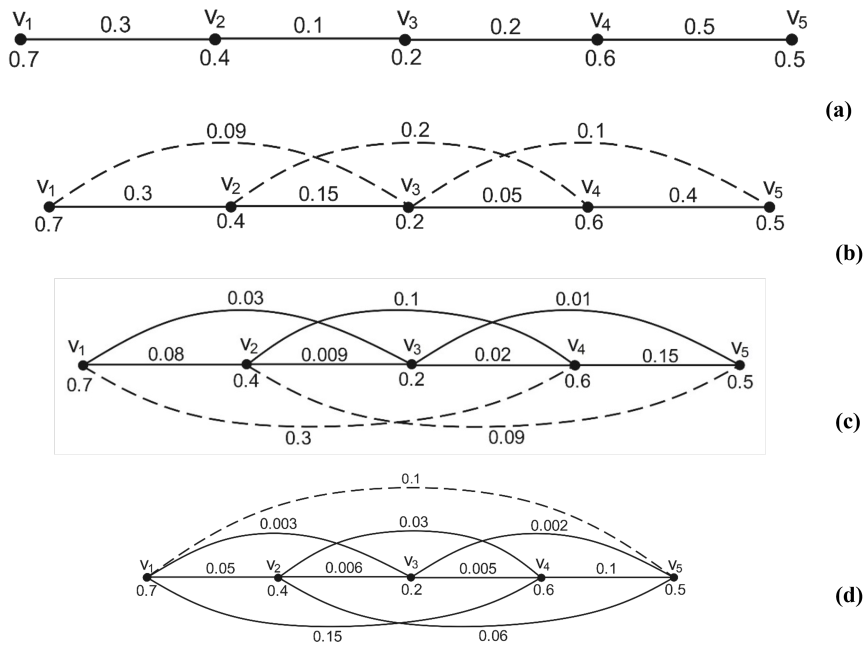


Fig 5. (a) 5 path power fuzzy graph${G_f}^1$ **(b)** (G_f^2) **5 path power fuzzy graph of** G_f^1 **(c)** (G_f^3) **- 5 path power fuzzy graph of** G_f^2 **(d)** $\left(G_f^4\right)$ **- 5-path power fuzzy graph of** $\left(G_f^3\right)$

3.2.3 Remark

For ${G_f}^k$,$size({G_f}^k) = \frac{1}{2}\sum_{i<j} i \neq j\; a_{ij}$ where a_{ij} is the element from $(A({G_f}^k))$

3.2.4 Lemma

The eigenvalues of the adjacency matrix of power fuzzy graph are bounded by $[-\sum_{i\neq j} a_{ij}, \sum_{i\neq j} a_{ij}]$.

Proof: Consider a power fuzzy graph ${G_f}^k$ with n vertices. Let $A\left({G_f}^k\right) = [a_{ij}]$ be an adjacency matrix of ${G_f}^k$ with $\lambda_i^{'}$s as eigenvalues. By the Theorem 2.10, every eigenvalue λ_i of $A\left({G_f}^k\right)$ lies within at least one of the $D(a_{ii}, R_i) = \{z \in C\ : |z - a_{ii}| \leq R_i\}$, where $R_i = \sum_{i\neq j} |a_{ij}|$. Since $A\left({G_f}^k\right)$ is a hollow matrix, $a_{ii} = 0$. Then for each row i and $a_{ii} = 0$, we get Gershgorin discs as $D(0, R_i) = \{z \in C\ : |z| \leq \sum_{i\neq j} |a_{ij}|\}$. This implies $-\sum_{i\neq j} a_{ij} \leq z \leq \sum_{i\neq j} a_{ij}$. Hence the eigenvalue lies in the interval $[-\sum_{i\neq j} a_{ij}, \sum_{i\neq j} a_{ij}]$.

3.2.5 Theorem

The adjacency matrix of power fuzzy graph is indefinite.

Proof: Consider a power fuzzy graph ${G_f}^k$ with n vertices. Let $A\left({G_f}^k\right) = [a_{ij}]$ be an adjacency matrix of ${G_f}^k$ with $\lambda_i^{'}$s as eigenvalues. By the Lemma 3.2.4, adjacency spectrum of ${G_f}^k$ can contain both positive and negative values. Suppose $A\left({G_f}^k\right)$ is not indefinite, then the matrix has either only positive eigenvalues or only negative eigenvalues. Since $a_{ii} = 0$ for all i, the trace of $A\left({G_f}^k\right)$ is also zero. This implies the sum of eigenvalues is zero. If $\lambda_i > 0\ \forall i$, then $trace > 0$ also if $\lambda_i < 0\ \forall i$, then $trace < 0$. This contradicts the fact that $trace = 0$. Hence $A\left({G_f}^k\right)$ is indefinite.

3.2.6 Example

The spectrum of the adjacency matrices in the Example 3.2.2 are $\{-0.5404, -0.3129, 0, 0.3129, 0.5404\}$, $\{-0.4851, -0.2525, -0.06924, 0.2677, 0.5391\}$, $\{-0.3358, -0.07913, -0.005252, 0.02357, 0.3967\}$, and

$\{-0.1516, -0.09168, -0.017, 0.0005096, 0.2598\}$. Here, the spectrum contains both positive and negative eigenvalues. Hence $A\left({G_f}^1\right)$, $A\left({G_f}^2\right)$, $A\left({G_f}^3\right)$ and $A\left({G_f}^4\right)$ are indefinite.

3.2.7 Theorem

The energy sequence of ${G_f}^k$ is $E_A\left({G_f}^2\right) > E_A\left({G_f}^3\right) > \cdots > E_A\left({G_f}^k\right) > \cdots > E_A({G_f}^{k_m})$ where $E_A\left({G_f}^k\right)$ is the energy.

Proof: From the Definition 2.7, $min(\sigma(u)^k, \sigma(v)^k) \le \mu_k(u,v) \le min(\sigma(u)^{k-1}, \sigma(v)^{k-1})$

$$\Longrightarrow \mu_2(v_i,v_j) > \mu_3(v_i,v_j) > \cdots > \mu_k(v_i,v_j) > \ldots > \mu_{k_m}(v_i,v_j) \tag{1}$$

$$\Longrightarrow a_{ij}\, of\, A({G_f}^2) > a_{ij}\, of\, A(G_f^3) > \cdots > a_{ij}\, of\, A(G_f^k) > \cdots > a_{ij}\, of\, A(G_f^{k_m}) \tag{2}$$

where $\mu_k(v_i,v_j)$ is the edge membership value and $A({G_f}^k)$ is the adjacency matrix of ${G_f}^k$. Let $\{\lambda_i\}_{i=1}^n$ be the eigenvalues for each ${G_f}^k$ and consider the absolute values of eigenvalues of ${G_f}^k$ in the decreasing order $\lambda_1 \le \lambda_2 \le \ldots \le \lambda_k \le \ldots \le \lambda_{k_m}$. Consider two power fuzzy graphs ${G_f}^t$ and ${G_f}^{t+1}$ such that $1 < t < k_m - 1$, then by the Theorem 2.11 for $i-th$ eigenvalue and any vector $v \ne 0 \in R^n$ consider a subspace Sp of dimension i we have,

$\lambda_i\left({G_f}^t\right) = \begin{matrix} min \\ dim(Sp)=i \end{matrix} \quad \begin{matrix} max \\ x \in Sp \end{matrix} \; \frac{x^T {G_f}^t x}{x^T x}$ and $\lambda_i\left({G_f}^{t+1}\right) = \begin{matrix} min \\ dim(Sp)=i \end{matrix} \quad \begin{matrix} max \\ x \in Sp \end{matrix} \; \frac{x^T {G_f}^{t+1} x}{x^T x}$.

By Equation (2), $x^T {G_f}^t x > x^T {G_f}^{t+1} x$

$$\Longrightarrow \lambda_i\left({G_f}^t\right) > \lambda_i\left({G_f}^{t+1}\right)\ \forall i$$

$$\Longrightarrow E_A\left({G_f}^t\right) > E_A\left({G_f}^{t+1}\right)$$

This proves the energy sequence of power fuzzy graph.

3.2.8 Note

In general, $E_A\left({G_f}^1\right) \not\geq E_A\left({G_f}^k\right) \forall k > 1$ since in maximal power fuzzy graph the $\mu_1(v_i,v_j)$ values are equal to $\mu_2(v_i,v_j)$.

3.2.9 Example

The energy of power fuzzy graphs in Figures 5(a), 5(b), 5(c) and 5(d) are 1.7066, 1.6136, 1.1331 and 0.5205 respectively. We can find $E_A\left({G_f}^2\right) > E_A\left({G_f}^3\right) > E_A\left({G_f}^4\right)$.

3.2.10 Remark

Let ${G_f}^k$ be the power fuzzy graph with n vertices and let σ_h be the highest vertex membership value. Then $E_A\left({G_f}^k\right) < \sigma_h(n+k+1)$.

3.2.11 Remark

If ${G_f}^{k_m}$ is a total power fuzzy graph with $\mu_{k_m}(v_i,v_j) = 1 \forall (v_i,v_j) \in E_{k_m}$ then $E_A\left({G_f}^{k_m}\right) = 2(k-1)$.

4 Conclusion

The adjacency sequence and energy of the power fuzzy graph has been studied with results and examples. A power fuzzy graph with all vertices adjacent to one another is vertex regular if adjacency sequence is constant. A general formula for size of a power fuzzy graph in terms of the elements of an adjacency matrix is derived. In power fuzzy graph systems, the adjacency matrix shows the capacity and reliability of connections between vertices, while the eigenvalues provide information about how loads are distributed uniformly throughout the network. The graph's energy indicates the failure spots and measures the network's flow potential.

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