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RESEARCH ARTICLE

On Solving Linear Complementarity Problem using Principal Pivoting Method

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Abstract

Objectives: A rudimentary framework called the linear complementarity problem (LCP) describes a wide range of real-world scenarios in which equilibrium conditions must be met while adhering to constraints. **Methods:** This study presents the Principal Pivoting Method (PPM), a novel method for solving linear Complementarity problems. PPM is a viable substitute for addressing large-scale complementarity problems. It is a powerful method for handling large-scale challenge circumstances while addressing LCPs. The PPM uses a principle-based pivoting strategy to navigate the solution space. It terminates in a finite number of iterations. It deals with degenerate cases. **Findings:** Under specific hypotheses, the PPM is guaranteed to converge to a solution if one exists. PPM's pivoting methods account for its numerical stability. In this paper, the process is analyzed using real-life scenarios. The algorithm goes with initialization, iteration, and termination. To ensure numerical stability, a pivot element is selected. Update the basis matrix through pivotal operations. The iterations show how well the algorithm goes perfectly without causing any wrongdoing. Using this method, the solution is obtained accurately in a smaller number of iterations. **Novelty:** A novelty of the work is finding a solution to a real-life application problem using this method. The comparison of my research work with the existing research work is a novel approach to illustrate the Principal Pivoting method in practical applications. Here, the crisp values are converted into intuitionistic triangular fuzzy numbers for analyzing data in real-life scenarios. This is a novel way to attempt a newly considered problem, which is taken as fuzzy coefficients transforming into a linear complementarity problem. Fuzzy arithmetic operations on the Function principle are utilized. The numerical trials proved the efficacy of the suggested algorithm.

Keywords: Fuzzy Linear Complementarity Problem (FLCP); Principle Pivoting Method (PPM); Intuitionistic Fuzzy set (IFS); Triangular Intuitionistic Fuzzy Number (TIFN)

1 Introduction

Fuzzy decision-making was first proposed by Bellman and Zadeh. Linear Complementarity Problems (LCPs)[1] are a fundamental class of mathematical optimization problems. It has numerous applications in the disciplines like operations research, economics, engineering, and computer science. In an LCP[2,3], a vector must satisfy a complementarity condition requiring certain variables to be non-negative and their related constraints to be non-positive. Because of their probable degeneracy and non-convex character, LCPs are still difficult to solve despite their significance. In 1967, the Principal Pivoting Method[4] was presented, an expansion of the Simplex Method for solving LCPs. The technique is predicated on the concept of pivoting, which entails switching variables and constraints to break the issue down into a series of easier sub-problems. It presents a viable substitute for resolving extensive Complementarity issues. However, we provide methods and algorithms to raise the reliability and effectiveness of LCP solvers. The approach is a dependable option for resolving complex problems because of its capacity to pivot on principal sub-matrices, which guarantees numerical stability and precision. The PPM has been widely used in Economic modeling and engineering design. In 1968, Lemke proposed a complementarity pivoting algorithm for solving linear complementarities. In 1997, K.G. Murthy[5] developed an algorithm for solving parametric LCPs. The Linear complementarity problem can be solved with the reference of Jenifer D H and Irene Hepzibah R[6–8]. Xi-Ming Fang et al[9,10] presented a direct method for solving LCP (M, q) and the Linear Complementarity problem by block principal pivoting transforms for lower and higher-order experiments. In 2018, Arumugam et al[11] presented a fuzzy linear complementarity problem without using artificial variables in hexagonal fuzzy numbers. The paper aims to compare another research to mine to emphasize that the Principal pivoting method gives reliable results to real-life applications.

In our research, the FLCP has been solved using the Principal Pivoting method with the arithmetic operations of Intuitionistic triangular fuzzy numbers. The paper aims to improve computational efficiency, enhance numerical stability, and investigate alternative pivoting rules. This paper is organized as follows: section 2 provides the necessary definition for the research and the algorithm for the Principal pivoting method. In section 3, the efficiency of the suggested method was demonstrated by solving a numerical example, which shows how well and quickly it can solve the LCP. Finally, section 4 concludes with a summary of key findings and contributions.

2 Methodology

2.1 Preliminaries

2.1.1 Intuitionistic Fuzzy Set

Let $Y = \{y_1, y_2, y_3, \ldots y_n\}$ an IFS is defined as $A = (y_i, t_A(y_i), f_A(y_i) : y_i \in Y)$ which assigns a membership degree $t_A(y_i)$ and a non-membership degree $f_A(y_i)$ under the condition $0 \le t_A(y_i) + f_A(y_i) \le 1$, for all $(y_i) \in Y$.

2.1.2 Triangular Intuitionistic Fuzzy Set

A Triangular Intuitionistic Fuzzy Number (TIFN) $\widetilde{J}$ is an IFS in R with the following membership function $t_{\widetilde{A}}(x)$ and non-membership function $f_{\widetilde{A}}(x)$

$$t_{\widetilde{A}}(x) = \begin{cases} \frac{x-j_1}{j_2-j_1}, & j_1 \le x \le j_2 \\ \frac{(x-j_3)}{(j_2-j_3)}, & j_2 \le x \le j_3 \\ 0, & otherwise \end{cases}$$

$$f_{\widetilde{A}}(x) = \begin{cases} \frac{j_2-x}{j_2-j_1'}, & j_1' \le x \le j_2 \\ \frac{x-j_2}{j_3'-j_2}, & j_2 \le x \le j_3' \\ 1 & otherwise \end{cases}$$

2.1.3 Arithmetic Operation of Intuitionistic Triangular Fuzzy number

Let $\widetilde{A}^{Int} = \{(\varsigma_1, \varsigma_2, \varsigma_3);(\varsigma_1', \varsigma_2, \varsigma_3')\}$ and $\widetilde{B}^{Int} = \{(\tau_1, \tau_2, \tau_3);(\tau_1', \tau_2, \tau_3')\}$ be two TIFN and $\gamma \neq 0$

1. **Addition:** $\widetilde{A}^{Int} + \widetilde{B}^{Int} = \langle(\varsigma_1+\tau_1, \varsigma_2+\tau_2, \varsigma_3+\tau_3);(\varsigma_1'+\tau_1', \varsigma_2+\tau_2, \varsigma_3'+\tau_3')\rangle$
2. **Subtraction:** $\widetilde{A}^{Int} - \widetilde{B}^{Int} = \langle(\varsigma_1-\tau_3, \varsigma_2-\tau_2, \varsigma_3-\tau_1);(\varsigma_1'-\tau_3', \varsigma_2-\tau_2, \varsigma_3'-\tau_1')\rangle$
3. **Multiplication:**

$\widetilde{A}^{Int} \times \widetilde{B}^{Int} = \langle(\varsigma_1\, \mathfrak{R}(B), \varsigma_2\, \mathfrak{R}(B), \varsigma_3\, \mathfrak{R}(B)); (\varsigma_1'\, \mathfrak{R}(B), \varsigma_2\, \mathfrak{R}(B), \varsigma_3'\, \mathfrak{R}(B))\rangle$,

Where $\mathfrak{R}(B) = \left(\frac{\tau_1+\tau_2+\tau_3}{3}\right), \mathfrak{R}\left(B'\right) = \left(\frac{\tau_1'+\tau_2+\tau_3'}{3}\right)$

4. **Division:** $\widetilde{A}^{Int} / \widetilde{B}^{Int} = \langle\left(\frac{\varsigma_1}{\mathfrak{R}(B)}, \frac{\varsigma_2}{\mathfrak{R}(B)}, \frac{\varsigma_3}{\mathfrak{R}(B)}\right); \left(\frac{\varsigma_1'}{\mathfrak{R}(B')}, \frac{\varsigma_2}{\mathfrak{R}(B')}\right), \frac{(\varsigma_3')}{\mathfrak{R}(B')}\,)\rangle$, ,

Where $\mathfrak{R}(B) = \left(\frac{\tau_1+\tau_2+\tau_3}{3}\right), \mathfrak{R}\left(B'\right) = \left(\frac{\tau_1'+\tau_2+\tau_3'}{3}\right)$

2.1.4 Graded Mean Integration Method

A Triangular Intuitionistic fuzzy number $\widetilde{T} = (q_1, q_2, q_3)$, then the defuzzification of fuzzy number is P($\widetilde{T}$) = $(q_1 + q_2 + q_3)$ / 3.

2.1.5 Fuzzy Linear Complementarity Problem

Given a square matrix M of order n and the column vector q$\in R^n$, the Linear Complementarity Problem (q, M).

$$W - MZ \;=\; q \tag{2.1.5.1}$$

$$W_j \geq 0, Z_j \geq 0, Z_0 \geq 0,\; j = 1, \ldots, n \tag{2.1.5.2}$$

$$W_j Z_j = 0,\; j = 1, \ldots, n \tag{2.1.5.3}$$

The parameters in Equations (2.1.5.1), (2.1.5.2) and (2.1.5.3) are fuzzy numbers. Then, by substituting fuzzy numbers for crisp parameters, the ensuing FLCP can be identified.

$$\widetilde{W} - \widetilde{M}\widetilde{Z} = \tilde{q} \tag{2.1.5.4}$$

$$\widetilde{W}_j \geq 0, \widetilde{Z}_j \geq 0, \widetilde{Z}_0 \geq 0,\; j = 1, \ldots, n \tag{2.1.5.6}$$

$$\widetilde{W}_j \widetilde{Z}_j = 0,\; j = 1, \ldots, n \tag{2.1.5.7}$$

The pair $(\widetilde{W}_j, \widetilde{Z}_j)$ is said to be a pair of Fuzzy Complementary variables.

2.2 Principal Pivoting Method: An Algorithm

Step 1: When addressing LCPs (q, M), this approach proves to be effective.

$$\begin{vmatrix} \widetilde{W} & \widetilde{Z} & \tilde{q} \\ \tilde{I} & -\widetilde{M} & \tilde{q} \end{vmatrix} \tag{2.2.1}$$

It goes along the Complementary basic vector for Equation (2.2.1) which end when a complementary feasible basic solution is obtained. It merely makes use of one main pivot step.

Step 2: Initialization

Start with an initial basic feasible solution. Compute the vector $r \;=\; Mz + q$ where k=0.

Step 3: Pivot selection

Let q = $(q_1, q_2, \ldots, q_n\,)$ be the updated RHS constant vector. If $\tilde{q} \geq 0$, the present complementary basic vector is feasible, and LCP (q, M) terminates.

If $\tilde{q} < 0$, Let $r = \{i : i \textit{ such that } \tilde{q}_i < 0\,\}$ make a pivot step in position r. Choose the pivot row according to the condition given above, that is select the most negative element in the column of q and replace the present basic variable in the complementary pair $(w_r z_r)$.

Step 4: Update

Update the basic feasible solution z. If the pivot element is zero, the procedure cannot proceed further and ends without solving the LCP. Otherwise the pivot step is carried out and moves to the next step.

Step 5: Repeat

Continue step 3 until either a pivot element cannot be selected or a complimentary solution is discovered. This method may include many sign changes for the variable during the algorithm, culminating in the final step of obtaining a complementary answer.

3 Results and Discussion

3.1 Application

Patients with diabetic conditions (D) are diagnosed in a hospital. The hospital offers the following three forms of treatment.

1. Medicament (M)
2. Lifestyle changes (LC)
3. Personalized treatment plans (PTP)

Four patients can receive Medicament (M), five patients can receive Lifestyle changes (LC), and one patients can receive personalized treatment plans (PTP).

Constraints:

1. Inadequate resources: Two patients can receive Medicament (M), one patients can receive Lifestyle Changes (LC), and one has received personalized treatment plans (PTP).
2. Treatment effectualness: The patients has taken (M) has one reduction, the patients has taken (LC) has two reduction, the Patients has taken (PTP) has one reduction.
3. Treatment priority: One patient prefer (M), one patient prefer (LC), and two patients prefer (PTP).

"The values from the patients mentioned above are resources, treatment effectualness, and treatment priority are taken as $\mathrm{X} \times 100$ patients".

The objective function is to allocate the treatment plans for patients to minimize the total disease burden.

Variables are represented as x_1, x_2, x_3

x_1: Number of patients receiving (M)

x_2: Number of patients receiving (LC)

x_3: Number of patients receiving (PTP)

The coefficient of the constraints can be taken as an Intuitionistic triangular fuzzy numbers and solved using the LCP solver (Matlab). The formulated LCP is as follows:

$$\mathrm{M}=\begin{bmatrix} \{(-3,-2,-1);(-2.5,-2,-1.5)\} & \{(-1,-1,-1);(-1.5,-1,-0.5)\} & \{(-1,-1,-1);(-1.5,-1,-0.5)\} \\ \{(-1,-1,-1);(-1.5,-1,-0.5)\} & \{(-3,-2,-1);(-2.5,-2,-1.5)\} & \{(-1,-1,-1);(-1.5,-1,-0.5)\} \\ \{(-3,-1,1);(-1.5,-1,-0.5)\} & \{(-3,-1,1);(-1.5,-1,-0.5)\} & \{(-3,-2,-1);(-2.5,-2,-0.5)\} \end{bmatrix},$$

$$\mathrm{q}=\begin{bmatrix} \{(-5,-4,-3);(-4.5,-4,-3.5)\} \\ \{(-6,-5,-4);(-5.5,-5,-4.5)\} \\ \{(-1,-1,-1);(-1.5,-1,-0.5)\} \end{bmatrix}$$

The proposed algorithm can solve the given Linear Complementarity problem. This solution minimizes the total disease burden while satisfying the constraints. The results are shown in Table 1.

The paper "The principal pivoting method for solving fuzzy quadratic programming problem" was suggested by Nagoor Gani et al, and is cited in reference 5. In that paper, they solved the fuzzy quadratic programming problem as a linear complementarity problem using the principal pivoting method. By taking different levels of alpha cut from [0,1], an example was proved and shown in the graph format. The comparison of that paper to my study is a unique way of presenting this method in real-life applications. Using the principal pivoting method, all the conditions are satisfied, giving the result undoubted. The objective function aims to minimize the patient's burden by providing healthy treatment plans. It is observed that allocating treatment plans for the patients is maximized by reducing the burden. The obtained solution is $x_1 = 1$ and $x_2 = 2$ the values are taken as $\mathrm{X} \times 100$ patients, which represents the patients healed from Medicament and Lifestyle changes of the given treatments. The obtained result presents the benefit of the patients from the treatment plans provided by the hospital, such as Medicament and lifestyle changes. A higher number of patients are benefiting from Lifestyle changes. The result shows that lifestyle changes are better than those of others. An alternative for lifestyle changes is Medicament. The arithmetic operations are coded in the Matlab program.

Table 1. Medicament, Lifestyle changes, and Personalized treatment plans for the Patients with Diabetic condition

BV	W_1	W_2	W_3	Z_1	Z_2	Z_3	q
W_1	{(1,1,1); (0.5,1,1.5)}	{(0,0,0); (0,0,0)}	{(0,0,0); (0,0,0)}	{(-3,-2,-1); (-2.5,-2,-1.5)}	{(-1,-1,-1); (-1.5,-1,-0.5)}	{(-1,-1,-1); (-1.5,-1,-0.5)}	{(-5,-4,-3); (-4.5,-4,-0.5)}
W_2	{(0,0,0); (0,0,0)}	{(1,1,1); (0.5,1,1.5)}	{(0,0,0); (0,0,0)}	{(-1,-1,-1); (-1.5,-1,-0.5)}	{(-3,-2,-1); (-2.5,-2,-1.5)}	{(-1,-1,-1); (-1.5,-1,-0.5)}	{(-6,-5,-4); (-5.5,-5,-4.5)}
W_3	{(0,0,0); (0,0,0)}	{(0,0,0); (0,0,0)}	{(1,1,1); (0.5,1,1.5)}	{(-3,-1,1); (-1.5,-1,-0.5)}	{(-3,-1,1); (-1.5,-1,-0.5)}	{(-3,-2,-1); (-2.5,-2,-1.5)}	{(-1,-1,-1); (-1.5,-1,-0.5)}
W_1	{(1,1,1); (0.5,1,1.5)}	{(0,0,0); (0,0,0)}	{(-0.5,-0.5,-0.5); (-0.2,-0.5,-0.7)}	{(-2.5,-1.5,-0.5); (-2.3,-1.5,-0.8)}	{(-0.5,-0.5,-0.5); (-1.3,-0.5,-0.3)}	{(0,0,0); (-1,0,1)}	{(-4.5,-3.5,-2.5); (-4.3,-3.5,-2.7)}
W_2	{(0,0,0); (0,0,0)}	{(1,1,1); (0.5,1,1.5)}	{(-0.5,-0.5,-0.5); (-0.2,-0.5,-0.7)}	{(-0.5,-0.5,-0.5); (-1.2,-0.5,0.2)}	{(-2.5,-1.5,-0.5); (-0.2,-1.5,-0.7)}	{(0,0,0); (-1,0,1)}	{(-5.5,-4.5,-3.5); (-5.2,-4.5,-3.7)}
Z_3	{(0,0,0); (0,0,0)}	{(0,0,0); (0,0,0)}	{(-0.5,-0.5,-0.5); (-0.2,-0.5,-0.7)}	{(-0.5,-0.5,-0.5); (0.7,0.5,0.2)}	{(1.5,0.5,-0.5); (0.7,0.5,0.2)}	{(1.5,1,0.5); (1.2,1,0.7)}	{(0.5,0.5,0.5); (0.7,0.5,0.2)}
Z_1	{(-0.67,-0.67,-0.67); (-1,-0.6,-0.3)}	{(0,0,0); (0,0,0)}	{(0.33,0.33,0.33); (0.1,0.3,0.5)}	{(1.67,1,0.33); (1.5,1,0.5)}	{(0.33,0.33,0.33); (-0.2,0.3,0.8)}	{(0,0,0); (-0.6,0,0.6)}	{(3,2.33,1.67); (2.8,2.3,1.8)}
W_2	{(-0.33,-0.33,-0.33); (-0.8,-0.3,0.2)}	{(1,1,1); (0.5,1,1.5)}	{(-0.33,-0.33,-0.33); (-0.3,-0.3,-0.3)}	{(0,0,0); (-1.5,0,1.5)}	{(-2.33,-1.33,-0.33); (-2.3,-1.3,-0.3)}	{(0,0,0); (0,0,0)}	{(-4.33,-3.33,-2.33); (-5.8,-3.3,-0.8)}
Z_3	{(-0.33,0.33,1); (0.,0.3,0.5)}	{(0,0,0); (0,0,0)}	{(-0.33,-0.67,-1); (-1,-0.7,-0.3)}	{(-2,0,2); (-0.5,0,0.5)}	{(1.67,0.33,-1); (0.7,0.3,0)}	{(1.5,1,0.5); (1.2,1,0.8)}	{(1.67,-0.67,-3); (-1.5,-0.7,0.2)}
Z_1	{(-0.75,-0.75,-0.75); (-0.6,-0.3,0.1)}	{(0.25,0.25,0.25); (-0.1,0.2,0.6)}	{(0.25,0.25,0.25); (0.2,0.2,0.3)}	{(1.67,1,0.33); (1.5,1,0.5)}	{(0,0,0); (-1,0,1)}	{(0,0,0); (-0.7,0,0.7)}	{(2.2,1.5,0.8); (-0.3,1.5,3.2)}
Z_2	{(0.25,0.25,0.25); (-0.1,0.2,0.6)}	{(-0.75,-0.75,-0.75); (-1,-0.7,-0.4)}	{(0.25,0.25,0.25); (0.2,0.2,0.2)}	{(0,0,0); (-1.1,0,1.1)}	{(1.75,1,0.25); (0.2,1,1.7)}	{(0,0,0); (0,0,0)}	{(3.25,1.5,1.75); (0.6,2.5,4.3)}
Z_3	{(-0.74,0.25,1.25); (0.2,0.2,0.3)}	{(1.25,0.25,-0.75); (-0.5,-0.2,0)}	{(-0.75,-0.75,-0.75); (-1.2,-0.7,0.3)}	{(-2,0,2); (-0.5,0,0.5)}	{(-2.67,0,2.67); (-0.6,0,0.6)}	{(1.5,1,0.5); (1.25,1,0.75)}	{(-7.18,-1.49,4.17); (-1.5,-1.5,-1.5)}
Z_1	{(-0.67,-0.67,-0.67); (-0.5,-0.1,0.2)}	{(0.33,0.33,0.33); (-0.2,0.1,0.6)}	{(0,0,0); (0,0,0)}	{(1.67,1,0.33); (1.5,1,0.5)}	{(0,0,0); (-1,0,1)}	{(0.33,0.33,0.33); (-0.3,0.3,0.9)}	{(1.7,1,0.3); (-0.7,1,2.7)}
Z_2	{(0.33,0.33,0.33); (-0.4,0.3,0.7)}	{(-0.66,-0.66,-0.66); (-1.2,-0.8,-0.5)}	{(0,0,0); (0,0,0)}	{(0,0,0); (-1.1,0.1.1)}	{(1.75,1,0.25); (1.7,1,0.2)}	{(0.33,0.33,0.33); (0.3,0.3,0.3)}	{(2.75,2,1.25); (3.8,2,0.1)}
W_3	{(1,-0.33,-1.67); (-0.4,-0.3,-0.2)}	{(1,-0.3,-1.7); (0,0.3,0.6)}	{(1,1,1); (0.4,1,1.5)}	{(-2.6,0,2.6); (-0.6,0,0.6)}	{(-3.5,0,3.5); (-0.8,0,0.8)}	{(-0.67,-1.3,-2); (-1.6,-1.3,-1)}	{(-5.6,2,9.6); (2,2,2)}

Table 1 shows the constraints in fuzzy coefficients. The values are taken asIntuitionistic triangular Fuzzy numbers. The table clearly shows how well the proposed algorithm works. Here, $w_1 w_2 w_3 z_1 z_2 z_3$ are the basic variables in the Complementary pairs, and q is a vector in Linear Complementarity problem (q, M). Using the Principal pivoting method the Intuitionistic Triangular fuzzy number has been solved to achieve the best optimum result.

The optimal solution is (w_1,w_2,w_3; z_1,z_2,z_3) = ({(0,0,0;0,0,0)},{(0,0,0;0,0,0)},{(-5.6,2,9.6;2,2,2)}:{(1.7,1,0.3;-0.7,1,2.7)},{(2.75,2,1.25;3.8,2,0.1)},{(0,0,0;0,0,0)})

4 Conclusion

It has been demonstrated that the principal pivoting method is a reliable and effective way to handle linear complementarity problems. The study offers a novel approach to illustrating the Principal Pivoting method's practical uses. The real-life problem results show that Lifestyle changes give the best choices above all the others, secondly medicament, and finally personalized treatment plans. This research demonstrated its efficacy in managing various problem sizes and structures as well as its flexibility in accommodating many pivoting rules and implementations. Sometimes it may require many iterations to converge. Also, it can get stuck in an infinite loop if the problem has no solution. But there is an assurance that it will solve a limited number of stages. It is capable of managing degenerate scenarios. Subsequent investigations may concentrate on enhancing its effectiveness and investigating novel uses.

References

1) Murthy KG. Linear Complementarity, Linear and Nonlinear Programming. Heldermann Verlag. 1997. Available from: https://public.websites.umich.edu/~murty/books/linear_complementarity_webbook/.
2) Bazaraa MS, Sherali HD, Shetty CM. Nonlinear Programming: Theory and Algorithms. Newyork. John Wiley and Sons. 2005. Available from: https://onlinelibrary.wiley.com/doi/book/10.1002/0471787779.
3) De Schutter B, Heemels WPMH, Bemporad A. On the equivalence of linear complementarity problems. *Operations research letters*. 2002;30:211–222. Available from: https://www.dcsc.tudelft.nl/~bdeschutter/pub/rep/00_15.pdf.
4) Gani AN, Kumar A. The Principal Pivoting method for solving Fuzzy Quadratic programming problem. *International journal of Pure and Applied Mathematics*. 2013;85(2):405–414. Available from: https://www.researchgate.net/publication/275596654_The_principal_pivoting_method_for_solving_fuzzy_quadratic_programming_problem.
5) Murty KG. Optimization for Decision Making: Linear and Quadratic Models. 1st ed. and others, editor;New York. Springer. 2010. Available from: http://www.springer.com/mathematics/book/978-1-4419-1290-9.
6) Hepzibah IR. An algorithmic approach to fuzzy linear and linear complementarity problems. 2010. Available from: https://shodhganga.inflibnet.ac.in/handle/10603/263806.
7) Jenifer DH, Hebzibah RI. On solving Fuzzy Linear Complementarity problem via Maximum index method. In: Referred Proceedings of 2nd International Conference on Emerging Trends in Science and Social science in Pattaya. 2023.
8) Jenifer DH, Hebzibah RI. A Methodology for solving Fuzzy Linear programming problem as a Fuzzy Linear complementarity problem. *Communications on Applied Nonlinear Analysis*. 2024;31(2):1–8. Available from: https://internationalpubls.com/index.php/cana/article/view/587.
9) Fang XM, Zhao HJ, Li J, Qiao Z. The Reduced Order Method for Solving the Linear Complementarity Problem with an M-Matrix. *Journal of Nonlinear Mathematical Physics*. 2022;29:204–217. Available from: https://link.springer.com/article/10.1007/s44198-022-00029-w.
10) Fang XM, Qiao ZJ, Zhao HJ. The Block Principal Pivoting Algorithm for the Linear Complementarity Problem with an M-Matrix. *Advances in Mathematical Physics*. 2019;2019:1–7. Available from: https://doi.org/10.1155/2019/2976768.
11) Arumugam K, Saleem RA, Pavithra S. Fuzzy Linear Complementary pivot algorithm without introducing any artificial variables. *International journal for Science and Advance Research in Technology*. 2018;4(3):136–140. Available from: https://ijsart.com/public/storage/paper/pdf/IJSARTV4I321211.pdf.