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RESEARCH ARTICLE

# Contra Functions of Soft Nano Alpha Star Generalized Closed Sets in Soft Nano Topological Spaces

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## Abstract

**Objective:** To introduce a new category of contra soft nano functions, labeled as contra soft nano $\alpha^*-$generalized continuous and contra soft nano $\alpha^*-$generalized irresolute functions in soft nano topological spaces and to identify and analyze their unique properties. This research aims to expand the theoretical framework by exploring these novel functions. **Method:** The definition of contra soft nano $\alpha^*-$generalized continuous and contra soft nano $\alpha^*-$generalized irresolute functions is obtained from the previously introduced definition of soft nano $\alpha^*-$generalized closed set. The study employs rigorous analysis of these contra soft nano functions and formal proof techniques to establish their properties. Comparative analysis with some existing function classes is conducted to highlight the distinct characteristics of these functions. **Findings:** The research identifies and confirms the unique features of contra soft nano $\alpha^*-$generalized continuous and contra soft nano $\alpha^*-$generalized irresolute functions under various operations, supported by detailed proofs and illustrative examples. **Novelty:** The research contributes to the mathematical literature by defining and exploring a new category of contra soft nano functions and comparing it with some of the existing functions. The counterparts of these properties are validated through illustrations. The formalization of their theoretical framework contributes novel insights and extends the boundaries of soft nano topological spaces.

**Keywords:** Soft Nano-Open Sets; Soft Nano $\alpha^*$ - Generalized Closed Sets; Soft Nano $\alpha^*$ - Generalized Open Sets; Contra Soft Nano $\alpha^*$ - Generalized Continuous Functions; Contra Soft Nano $\alpha^*$ - Generalized Irresolute Functions

## 1 Introduction

In the pioneering effort, Ganster and Reilly unveiled the LC-continuous functions, while Dontchey brought forth contra continuity as a robust variant of LC-continuity. In 2020, Lellis Thivagar et al [1] Pioneered the concept of contra functions in nano topological spaces and the relevant research papers in nano topology providing the foundation for our study have been exclusively documented in [2,3] and soft topology in [4]. Subsequently, contra functions in soft nano topology were introduced by Patil and Benakanawari [5] In 2020.

While contra functions have been extensively studied in various topological spaces [6–9], their specific application and behavior in soft nano topological spaces remain underexplored. The current literature lacks comprehensive studies that integrate these functions into this emerging field. There is a need to investigate the characteristics and behavior of contra functions within the framework of soft nano topological spaces. This includes defining the properties, potential applications, and implications of these functions in this specific context. Understanding contra functions in soft nano topological spaces is crucial for advancing theoretical mathematics and its practical applications in technology and science.

This paper introduces a new class of contra functions namely contra soft nano $\alpha^{*}-$generalized continuous and contra soft nano $\alpha^{*}-$generalized irresolute functions and exercised with theorems and appropriate examples. Furthermore, the essential characteristics, properties, and conditions of these functions are established by formal proof techniques, providing appropriate exemplifications for counterparts, the interrelationship between these functions and composition theorems are also established. Comparative analysis with some existing function classes is conducted to highlight the distinct characteristics of these functions.

Existing contra functions in soft nano topology have been studied extensively, but a new type of contra function in soft nano topology is introduced through soft nano alpha star generalized closed sets. This is significant because it broadens the scope of contra functions, offering a strong framework. Theorems 4.1.3 to theorem 4.1.6 compare the previously introduced contra functions with the new type, demonstrating that the new contra function is superior. Unlike the older versions, the new contra function encompasses both previously introduced types, making it more robust and versatile. This advancement opens up new possibilities for further research and applications in the field of soft nano topology, providing a more comprehensive understanding of contra functions.

## 2 Methodology

This paper introduces and extends the concepts of contra-soft nano $\alpha^{*}-$generalized continuous and contra soft nano $\alpha^{*}-$generalized irresolute functions within the framework of soft nano topological spaces. It explores the connection between the newly defined functions and the previously established contra soft nano continuous through the application of definitions. Additionally, the paper provides examples to demonstrate the converse parts of these relationships and other properties.

## 3 Preliminaries

### Definition 3.1:

Let $\mathcal{U}$ be a non-empty finite set of objects called the universe and $A$ be the set of parameters. Let $R$ be a soft equivalence relation on $\mathcal{U}$ . The triplet $(\tau_R X, \mathcal{U}, A)$ is said to be the soft approximation space. Let $X \subseteq_{S_t N} \mathcal{U}$ . Then

1. The soft lower approximation of $X$ with respect to $R$ and the set of parameters $A$ is the set of all objects, which can be for certain classified as $X$ with respect to $R$ and it is denoted by $(L_R(X), A)$. That is, $(L_R(X), A) = U\{R(k) : R(k) \subseteq StNX\}$ , where $R(k)$ denote the equivalence class determined by $k \in \mathcal{U}$ .
2. The soft upper approximation of $X$ with respect to $R$ and the set of parameters $A$ is the set of all objects, which can be possibly classified as $X$ with respect to $R$ and it is denoted by $(\mathcal{U}_R(X), A)$ That is, $\{\mathcal{U}_R(X), A) = U(R(k) : R(k) \cap X \neq \phi\}$
3. The soft boundary region of $X$ with respect to $R$ and the set of parameters $A$ is the set of all objects, which can be classified neither inside $X$ nor as outside $X$ with respect to $R$ and it is denoted by $(B_R(X), A)$ . That is, $(B_R(X), A) = (\mathcal{U}_R(X), A) - (L_R(X), A)$.

### Definition 3.2:

Let $\mathcal{U}$ be a non-empty universal set and $A$ be the set of parameters. Let $R$ be a soft equivalence relation on $\mathcal{U}$ . Let $X \subseteq_{S_t N} \mathcal{U}$ . Let $(\tau_R X, \mathcal{U}, A) = (\phi, (\mathcal{U}, A),\ (L_R(X), A), (\mathcal{U}_R(X), A), (B_R(X), A)\}$. Then $(\tau_R X, \mathcal{U}, A)$ is a Soft topology on $(\mathcal{U}, A)$, called as the soft nano topology with respect to $X$. Elements of soft nano topology are called soft nano open sets and $(\tau_R X, \mathcal{U}, A)$ is called soft nano topological space ($S_t NTS$). The complements of soft nano open sets are called soft nano closed sets.

### Definition 3.3:

A subset $(R,A)$ of a $S_tNTS(\tau_R X,\mathcal{U},A)$ is called a soft nano $\alpha^*-$generalized closed set ($S_tN\alpha^*g-$closed) then $S_tN\alpha cl(R,A) \subseteq_{S_tN} S_tNint^*(S,A)$ whenever $(R,A)\subseteq_{S_tN}(S,A)$ and $(S,A) \in S_tN\alpha O(\tau_R X,\mathcal{U},A)$. The class of soft nano $\alpha^*-$generalized closed sets in $S_tN\alpha O(\tau_R X,\mathcal{U},A)$ is denoted by $S_tN\alpha^*gC(\tau_R X,\mathcal{U},A)$. The complement of $S_tN\alpha^*g-$closed set is called $S_tN\alpha^*g-$open set.

### Definition 3.4:

Let $S_tN\alpha^*gC(\tau_R X,\mathcal{U},A)$ and $(\sigma_R Y,V,B)$ be two $S_tNTS$. A function $f:(\tau_R X,\mathcal{U},A)\rightarrow(\sigma_R Y,V,B)$ is called soft nano continuous ($S_tN-$continuous) if the inverse image of every soft nano open set in $(\sigma_R Y,V,B)$ is soft nano open in $(\tau_R X,\mathcal{U},A)$.

### Definition 3.5:

Let $(\tau_R X,U,A)$ and $(\sigma_R Y,V,B)$ be two $S_tNTS$. A function $f:(\tau_R X,U,A)\rightarrow(\sigma_R Y,V,B)$ is called

a) soft nano $\alpha^*-$generalized continuous ($S_tN\alpha^*g-$continuous) if the inverse image of every soft nano closed set in $(\sigma_R Y,V,B)$ is $S_tN\alpha^*g-$closed in $(\tau_R X,U,A)$.

b) soft nano $\alpha^*-$generalized irresolute ($S_tN\alpha^*g-$irresolute) if the inverse image of every $S_tN\alpha^*g-$closed set in $(\sigma_R Y,V,B)$ is soft $S_tN\alpha^*g-$closed in $(\tau_R X,U,A)$.

### Definition 3.6:

A $S_tNTS(\tau_R X,U,A)$ is called $S_tN\alpha^*gT_{1/2}-$space if every $S_tN\alpha^*g-$closed of $(\tau_R X,U,A)$ is soft nano closed in $(\tau_R X,U,A)$.

### Definition 3.7:

A function $f:(\tau_R X,U,A)\rightarrow(\sigma_R Y,V,B)$ is a

a) Soft nano contra continuous ($S_tN-C-$continuous) if the inverse image of every soft nano open set in $(\sigma_R Y,V,B)$ is soft nano closed in $(\tau_R X,U,A)$.

b) Soft nano contra$-g\omega-$continuous ($S_tN-C-g\omega-$ continuous) if the inverse image of every $S_tNg\omega-$open set in $(\sigma_R Y,V,B)$ is soft nano closed in $(\tau_R X,U,A)$.

### Result 3.8:

a) Every soft nano open set is $S_tN\alpha^*g-$open.

b) Every $S_tNg\omega-$open set is $S_tN\alpha^*g-$open.

### Result 3.9:

a) $S_tN\alpha^*gint(R,A)=(R,A)$ iff $(R,A)$ is a $S_tN\alpha^*g-$open set in $(\tau_R X,U,A)$.

b) $S_tN\alpha^*gcl(R,A)=(R,A)$ iff $(R,A)$ is a $S_tN\alpha^*g-$closed set in $(\tau_R X,U,A)$.

### Result 3.10:

a) A function $f:(\tau_R X,U,A)\rightarrow(\sigma_R Y,V,B)$ is $S_tN\alpha^*g-$continuous iff $f^{-1}(R,B)\in S_tN\alpha^*gO(\tau_R X,U,A)$ for every $(R,B)\in S_tNO(\sigma_R Y,V,B)$.

b) A function $f:(\tau_R X,U,A)\rightarrow(\sigma_R Y,V,B)$ is $S_tN\alpha^*g-$irresolute iff $f^{-1}(R,B)\in S_tN\alpha^*gO(\tau_R X,U,A)$ for every $(R,B)\in S_tN\alpha^*gO(\sigma_R Y,V,B)$.

## 4 Result and discussion

### 4.1 Contra soft nano $\alpha^*$- generalized continuous functions

In this segment, we define a new class of soft nano function termed contra soft nano $\alpha^*-$generalized continuous function. Also, discuss the properties of these functions along with the comparison of other soft nano functions. The counterparts of the properties are validated through suitable examples and composition theorems are also employed.

***Definition 4.1.1 :***

Let $(\tau_R X, U, A)$ and $(\sigma_R Y, V, B)$ be two $S_t NTS$. A function $f : (\tau_R X, U, A) \to (\sigma_R Y, V, B)$ is called contra soft nano $\alpha^*-$generalized continuous ($CS_t N\alpha^* g-$continuous) if the inverse image of every soft nano open set in $(\sigma_R Y, V, B)$ is $S_t N\alpha^* g-$closed in $(\tau_R X, U, A)$.

Equivalently, a function $f : (\tau_R X, U, A) \to (\sigma_R Y, V, B)$ is $CS_t N\alpha^* g-$continuous iff $f^{-1}(R, B) \in S_t N\alpha^* gO(\tau_R X, U, A)$ for every $(R, B) \in S_t NC(\sigma_R Y, V, B)$.

The following soft nano sets are used in the examples.

Let $U = \{i, j, k, l\}$ and $A = \{m_1, m_2, m_3\}$. Then the soft nano sets are

$(K_1, A) = \{(m_1, \phi), (m_2, \phi), (m_3, \phi)\} = \phi$

$(K_2, A) = \{(m_1, \{i\}), (m_2, \{i\}), (m_3, \{i\})\}$

$(K_3, A) = \{(m_1, \{j\}), (m_2, \{j\}), (m_3, \{j\})\}$

$(K_4, A) = \{(m_1, \{k\}), (m_2, \{k\}), (m_3, \{k\})\}$

$(K_5, A) = \{(m_1, \{l\}), (m_2, \{l\}), (m_3, \{l\})\}$

$(K_6, A) = \{(m_1, \{i, j\}), (m_2, \{i, j\}), (m_3, \{i, j\})\}$

$(K_7, A) = \{(m_1, \{i, k\}), (m_2, \{i, k\}), (m_3, \{i, k\})\}$

$(K_8, A) = \{(m_1, \{i, l\}), (m_2, \{i, l\}), (m_3, \{i, l\})\}$

$(K_9, A) = \{(m_1, \{j, k\}), (m_2, \{j, k\}), (m_3, \{j, k\})\}$

$(K_{10}, A) = \{(m_1, \{j, l\}), (m_2, \{j, l\}), (m_3, \{j, l\})\}$

$(K_{11}, A) = \{(m_1, \{k, l\}), (m_2, \{k, l\}), (m_3, \{k, l\})\}$

$(K_{12}, A) = \{(m_1, \{i, j, k\}), (m_2, \{i, j, k\}), (m_3, \{i, j, k\})\}$

$(K_{13}, A) = \{(m_1, \{i, j, l\}), (m_2, \{i, j, l\}), (m_3, \{i, j, l\})\}$

$(K_{14}, A) = \{(m_1, \{i, k, l\}), (m_2, \{i, k, l\}), (m_3, \{i, k, l\})\}$

$(K_{15}, A) = \{(m_1, \{j, k, l\}), (m_2, \{j, k, l\}), (m_3, \{j, k, l\})\}$

$(K_{16}, A) = \{(m_1, \{i, j, k, l\}), (m_2, \{i, j, k, l\}), (m_3, \{i, j, k, l\})\} = (U, A)$

Similarly, the soft nano sets obtained by taking $V = \{m, n, o, p\}$ and $B = \{n_1, n_2, n_3\}$ are denoted by $(G_1, B)$, $(G_2, B)$, $(G_3, B)$,………, $(G_{16}, B)$ and the soft nano sets obtained by taking $W = \{q, r, s, t\}$ and $G = \{r_1, r_2, r_3\}$ are denoted by $(H_1, G)$, $(H_2, G)$, $(H_3, G)$,……..., $(H_{16}, G)$.

***Example 4.1. 2:***

Let $U = \{i, j, k, l\}$, $A = \{m_1, m_2, m_3\}$ and $X = \{i, k\} \subseteq_{S_t N} U$ with $U \backslash R = \{\{j\}, \{k\}, \{i, l\}\}$. Then $S_t N\alpha^* gC(\tau_R X, U, A) = \{\phi, (U, A), (K_3, A), (K_6, A), (K_9, A), (K_{10}, A), (K_{12}, A), (K_{13}, A), (K_{15}, A)\}$. Let $V = \{m, n, o, p\}$, $B = \{n_1, n_2, n_3\}$ and $Y = \{m, n\} \subseteq V$ with $V \backslash R = \{\{m, n\}, \{o, p\}\}$. Then $S_t NO(\sigma_R Y, V, B) = \{\phi, (V, B), (G_6, B)\}$. Define $f : (\tau_R X, U, A) \to (\sigma_R Y, V, B)$ by $f(i) = o$, $f(j) = m$, $f(k) = p$, $f(l) = n$ and $\widetilde{p} : A \to B$ by $\tilde{p}(m_1) = n_1, \tilde{p}(m_2) = n_2, \tilde{p}(m_3) = n_3$. Then $f^{-1}(m) = j, f^{-1}(n) = l, f^{-1}(o) = i, f^{-1}(p) = k$. Thus $f : (\tau_R X, U, A) \to (\sigma_R Y, V, B)$ is $CS_t N\alpha^* g-$continuous.

***Theorem 4.1 .3: (Comparison theorem)***

Every $S_t N - C-$continuous function is $CS_t N\alpha^* g-$continuous.

**Proof:** Let $(R, B) \in S_t NO(\sigma_R Y, V, B)$. Since $f$ is $S_t N - C-$continuous, $f^{-1}(R, B) \in S_t NC(\tau_R X, U, A)$, which implies $f^{-1}(R, B) \in S_t N\alpha^* gC(\tau_R X, U, A)$. Hence $f$ is $CS_t N\alpha^* g-$continuous.

***Theorem 4.1.4 :***

If $f : (\tau_R X, U, A) \to (\sigma_R Y, V, B)$ is $CS_t N\alpha^* g-$continuous and $(\tau_R X, U, A)$ is a $S_t N\alpha^* gT_{\frac{1}{2}}-$space then $f$ is $S_t N - C-$continuous.

**Proof:** Let $(R, B) \in S_t NO(\sigma_R Y, V, B)$. Since $f$ is $CS_t N\alpha^* g-$continuous, $f^{-1}(R, B) \in S_t N\alpha^* gC(\tau_R X, U, A)$. Since $(\tau_R X, U, A)$ is a $S_t N\alpha^* gT_{1/2}-$space, $f^{-1}(R, B) \in S_t NC(\tau_R X, U, A)$. Hence $f$ is $S_t N - C-$continuous.

***Theorem 4.1.5 :***

Every $S_t N - C - g\omega-$continuous function is $CS_t N\alpha^* g-$continuous.

**Proof:** Let $(R, B) \in S_t NO(\sigma_R Y, V, B)$. Then $(R, B) \in S_t Ng\omega O(\sigma_R Y, V, B)$. Since $f$ is $S_t N - C - g\omega-$continuous, $f^{-1}(R, B) \in S_t NC(\tau_R X, U, A)$, which implies $f^{-1}(R, B) \in S_t N\alpha^* gC(\tau_R X, U, A)$. Hence $f$ is $CS_t N\alpha^* g-$continuous.

### *Theorem 4.1.6 :*

If $f:(\tau_R X,U,A)\to(\sigma_R Y,V,B)$ is $CS_tN\alpha^*g-$continuous and $(\tau_R X,U,A),(\sigma_R Y,V,B)$ are $S_tN\alpha^*gT_{\frac{1}{2}}-$space then $f$ is $S_tN-C-g\omega-$continuous.

**Proof:** Let $(R,B)\in S_tNg\omega O(\sigma_R Y,V,B)$. By result 3.8 b), $(R,B)\in S_tN\alpha^*gO(\sigma_R Y,V,B)$. Since $(\sigma_R Y,V,B)$ is $S_tN\alpha^*gT_{\frac{1}{2}}-$space, $(R,B)\in S_tNO(\sigma_R Y,V,B)$. Using $f$ is $CS_tN\alpha^*g-$continuous, we get $f^{-1}(R,B)\in S_tN\alpha^*gC(\tau_R X,U,A)$. Since $(\tau_R X,U,A)$ is a $S_tN\alpha^*gT_{1/2}-$space, $f^{-1}(R,B)\in S_tNC(\tau_R X,U,A)$. Hence $f$ is $S_tN-C-g\omega-$continuous.

### *Theorem 4 .1 .7 :*

Let $f:(\tau_R X,U,A)\to(\sigma_R Y,V,B)$ is $CS_tN\alpha^*g-$continuous function then for each soft nano set $(S,A)$ of $(\tau_R X,U,A)$ and $(Q,B)\in S_tNO(\sigma_R Y,V,B)$ such that $f(S,A)\subseteq_{S_tN}(Q,B)$, there exists $(P,A)\in S_tN\alpha^*gC(\tau_R X,U,A)$ such that $(S,A)\subseteq_{S_tN}(P,A)$ and $f(P,A)\subseteq_{S_tN}(Q,B)$.

**Proof:** Let $f:(\tau_R X,U,A)\to(\sigma_R Y,V,B)$ is $CS_tN\alpha^*g-$continuous function and $(S,A)$ be a soft nano set of $(\tau_R X,U,A)$ and $(Q,B)\in S_tNO(\sigma_R Y,V,B)$ such that $f(S,A)\subseteq_{S_tN}(Q,B)$. Since $f$ is $CS_tN\alpha^*g-$continuous, $f^{-1}(Q,B)\in S_tN\alpha^*gC(\tau_R X,U,A)$ and $(S,A)\subseteq_{S_tN}f^{-1}(Q,B)$. Take $(P,A)=f^{-1}((Q,B))$. Then $(P,A)\in S_tN\alpha^*gC(\tau_R X,U,A)$ and $(S,A)\subseteq_{S_tN}(P,A)$. Also, $f(P,A)=f(f^{-1}((Q,B)))\subseteq_{S_tN}(Q,B)$. Thus $(P,A)\in S_tN\alpha^*gC(\tau_R X,U,A)$ such that $(S,A)\subseteq_{S_tN}(P,A)$ and $f(P,A)\subseteq_{S_tN}(Q,B)$.

### *Theorem 4 .1 .8 :*

Let $f:(\tau_R X,U,A)\to(\sigma_R Y,V,B)$ be a $CS_tN\alpha^*g-$continuous function. Then

a) For each soft nano set $(R,A)$ in $(\tau_R X,U,A)$, $f(S_tN\alpha^*gint(R,A))\subseteq_{S_tN}S_tNcl(f(R,A))$.

b) For each soft nano set $(R,B)$ in $(\sigma_R Y,V,B)$, $S_tN\alpha^*gint(f^{-1}(R,B))\subseteq_{S_tN}f^{-1}(S_tNcl(R,B))$.

c) For each soft nano set $(R,A)$ in $(\tau_R X,U,A)$, $S_tNint(f(R,A))\subseteq_{S_tN}f(S_tN\alpha^*gcl(R,A))$.

d) For each soft nano set $(R,B)$ in $(\sigma_R Y,V,B)$, $f^{-1}(S_tNint(R,B))\subseteq_{S_tN}S_tN\alpha^*gcl(f^{-1}(R,B))$.

**Proof:** a) Let $f(R,A)$ be a soft nano set in $(\sigma_R Y,V,B)$. Then $S_tNcl(f(R,A))\in S_tNC(\sigma_R Y,V,B)$. Since $f$ is $CS_tN\alpha^*g-$continuous, $f^{-1}(S_tNcl(f(R,A)))\in S_tN\alpha^*gO(\tau_R X,U,A)$. By result 3.9 a), $S_tN\alpha^*gint(f^{-1}(S_tNcl(f(R,A))))=f^{-1}(S_tNcl(f(R,A)))$ (1). Since $f(R,A)\subseteq_{S_tN}S_tNcl(f(R,A))$, $(R,A)\subseteq_{S_tN}f^{-1}(S_tNcl(f(R,A)))$ which implies $S_tN\alpha^*gint(R,A)\subseteq_{S_tN}S_tN\alpha^*gint(f^{-1}(S_tNcl(f(R,A))))$. Substituting (1) we get $S_tN\alpha^*gint(R,A)\subseteq_{S_tN}f^{-1}(S_tNcl(f(R,A)))$. Hence $f(S_tN\alpha^*gint(R,A))\subseteq_{S_tN}S_tNcl(f(R,A))$ for each soft nano set $(R,A)$ in $(\tau_R X,U,A)$.

The proofs for b), c) and d) closely resemble that of a).

### *Remark 4.1.9 :*

The converse of the preceding theorem does not hold as witnessed in the succeeding example.

### *Example 4.1.10 :*

a) Let $U=\{i,j,k,l\}$, $A=\{m_1,m_2,m_3\}$ and $X=\{j,l\}\subseteq_{S_tN}U$ with $U\backslash R=\{\{i\},\{j,k\},\{l\}\}$. Then $S_tN\alpha^*gC(\tau_R X,U,A)=\{\phi,(U,A),(K_2,A),(K_6,A),(K_7,A),(K_8,A),(K_{12},A),(K_{13},A),(K_{14},A)\}$. Let $V=\{m,n,o,p\}$, $B=\{n_1,n_2,n_3\}$ and $Y=\{o,p\}\subseteq_{S_tN}V$ with $V\backslash R=\{\{m\},\{o\},\{n,p\}\}$. Then $S_tNO(\sigma_R Y,V,B)=\{\phi,(V,B),(G_4,B),(G_{10},B),(G_{15},B)\}$. Define $f:(\tau_R X,U,A)\to(\sigma_R Y,V,B)$ by $f(i)=o$, $f(j)=m$, $f(k)=p$, $f(l)=n$ and $\widetilde{p}:A\to B$ by $\tilde{p}(m_1)=n_1,\tilde{p}(m_2)=n_2,\tilde{p}(m_3)=n_3$. Then $f^{-1}(m)=j,f^{-1}(n)=l,f^{-1}(o)=i,f^{-1}(p)=k$. Here for each soft nano set $(R,A)$ in $(\tau_R X,U,A)$, we have $f(S_tN\alpha^*gint(R,A))\subseteq_{S_tN}S_tNcl(f(R,A))$ but $f$ is not $CS_tN\alpha^*g-$continuous since $f^{-1}(G_{10},B)=(K_{11},A)\tilde{\notin}S_tN\alpha^*gC(\tau_R X,U,A)$.

b) Let $U=\{i,j,k,l\}$, $A=\{m_1,m_2,m_3\}$ and $X=\{j,k\}\subseteq_{S_tN}U$ with $U\backslash R=\{\{i\},\{k\},\{j,l\}\}$. Then $S_tN\alpha^*gC(\tau_R X,U,A)=\{\phi,(U,A),(K_2,A),(K_6,A),(K_7,A),(K_8,A),(K_{12},A),(K_{13},A),(K_{14},A)\}$. Let $V=\{m,n,o,p\}$, $B=\{n_1,n_2,n_3\}$ and $Y=\{m,o\}\subseteq_{S_tN}V$ with $V\backslash R=\{\{o\},\{m,n,p\}\}$. Then $S_tNO(\sigma_R Y,V,B)=\{\phi,(V,B),(G_4,B)\}$. Define $f:(\tau_R X,U,A)\to(\sigma_R Y,V,B)$ by $f(i)=m$, $f(j)=n,f(k)=o,f(l)=p$ and $\widetilde{p}:A\to B$ by $\tilde{p}(m_1)=n_1,\tilde{p}(m_2)=n_2,\tilde{p}(m_3)=n_3$. Then $f^{-1}(m)=i,f^{-1}(n)=j,f^{-1}(o)=k,f^{-1}(p)=l$. Here for each soft nano set $(R,B)$ in $(\sigma_R Y,V,B)$, we have $S_tN\alpha^*gint(f^{-1}(R,B))\subseteq_{S_tN}f^{-1}(S_tNcl(R,B))$ but $f$ is not $CS_tN\alpha^*g-$continuous since $f^{-1}(G_4,B)=(K_4,A)\tilde{\notin}S_tN\alpha^*gC(\tau_R X,U,A)$.

c) Let $U=\{i,j,k,l\}$, $A=\{m_1,m_2,m_3\}$ and $X=\{i,k\}\subseteq_{S_tN} U$ with $U\backslash R=\{\{j\},\{k\},\ \{i,l\}\}$. Then $S_tN\alpha^*gC(\tau_R X,U,A)=\{\phi,(U,A),(K_3,A),(K_6,A),(K_9,A),\ (K_{10},A),\ (K_{12},A),\ (K_{13},A),(K_{15},A)\}$. Let $V=\{m,n,o,p\}$, $B=\{n_1,n_2,n_3\}$ and $Y=\{n,p\}\subseteq_{S_tN} V$ with $V\backslash R=\{\{m\},\{n\},\{o,p\}\}$. Then $S_tNO(\sigma_R Y,V,B)=\{\phi,(V,B),(G_3,B),\ (G_{11},B),(G_{15},B)\}$. Define $f:(\tau_R X,U,A)\to(\sigma_R Y,V,B)$ by $f(i)=p$, $f(j)=n$, $f(k)=m$, $f(l)=o$ and $\widetilde{p}:A\to B$ by $\tilde{p}(m_1)=n_1,\tilde{p}(m_2)=n_2,\tilde{p}(m_3)=n_3$. Then $f^{-1}(m)=k,f^{-1}(n)=j,f^{-1}(o)=l,f^{-1}(p)=i$. Here for each soft nano set $(R,A)$ in $(\tau_R X,U,A)$, we have $S_tNint(f(R,A))\subseteq_{S_tN} f(S_tN\alpha^*gcl(R,A))$ but $f$ is not $CS_tN\alpha^*g-$continuous since $f^{-1}(G_{11},B)=(K_8,A)\notin S_tN\alpha^*gC(\tau_R X,U,A)$.

d) Let $U=\{i,j,k,l\}$, $A=\{m_1,m_2,m_3\}$ and $X=\{i,j\}\subseteq_{S_tN} U$ with $U\backslash R=\{\{i\},\ \{k\},\ \{j,l\}\}$. Then $S_tN\alpha^*gC(\tau_R X,U,A)=\{\phi,(U,A),(K_4,A),(K_7,A),(K_9,A),\ (K_{11},A),\ (K_{12},A),\ (K_{14},A),\ (K_{15},A)\}$. Let $V=\{m,n,o,p\}$, $B=\{n_1,n_2,n_3\}$ and $Y=\{n,o,p\}\subseteq_{S_tN} V$ with $V\backslash R=\{\{m,n\},\{o,p\}\}$. Then $S_tNO(\sigma_R Y,V,B)=\{\phi,(V,B),(G_6,B),\ (G_{11},B)\}$. Define $f:(\tau_R X,U,A)\to(\sigma_R Y,V,B)$ by $f(i)=o$, $f(j)=p,f(k)=m,\ \ f(l)=n$ and $\widetilde{p}:A\to B$ by $\tilde{p}(m_1)=n_1,\tilde{p}(m_2)=n_2,\tilde{p}(m_3)=n_3$. Then $f^{-1}(m)=k,f^{-1}(n)=l,f^{-1}(o)=i,f^{-1}(p)=j$. Here for each soft nano set $(R,B)$ in $(\sigma_R Y,V,B)$, we have $f^{-1}(S_tNint(R,B))\subseteq_{S_tN} S_tN\alpha^*gcl(f^{-1}(R,B))$ but $f$ is not $CS_tN\alpha^*g-$continuous since $f^{-1}(G_{11},B)=(K_6,A)\notin S_tN\alpha^*gC(\tau_R X,U,A)$.

### *Theorem 4.1.11 :*

If $f:(\tau_R X,U,A)\to(\sigma_R Y,V,B)$ is $CS_tN\alpha^*g-$continuous and $g:(\sigma_R Y,V,B)\to(\gamma_R Z,W,G)$ is $S_tN-$continuous then $g\circ f$ is $CS_tN\alpha^*g-$continuous.

**Proof:** The theorem's proof is self-evident.

### *Remark 4.1.12 :*

Composition of two $CS_tN\alpha^*g-$continuous function need not be $CS_tN\alpha^*g-$continuous which is shown in the succeeding example.

### *Example 4.1.13 :*

Let $U=\{i,j,k,l\}$, $A=\{m_1,m_2,m_3\}$ and $X=\{i,k\}\subseteq_{S_tN} U$ with $U\backslash R=\{\{j\},\{k\},\{i,l\}\}$. Then $S_tN\alpha^*gC(\tau_R X,U,A)=\{\phi,(U,A),(K_3,A),(K_6,A),\ (K_9,A),\ (K_{10},A),(K_{12},A),\ (K_{13},A),(K_{15},A)\}$. Let $V=\{m,n,o,p\}$, $B=\{n_1,n_2,n_3\}$ and $Y=\{m,n\}\subseteq_{S_tN} V$ with $V\backslash R=\{\{m,n\},\{o,p\}\}$. Then $S_tNO(\sigma_R Y,V,B)=\{\phi,\ (V,B),(G_6,B)\}$ and $S_tN\alpha^*gC(\sigma_R Y,V,B)=\{\phi,(V,B),(G_4,B),(G_5,B),(G_{11},B),\ (G_{14},B),(G_{15},B)\}$. Let $W=\{q,r,s,t\}$, $G=\{r_1,r_2,r_3\}$ and $Z=\{r,t\}\subseteq_{S_tN} W$ with $W\backslash R=\{\{s\},\{q,r,t\}\}$. Then $S_tNO(\gamma_R Z,W,G)=\{\phi,(W,G),(H_{13},G)\}$. Define $f:(\tau_R X,U,A)\to(\sigma_R Y,V,B)$ by $f(i)=o$, $f(j)=m,f(k)=p,f(l)=n$ and $\widetilde{p}:A\to B$ by $\tilde{p}(m_1)=n_1,\tilde{p}(m_2)=n_2,\tilde{p}(m_3)=n_3$. Then $f^{-1}(m)=j,f^{-1}(n)=l,f^{-1}(o)=i,f^{-1}(p)=k$. Thus $f$ is $CS_tN\alpha^*g-$continuous. Also define $g:(\sigma_R Y,V,B)\to(\gamma_R Z,W,G)$ by $g(m)=s$, $g(n)=q,g(o)=t,g(p)=r$ and $\widetilde{q}:B\to G$ by $\tilde{q}(n_1)=r_1,\tilde{q}(n_2)=r_2,\tilde{q}(n_3)=r_3$. Then $g^{-1}(q)=n,g^{-1}(r)=p,g^{-1}(s)=m,g^{-1}(t)=o$. Thus $g$ is $CS_tN\alpha^*g-$continuous. Take $(H_{13},G)\in S_tNO(\gamma_R Z,W,G)$. Then $(g\circ f)^{-1}(H_{13},G)=f^{-1}(g^{-1}(H_{13},G))=f^{-1}(G_{15},B)=(K_{14},A)$ which is not $S_tN\alpha^*g-$closed in $(\tau_R X,U,A)$. Hence $g\circ f:(\tau_R X,U,A)\to(\gamma_R Z,W,G)$ is not $CS_tN\alpha^*g-$continuous.

**4.2 Contra soft nano $\alpha^*-$ generalized irresolute functions**

In this segment, we introduce a new class of soft nano function termed as contra soft nano $\alpha^*-$generalized irresolute function, compare it with other soft nano functions and also discuss its properties.

### *Definition 4.2 .1:*

Let $(\tau_R X,U,A)$ and $(\sigma_R Y,V,B)$ be two $S_tNTS$. A function $f:(\tau_R X,U,A)\to(\sigma_R Y,V,B)$ is called contra soft nano $\alpha^*-$generalized irresolute ($CS_tN\alpha^*g-$irresolute) if the inverse image of every $S_tN\alpha^*g-$open set in $(\sigma_R Y,V,B)$ is $S_tN\alpha^*g-$closed in $(\tau_R X,U,A)$.

Equivalently $f:(\tau_R X,U,A)\to(\sigma_R Y,V,B)$ is $CS_tN\alpha^*g-$irresolute iff $f^{-1}(R,B)\in S_tN\alpha^*gO(\tau_R X,U,A)$ for every $(R,B)\in S_tN\alpha^*gC(\sigma_R Y,V,B)$.

***Example 4.2 .2:***

Let $U = \{i,j,k,l\}$, $A = \{m_1,m_2,m_3\}$ and $X = \{j,k,l\} \subseteq_{S_tN} U$ with $U \backslash R = \{\{i,j\},\{k,l\}\}$. Then $S_tN\alpha^*gC(\tau_RX,U,A) = \{\phi,(U,A),(K_2,A),(K_3,A),(K_4,A),(K_5,A),(K_6,A),(K_7,A),(K_8,A),(K_9,A),(K_{10},A),(K_{11},A),(K_{12},A),(K_{13},A),(K_{14},A),(K_{15},A)\}$.

Let $V = \{m,n,o,p\}$, $B = \{n_1,n_2,n_3\}$ and $Y = \{n,p\} \subseteq_{S_tN} V$ with $V \backslash R = \{\{m\},\{n\},\{o,p\}\}$. Then $S_tN\alpha^*gO(\sigma_RY,V,B) = \{\phi,(V,B),(G_3,B),(G_4,B),(G_5,B),(G_9,B),(G_{10},B),(G_{11},B),(G_{15},B)\}$. Define $f:(\tau_RX,U,A) \to (\sigma_RY,V,B)$ by $f(i) = p$, $f(j) = o$, $f(k) = n$, $f(l) = m$ and $\widetilde{p}:A \to B$ by $\tilde{p}(m_1) = n_1$, $\tilde{p}(m_2) = n_2$, $\tilde{p}(m_3) = n_3$. Then $f^{-1}(m) = l$, $f^{-1}(n) = k$, $f^{-1}(o) = j$, $f^{-1}(p) = i$. Thus $f:(\tau_RX,U,A) \to (\sigma_RY,V,B)$ is $CS_tN\alpha^*g-$irresolute.

***Theorem 4.2 .3:***

Every $CS_tN\alpha^*g-$irresolute function is $CS_tN\alpha^*g-$continuous but not conversely.

**Proof:** Let $(R,B) \in S_tNO(\sigma_RY,V,B)$. By result 3.8 a), $(R,B) \in S_tN\alpha^*gO(\sigma_RY,V,B)$. Since $f$ is $CS_tN\alpha^*g-$irresolute, $f^{-1}(R,B) \in S_tN\alpha^*gC(\tau_RX,U,A)$. Hence $f$ is $CS_tN\alpha^*g-$continuous.

***Example 4.2 .4:***

Let $\mathcal{U} = \{,,,\}$, $\varepsilon = \{_1,_2,_3\}$ and $\mathcal{X} = \{,\} \subseteq \mathcal{U}$ with $\mathcal{U}\backslash\mathcal{R} = \{\{\},\{,\},\{\}\}$. Then $S_t\mathcal{N}\alpha^*gC(\tau_RX,\mathcal{U},\varepsilon) = \{\phi,(\mathcal{U},\varepsilon),(\mathcal{K}_5,\varepsilon),(\mathcal{K}_8,\varepsilon),(\mathcal{K}_{10},\varepsilon),(\mathcal{K}_{11},\varepsilon),(\mathcal{K}_{13},\varepsilon),(\mathcal{K}_{14},\varepsilon),(\mathcal{K}_{15},\varepsilon)\}$. Let $\mathcal{V} = \{,,,\}$, $\mathcal{F} = \{_1,_2,_3\}$ and $\mathcal{Y} = \{,\} \subseteq \mathcal{V}$ with $\mathcal{V}\backslash\mathcal{R} = \{\{\},\{\},\{,\}\}$. Then $S_tNO(\sigma_R\mathcal{Y},V,\mathcal{F}) = \{\phi,(V,\mathcal{F}),(\mathcal{G}_9,\mathcal{F})\}$ and $S_tN\alpha^*gO(\sigma_R\mathcal{Y},V,\mathcal{F}) = \{\phi,(V,\mathcal{F}),(\mathcal{G}_3,\mathcal{F}),(\mathcal{G}_4,\mathcal{F}),(\mathcal{G}_9,\mathcal{F}),(\mathcal{G}_{12},\mathcal{F}),(\mathcal{G}_{15},\mathcal{F})\}$.Define $f:(\tau_RX,\mathcal{U},\varepsilon) \to (\sigma_R\mathcal{Y},V,\mathcal{F})$ by $f(i) = o$, $f() = ,f() = ,\ f() =$ and $\widetilde{p}:\varepsilon \to \mathcal{F}$ by $\tilde{p}(_1) = _1,\tilde{p}(_2) = _2,\ \tilde{p}(_3) = _3$. Then $f^{-1}() = ,f^{-1}() = ,f^{-1}() = ,f^{-1}() =$. Then $f$ is $CS_tN\alpha^*g-$continuous. Even though $(\mathcal{G}_4,\mathcal{F}) \in S_tN\alpha^*gO(\sigma_R\mathcal{Y},V,\mathcal{F})$, $f^{-1}(\mathcal{G}_4,\mathcal{F}) = (\mathcal{K}_2,\varepsilon) \notin S_tN\alpha^*gC(\tau_RX,\mathcal{U},\varepsilon)$. Hence $f$ is not $CS_tN\alpha^*g-$irresolute. Thus $f$ is $CS_tN\alpha^*g-$continuous but not $CS_tN\alpha^*g-$irresolute.

***Theorem 4.2 .5:***

The composition of two $CS_tN\alpha^*g-$irresolute functions is $S_tN\alpha^*g-$irresolute.

**Proof:** Let $f:(\tau_RX,U,A) \to (\sigma_RY,V,B)$ and $g:(\sigma_RY,V,B) \to (\gamma_RZ,W,G)$ be two $CS_tN\alpha^*g-$irresolute functions and let $(S,G) \in S_tN\alpha^*gO(\gamma_RZ,W,G)$. Since $g:(\sigma_RY,V,B) \to (\gamma_RZ,W,G)$ is $CS_tN\alpha^*g-$irresolute, $g^{-1}(S,G) \in S_tN\alpha^*gC(\sigma_RY,V,B)$. Again since $f:(\tau_RX,U,A) \to (\sigma_RY,V,B)$ is $CS_tN\alpha^*g-$irresolute, $f^{-1}(g^{-1}(S,G)) \in S_tN\alpha^*gO(\tau_RX,U,A)$ which implies $(g\circ f)^{-1}(S,G) \in S_tN\alpha^*gO(\tau_RX,U,A)$. Hence by result 3.10 b), $g\circ f : (\tau_RX,U,A) \to (\gamma_RZ,W,G)$ is $S_tN\alpha^*g-$irresolute.

***Theorem 4.2.6 :***

Let $f:(\tau_RX,U,A) \to (\sigma_RY,V,B)$ and $g:(\sigma_RY,V,B) \to (\gamma_RZ,W,G)$ be any two soft nano functions.

a) If $f$ is $CS_tN\alpha^*g-$irresolute and $g$ is $CS_tN\alpha^*g-$continuous then $g\circ f$ is $S_tN\alpha^*g-$continuous.

b) If $f$ is $CS_tN\alpha^*g-$irresolute and $g$ is $S_tN\alpha^*g-$continuous then $g\circ f$ is $CS_tN\alpha^*g-$continuous.

c) If $f$ is $S_tN\alpha^*g-$irresolute and $g$ is $CS_tN\alpha^*g-$irresolute then $g\circ f$ is $CS_tN\alpha^*g-$irresolute.

**Proof:** The theorem's proof is self-evident.

***Theorem 4.2 .7:***

Let $f:(\tau_RX,U,A) \to (\sigma_RY,V,B)$ is $CS_tN\alpha^*g-$irresolute function then for each soft nano set $(S,A)$ of $(\tau_RX,U,A)$ and $(Q,B) \in S_tN\alpha^*gO(\sigma_RY,V,B)$ such that $f(S,A) \subseteq_{S_tN} (Q,B)$ there exists $(P,A) \in S_tN\alpha^*gC(\tau_RX,U,A)$ such that $(S,A) \subseteq_{S_tN} (P,A)$ and $f(P,A) \subseteq_{S_tN} (Q,B)$.

**Proof:** The proof of this theorem is similar to theorem 4.1.7

***Remark 4.2 .8:***

If $f$ is $CS_tN\alpha^*g-$irresolute then theorem 4.1.8 holds. The proof follows from theorem 4.2.3

## 5 Conclusion

This study underscores the significance of contra soft nano $\alpha^*-$generalized continuous and contra soft nano $\alpha^*-$generalized irresolute functions by discussing their properties and investigating the relationship between them, supported by pertinent examples. It also delves into how these functions interact when composed with other soft nano functions, providing a comprehensive understanding of their behaviour. The next phase of this research will incorporate new concepts, including connectedness, compactness, separation axioms, and other related ideas through soft nano $\alpha^*-$generalized closed sets. Soft nano sets find diverse applications, including machine reasoning, artificial neural networks, and databases for smart computing, among others. Defining the CORE of the soft nano sets allows the identification of key attributes of a disease, providing valuable insights and also in decision-making problems. These applications demonstrate the broad potential and practical relevance of the theoretical concepts introduced, which will be elaborated in future research.

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