INDIAN JOURNAL OF SCIENCE AND TECHNOLOGY

RESEARCH ARTICLE

Check for updates

OPEN ACCESS

Received: 01-11-2024

Accepted: 30-11-2024

Published: 19-12-2024

Citation: Kauser SA, Alsaeedi M (2024) Status Sombor Indices of Unitary Cayley Graph. Indian Journal of Science and Technology 17(46): 4861-4866. https://doi.org/10.17485/IJST/v17i46.3569

* **Corresponding author**.

asmaxlnc1@gmail.com

Funding: None

Competing Interests: None

Published By Indian Society for Education and Environment (iSee)

ISSN
Print: 0974-6846
Electronic: 0974-5645

Status Sombor Indices of Unitary Cayley Graph

Syeda Asma Kauser[1]*, Mashaer Alsaeedi[2]

1 Assistant Professor, Humanities & Sciences, Global institute for engineering and technology, Hyderabad, Telangana, India
2 Assistant Professor, Department of Sciences, Saudi Arabia

Abstract

Objectives: The objective of the study is to obtain the values of status Sombor indices for the unitary Cayley graph. **Methods**: The unitary Cayley graph X_n has vertex set $Z_n = \{0, 1, 2, \ldots n-1\}$. Vertices a, and b are adjacent, if $gcd(a-b,\ n)\ =\ 1$. The status $\sigma(u)$ of a vertex u in a graph, G is the sum of distances of all other vertices from u in G. In this paper, we obtain the values of the Status Sombor index, Modified Status Sombor index, and their corresponding exponentials and compute exact formulae for unitary Cayley graph X_n. **Findings:** Different values of status Sombor indices have been obtained for unitary Cayley graphs of various orders. **Novelty:** The values of status Sombor indices have been extensively studied by various researchers recently and most of the research has been done for different types of graphs in chemical graph theory. The present work reveals that the unitary Cayley graph is one of the graphs that is extensively studied in both the areas of group theory and graph theory. We derive the values of status Sombor indices that are dependent on the status and degree of the vertices of the unitary Cayley graph.

Mathematics Subject Classification: 05C09**,** 05C25, 05C50.

Keywords: Unitary Cayley graphs; Status Sombor index; Modified status Sombor index; Status Sombor exponential; Modified status Sombor exponential

1 Introduction

A graph $G = (V, E)$ in which $V\ (G)$ is the set of vertices of, and $E(G)$ is the set of edges. An edge $\{u, v\}$ is an unordered pair of distinct vertices; the vertices u and v are the endpoints of the edge. The vertices of a graph can be pictured as dots, and the edges as line segments whose endpoints are vertices. An edge e is said to be incident at a vertex v if v is an endpoint of e. Two vertices are said to be adjacent if there is an edge between them. Adjacent vertices are sometimes referred to as neighbors. The degree $deg(u)$ of a vertex u in a graph G is the number of vertices adjacent to u. A graph is simple if there is at most one edge between any two vertices, and the endpoints of each edge are distinct.

Researchers have introduced many regular graphs using Cayley graphs with more number of vertices for a given diameter and for a given number of edges per vertex were studied previously. This leads to the construction of larger networks, while meeting design criteria of a fixed number of nearest neighbors and a fixed maximum communication time between arbitrary vertices. An immediate example of a Cayley graph is a unitary Cayley graph. More information about Cayley graphs can be found in the books on algebraic graph theory by Biggs[1]. and by Godsil and Royle[2]. Unitary Cayley graphs are highly symmetric. They have some remarkable properties connecting graph theory and number theory.

All graphs discussed in this paper are simple graphs. The unitary Cayley graph $X_n$has vertex set $Z_n = \{0, 1, 2, ...n-1\}$ where the vertices a, and b are adjacent, if $gcd(a-b,\ n)\ =\ 1$.

If we represent the elements of Z_n by the integers $0,\ 1,\ ...,\ n\ -\ 1$, then it is well known that X_n has vertex set $V(X_n) = Z_n = \{0, 1, ...n-1\}$and edge set

$$E(X_n) = \{\{a, b\} : a, b \in Z_n, gcd(a-b, n) = 1\}.$$

A numerical parameter mathematically derived from the graph structure is called a topological index or graph index. Topological indices uniquely characterize molecular topology and are extensively applied in chemical graph theory. Application of ideas from one scientific field to another one often gives a new view on the problems. The real numbers are derived from the structure of a graph, which is invariant under graph isomorphism. These invariants under some properties are called graph indices. These indices reflect the chemical and physical properties of molecules. Many such invariants have been introduced.

Mostly we can find the applications of graph indices[3] in various fields of Science and Technology in[4,5]. One can find that graph indices are extensively studied in[6–8]. The status, denoted by$\sigma(u)$, of a vertex u in G is the sum of distances of all other vertices from u in G.Degree-based and distance-based topological indices have been extensively studied in[6–9].

In 2021, I. Gutman introduced a novel degree-based topological index in[10], called the Sombor index which has received a lot of attention within mathematics and chemistry. For example, the chemical applicability of the Sombor index, especially the predictive and discriminative potentials was investigated in[11,12]. Thebounds for topological invariants of a weighted graph using traces were studied in[13]. The study of some new multiplicative indices of unitary Cayley graphs has been done in[14]. The multiplicative indices are based on the distances of vertices in a graph.

2 Methodology

The unitary Cayley graph $X_n$has vertex set $Z_n = \{0, 1, 2, ...n-1\}$ vertices a, and b are adjacent, if $gcd(a-b,\ n)\ =\ 1$. The status $\sigma(u)$of a vertex u in a graph, G is the sum of distances of all other vertices from u in G. In this paper, we obtain the values of the Status Sombor index, Modified Status Sombor index, and their corresponding exponentials and compute exact formulae for unitary Cayley graph X_n. Different values of status Sombor indices have been obtained for unitary Cayley graphs for different numbers of vertices (odd and even number of vertices).

3 Results and Discussions

The values of status Sombor indices have been extensively studied by various researchers recently and most of the research has been done for different types of graphs in chemical graph theory. The present work reveals that the unitary Cayley graph is one of the graphs that is extensively studied in both the areas of group theory and graph theory. We derive the values of status Sombor indices that are dependent on the status and degree of the vertices of the unitary Cayley graph.

Lemma 1[14] **:** The general multiplicative first status index of a unitary Cayley graph where $n = p$ is a prime number is $S_1^a II(X_n) = 2(n-1)^{\frac{an(n-1)}{2}}$ with $\frac{n(n-1)}{2}$ edges and the status of every vertex $\sigma(u) = n-1$.

Lemma 2[14] **:** The general multiplicative first status index of a unitary Cayley graph where $n = 6$ is $S_1^a II(X_n) = (\frac{n^2}{2})^{an}$ with n edges and the status of every vertex is $\sigma(u) = \frac{n^2}{4}$.

The status Sombor index and its related parameters are given by Kulli[15].

Status Sombor Indices of Unitary Cayley Graph:

Firstly, we study the case when $X_n$is a unitary Cayley graph where $n = p$ is a prime number, then $X_n = K_p$ is a complete graph on p vertices.

Lemma 3: The status Sombor index of a unitary Cayley graph $X_n$where $n = p$ is a prime number is

$SSO(X_n) = \frac{n(n-1)^2}{\sqrt{2}}$.

Proof: Given X_n is a unitary Cayley graph where $n = p$ is a prime number, then $X_n = K_p$ is a complete graph on p vertices. In a complete graph, we have $\frac{n(n-1)}{2}$ edges and the status of every vertex $\sigma(u) = n-1$. From [15]

$$SSO(X_n) = \sum_{uv \in E(G)} \sqrt{\sigma(u)^2 + \sigma(v)^2}$$

$$= \frac{n(n-1)}{2}\sqrt{(n-1)^2 + (n-1)^2}$$

$$\frac{n(n-1)^2}{\sqrt{2}}$$

Observations:

3.1 From [15], the status Sombor exponential of a unitary Cayley graph X_n where $n = p$ is a prime number is

$$SSO(X_n, x) = \sum_{uv \in E(X_n)} x^{\sqrt{\sigma(u)^2+\sigma(v)^2}} = \frac{n(n-1)}{2} x^{\sqrt{2}(n-1)}$$

3.2 From [15] the modified status Sombor index of a unitary Cayley graph X_n where $n = p$ is a prime number is

$$m_{SSO(X_n)} = \sum_{uv \in E(X_n)} \frac{1}{\sqrt{\sigma(u)^2 + \sigma(v)^2}} = \frac{n}{2\sqrt{2}} = \frac{\sqrt{2}n}{4}$$

3.3 From [15] the modified status Sombor exponential of a unitary Cayley graph X_n where $n = p$ is a prime number is

$$m_{SSO(X_n, x)} = \sum_{uv \in E(X_n)} x^{\frac{1}{\sqrt{\sigma(u)^2+\sigma(v)^2}}} = \frac{n(n-1)}{2} x^{\frac{1}{\sqrt{2}(n-1)}}$$

3.4 From [16,17] the forgotten topological index and F-status index of a unitary Cayley graph X_n where $n = p$ is a prime number is $n(n-1)^3$

We continue our study of Sombor indices of unitary Cayley graphs for n as any even number. Specifically when $n = 6$,X_n becomes a cycle.

Lemma 4: The status Sombor index of a unitary Cayley graph X_n where $n = 6$ is it an even number is $SSO(X_n) = \frac{\sqrt{2}n^3}{4}$.

Proof: When $n = 6$,X_n is a cycle, then it has n edges and the status of every vertex is $\sigma(u) = \frac{n^2}{4}$.

From [15]

$$SSO(X_n) = \sum_{uv \in E(G)} \sqrt{\sigma(u)^2 + \sigma(v)^2} = n\left(\sqrt{\left(\frac{n^2}{4}\right)^2 + \left(\frac{n^2}{4}\right)^2}\right) = n\left(\sqrt{2}\frac{n^2}{4}\right) = \frac{\sqrt{2}n^3}{4}$$

Observations:

4.1 The status Sombor exponential [15] of a unitary Cayley graph X_n where $n = 6$ is it an even number is

$$SSO(X_n, x) = \sum_{uv \in E(X_n)} x^{\sqrt{\sigma(u)^2+\sigma(v)^2}} = n x^{\sqrt{2}\frac{n^2}{4}}$$

4.2 The modified status Sombor index [15] of a unitary Cayley graph X_n where $n = 6$ is it an even number is

$$m_{SSO(X_n)} = \sum_{uv \in E(X_n)} \frac{1}{\sqrt{\sigma(u)^2 + \sigma(v)^2}} = n\left(\frac{1}{\sqrt{2}\frac{n^2}{4}}\right) = \frac{2\sqrt{2}}{n}.$$

4.3 The modified status Sombor exponential[15] of a unitary Cayley graph X_n where $n = p$ is a prime number is

$$m_{SSO(X_n,x)} = \sum_{uv\in E(X_n)} x^{\frac{1}{\sqrt{\sigma(u)^2+\sigma(v)^2}}} = nx^{\frac{2\sqrt{2}}{n^2}}.$$

4.4 The forgotten topological index[16] of a unitary Cayley graph X_n where $n = 6$ is an even number in a cycle and we know that the degree of each vertex in a cycle is 2. Then,

$$F(X_n) = \sum_{uv\in E(X_n)} \left(d_G(u)^2 + d_G(v)^2\right) = n\left(2^2+2^2\right) = 8n.$$

4.5 The F-status index[17] of a unitary Cayley graph X_n where $n = 6$ is an even number is

$$FS(X_n) = \sum_{uv\in E(G)} \left(\sigma(u)^2+\sigma(v)^2\right) = \frac{n^5}{8}$$

Further, we study the Sombor indices of unitary Cayley graphs for n as any even number such that n = 2p where p is a prime and p ≠ 2, 3.

From[14] this type of graph has $2n$ edges, every vertex has $deg(u) = n-1$ and the status of every vertex u in X_n is $\sigma(u) = \frac{3}{2}n$.

Lemma 5: The status Sombor index of a unitary Cayley graph X_n where $n = 2p$ where p is a prime and p ≠ 2, 3 then $SSO(X_n) = 3\sqrt{2}n^2$.

Proof: When $n = 2p$ where p is a prime and p ≠ 2, 3 the status of every vertex u in X_n is $\sigma(u) = \frac{3}{2}n$.

Thus

$$SSO(X_n) = \sum_{uv\in E(G)} \sqrt{\sigma(u)^2+\sigma(v)^2} = 2n\sqrt{\left(\frac{3}{2}n\right)^2+\left(\frac{3}{2}n\right)^2} = 2n\sqrt{2\frac{9}{4}n^2} = 3\sqrt{2}n^2$$

Observations:

5.1 The status Sombor exponential of a unitary Cayley graph X_n where $n = 2p$ where p is a prime and p ≠ 2, 3 is

$$SSO(X_n,x) = \sum_{uv\in E(X_n)} x^{\sqrt{\sigma(u)^2+\sigma(v)^2}} = 2nx^{\frac{3\sqrt{2}n}{2}}$$

5.2 The modified status Sombor index of a unitary Cayley graph X_n where $n = 2p$ where p is a prime and p ≠ 2, 3 is

$$m_{SSO(X_n)} = \sum_{uv\in E(X_n)} \frac{1}{\sqrt{\sigma(u)^2+\sigma(v)^2}} = 2n\left(\frac{1}{\frac{3\sqrt{2}n}{2}}\right) = \frac{2\sqrt{2}}{3}.$$

5.3 The modified status Sombor exponential of a unitary Cayley graph X_n where $n = 2p$ where p is a prime and p ≠ 2, 3 is

$$m_{SSO(X_n,x)} = \sum_{uv\in E(X_n)} x^{\frac{1}{\sqrt{\sigma(u)^2+\sigma(v)^2}}} = 2nx^{\frac{3\sqrt{2}n}{2}}.$$

5.4 The forgotten topological index of a unitary Cayley graph X_n where $n = 2p$ where p is a prime and p ≠ 2, 3 is

$$FS(X_n) = \sum_{uv\in E(X_n)} \left(d_G(u)^2+d_G(v)^2\right) = 2n\left((n-1)^2+(n-1)^2\right) = 4n(n-1)^2.$$

5.5 The F-status index of a unitary Cayley graph X_n where $n = 2p$ where p is a prime and p ≠ 2, 3 is

$$FS(X_n) = \sum_{uv\in E(G)} \left(\sigma(u)^2+\sigma(v)^2\right) = 2n\left(\left(\frac{3}{2}n\right)^2+\left(\frac{3}{2}n\right)^2\right) = 9n^3$$

Lastly, we study the Sombor indices of unitary Cayley graphs for n as any even number such that n = pa where p is a prime, p = 2 and a = 3, 4...

From [14] this type of graph has $2n$ edges, every vertex has $deg(u) = \frac{n}{2}$ and the status of every vertex u in X_n is $\sigma(u) = \frac{3}{2}n - 2$.

Lemma 6: The status Sombor index of a unitary Cayley graph X_n where =pa where p is a prime, p = 2 and a= 3, 4 ... then $SSO(X_n) = \sqrt{2}n(3n-4)$.

Proof: When = pawhere p is a prime, p= 2 and a= 3, 4 ... The status of every vertex u in X_n is $\sigma(u) = \frac{3}{2}n - 2$.

Thus

$$SSO(X_n) = \sum_{uv\in E(G)} \sqrt{\sigma(u)^2 + \sigma(v)^2} = 2n\left(\sqrt{\left(\frac{3}{2}n-2\right)^2 + \left(\frac{3}{2}n-2\right)^2}\right) = 2n\sqrt{2\left(\frac{3}{2}n-2\right)^2}$$

$$= \sqrt{2}n(3n-4).$$

Observations:

6.1 The status Sombor exponential of a unitary Cayley graph X_n where =pa where p is a prime,

$$p = 2\, and\, \alpha = 3,\, 4\ldots\ is\, SSO(X_n, x) = \sum_{uv\in E(X_n)} x^{\sqrt{\sigma(u)^2+\sigma(v)^2}} = 2nx^{\sqrt{2}\left(\frac{3n-4}{2}\right)}$$

6.2 The modified status Sombor index of a unitary Cayley graph X_n where =pa where p is a prime, $p = 2\, and\, \alpha = 3,\, 4\ldots\ is$

$$m_{SSO(X_n)} = \sum_{uv\in E(X_n)} \frac{1}{\sqrt{\sigma(u)^2 + \sigma(v)^2}} = 2n\left(\frac{1}{\sqrt{2}\left(\frac{3n-4}{2}\right)}\right) = \frac{2\sqrt{2}n}{(3n-4)}.$$

6.3 The modified status Sombor exponential of a unitary Cayley graph X_n where =pa where p is a Prime, $p = 2\, and\, \alpha = 3,\, 4\ldots\ is$

$$m_{SSO(X_n,x)} = \sum_{uv\in E(X_n)} x^{\frac{1}{\sqrt{\sigma(u)^2+\sigma(v)^2}}} = 2nx^{\frac{2}{\sqrt{2}(3n-4)}}.$$

6.4 The forgotten topological index of a unitary Cayley graph X_n where =pa where p is a Prime, $p = 2\, and\, \alpha = 3,\, 4\ldots\ is$

$$FS(X_n) = \sum_{uv\in E(X_n)} \left(d_G(u)^2 + d_G(v)^2\right) = 2n\left(\left(\frac{n}{2}\right)^2 + \left(\frac{n}{2}\right)^2\right) = n^3.$$

6.5 The F-status index of a unitary Cayley graph X_n where $n = 2p$ where p is a prime and p $\neq$ 2, 3 is

$$FS(X_n) = \sum_{uv\in E(G)} \left(\sigma(u)^2 + \sigma(v)^2\right) = 2n\left(\left(\frac{3}{2}n-2\right)^2 + \left(\frac{3}{2}n-2\right)^2\right) = n(3n-4)^2$$

Comparison: A study of status Sombor indices of Unitary Cayley graph is made and the results are obtained for different values of vertices by comparing the results obtained in [15]. Moreover, the values of the status of vertices have been used from the results given in [14].

References

1) Biggs N. Algebraic graph theory. . *Second EditionCambridge Mathematical Library*. 1993. Available from: https://superoles.wordpress.com/wp-content/uploads/2015/09/n-biggs-algebraic-graph-theory-1993.pdf.
2) Godsil C, Royle G. Algebraic graph theory. *Graduate Texts in Mathematics*. 2001;207. Available from: https://books.google.co.in/books/about/Algebraic_Graph_Theory.html?id=pYfJe-ZVUyAC&redir_esc=y.

3) Devillers J, Balaban AT. Topological Indices and Related Descriptors in QSAR and QSPR. Amsterdam, The Netherlands. Gordan and Breach Science Publishers. 1999. Available from: https://pubs.acs.org/doi/10.1021/ci010441h.
4) and MK. Molecular Descriptors in QSAR/QSPR; . New York, NY, USA. Wiley-Interscience: New York. 2000. Available from: https://www.wiley.com/en-us/Molecular+Descriptors+in+QSAR/QSPR+-p-9780471351689.
5) Diudea MV. QSPR/QSAR Studies by Molecular Descriptors. Huntington, NY, USA. Nova Science Publishers. 2000. Available from: https://doi.org/10.1021/ci0104432.
6) Gutman I. Degree-based topological indices. *Croat Chem Acta*. 2014;86:351–361. Available from: http://dx.doi.org/10.5562/cca2294.
7) Xu K, Liu M, Das KC, Gutman I, Furtula B. A survey on graphs extremal with respect to distance-based topological indices. *MATCH Commun Math Comput Chem*. 2014;71:461–508. Available from: https://match.pmf.kg.ac.rs/electronic_versions/Match71/n3/match71n3_461-508.pdf.
8) Rada J, Cruz R. Vertex-degree-based topological indices over graphs. *MATCH Commun Math Comput Chem*. 2014;72:603–616. Available from: https://match.pmf.kg.ac.rs/electronic_versions/Match72/n3/match72n3_603-616.pdf.
9) Das KC, Gutman I, Nadjafi-Arani MJ. Relations between distance-based and degree-based topological indices. *Appl Math Comput*. 2015;270:142–147. Available from: https://doi.org/10.1016/j.amc.2015.08.061.
10) Gutman I. Geometric approach to degree-based topological indices: Sombor indices. *MATCH Commun Math Comput Chem*. 2021;86:11–16. Available from: https://match.pmf.kg.ac.rs/electronic_versions/Match86/n1/match86n1_11-16.pdf.
11) Redžepović I. Chemical applicability of Sombor indices. *J Serb Chem Soc*. 2021;86:445–457. Available from: https://doi.org/10.2298/JSC201215006R.
12) Deng H, Tang Z, Wu R. Molecular trees with extremal values of Sombor indices. *Int J Quantum Chem*. 2021;121. Available from: https://doi.org/10.1002/qua.26622.
13) Sevgi E, Kizilirmak GÖ, Büyükköse S. The bounds for topological invariants of a weighted graph using traces. *Open Journal of Discrete Applied Mathematics*. 2023;6:30–34. Available from: https://doi.org/10.30538/psrp-odam2023.0091.
14) Asma KS. Some Multiplicative Indices of Unitary Cayley Graphs. *Global Journal of Pure and Applied Mathematics*;2023(2). Available from: https://doi.org/10.37622/GJPAM/19.2.2023.319-328.
15) Kulli VR. Status somber indices. *International journal of mathematics and computer research*. 2022;(6):2726–2730. Available from: https://doi.org/10.47191/ijmcr/v10i6.05.
16) Furtula B, Gutman I. A forgotten topological index. *J Math Chem*. 2015;53:1184–1190. Available from: https://link.springer.com/article/10.1007/s10910-015-0480-z.
17) Kulli VR. The (a, b)-status index of graphs. *Annals of Pure and Applied Mathematics*;21(2). Available from: https://doi.org/10.22457/apam.v21n2a5646.