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RESEARCH ARTICLE

Optimization of Energy Involved in Rolling Stock of a Sub-Urban Rail Transport System

Mohammad Ishaq[1]*, **Praveen Kumar Shukla**[2], **Haroon Ashfaq**[3]

1 Research Scholar, Department of Electrical Engineering, Babu Banarasi Das University, 226028, Lucknow, India
2 Professor and Dean, Department of Computer Science and Engineering, School of Engineering, Babu Banarasi Das University, 226028, Lucknow, India
3 Professor, Department of Electrical Engineering, Jamia Millia Islamia, 110025, New Delhi, India

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* **Corresponding author**.

ishaq.dmrc@gmail.com

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Abstract

Objectives: This research is based on the real-world study of Mumbai Metro, which focuses on enhancing the efficiency of metro rail systems by optimizing the timetable to synchronize train movements. The primary goal is to minimize total energy consumption by maximizing the exchange of regenerative-braking energy. **Methods:** Mumbai Metro Rolling Stock is having regenerative braking systems, allowing surplus energy regenerated during braking to be transferred to other trains in the same electrical rail section. To achieve this optimization, a graphical method is used to coordinate the time synchronization of the braking train arriving at a station with the accelerating train departing from the same or different station. This coordination ensures that regenerative-braking energy is effectively utilized and distributed among trains within the system. By quantifying the energy savings resulting from synchronization, this model provides an incentive for implementing more efficient train scheduling practices. The application of this model was demonstrated through the design of schedules for the Mumbai Metro. The implementation resulted in measurable energy savings, showcasing a clear correlation between train movement synchronization and reduced energy consumption. Importantly, this optimization strategy does not compromise overall passenger service quality and can be implemented with relatively low investment costs. In conclusion, adjusting the timetable in force based on principles of time synchronization and energy optimization has the potential to yield considerable energy savings for metro rail systems. **Findings:** This research emphasizes the importance of strategic train scheduling in maximizing the efficiency and sustainability of public transportation networks using regeneration of electrical energy. By implementing the optimized timetable, could lead to approximately a 5.03 % increase in regenerative energy utilization which would save 2.2 % in total energy consumption. **Novelty:** This study introduces a novel method to assess energy cooperation between trains using timetable optimization because this is the first real-world study of its kind conducted on any Indian metro system in which if one train is in braking mode then another train in the same electrical section should be in motoring mode so as to avoid the losses of the regenerated energy.

Keywords: Optimization; Headway; Dwell time; Coasting period; Traction energy

1 Introduction

With rising energy costs and growing environmental concerns, there is a growing emphasis on energy-efficient train operations. This involves optimizing speed profiles and timetables between consecutive stations to minimize energy consumption and reduce operational costs. Extending coasting periods[1] can decrease energy use, but maintaining a strict schedule remains essential.

Previous research has explored energy-saving operations through Automatic Train Operation (ATO) having a hidden time[2] for coasting. This introduces an integrated model for optimizing both train scheduling and train trajectory[3], aiming to minimize energy consumption in an AC-based traction system. Particle swarm optimization models and other genetic algorithm models[4] are used to solve energy consumption optimization problems. Non-sorting genetic algorithms may be used for multi-objective optimization problems[5] considering flexible train operation levels, speed profiles, and passenger demands. This is how ATO has become more relevant, offering precise control that enhances speed profiles and scheduling for energy efficiency.

Regenerative braking, which recovers energy without mechanical braking, is key to energy-saving strategies. Reduction in traction energy consumption[6] and maximization of regenerative braking energy exchange utilization are critical methods for optimizing energy use in metro systems. These methods are linked to driving strategy and timetable adjustments. This highlights the importance of balancing passenger demand with energy efficiency, focusing on minimizing energy consumption across the whole metro route while maximizing the reuse of regenerative braking energy[7]. There are various algorithms used for the maximization of regenerated energy[8], which will in turn increase the system's efficiency. Hybrid onboard energy storage systems[9] may also be used for restoring the regenerated electrical energy to avoid wastage. This storage process will also optimize energy use. Onboard advisory systems[10] are also helpful during manual driving operations of metro railways using minimal mechanical braking. This will also avoid excessive energy consumption and, hence will optimize energy use.

Optimizing metro timetables is essential for efficient, reliable, and sustainable train operations. Metro systems serve millions, reducing traffic congestion and pollution. Effective operations rely on precise timetable circulation planning[11] to enhance efficiency and passenger satisfaction. Recently, there has been a focus on energy-efficient timetables, especially by synchronizing train movements to maximize regenerative braking[12], which captures braking energy and reuses it for acceleration, lowering external power use and emissions. Timetable optimization balances passenger demand, service reliability, and costs, using advanced models and real-time data analytics to schedule train movements and optimize factors like dwell times and headways[13]. Simulation techniques also help operators assess timetable changes to optimize energy consumption.

Dynamic passenger flow[14], which affects train mass, significantly influences energy consumption in metro systems. A comprehensive model optimizes train scheduling and circulation planning[15] for energy efficiency, using a linear programming approach[16] to increase the synchronization time and hence maximize regenerative energy savings.

In modern metro systems, braking is typically provided by the pneumatic braking system as well as the regenerative braking system. Pneumatic braking systems reduce the train's speed by applying friction on brake blocks, converting the train's kinetic energy into heat, which is wasted in the environment. In the case of regenerative braking systems, the train's kinetic energy is converted into electrical energy using the traction motors (TM) which in turn slows the train. In summary, optimizing metro timetables integrates energy efficiency, passenger satisfaction, and operational reliability. By using advanced modeling and real-time data, metro operators can create sustainable, efficient schedules, maximizing regenerative braking energy reuse when another train consumes power in the same section.

2 Methodology

2.1 Power flow in the traction system of a Rolling Stock

Power flow in a rolling stock of an AC traction system involves the following stages:

1. Power Supply: Overhead lines are set up to supply electrical power.
2. Power Collection: Power for the train is collected by pantograph shoes.
3. Transformer: Single phase AC transformer is used to supply the powers of various voltage levels as per the requirements to the traction system and to auxiliary system of rolling stock.
4. Power Conversion: AC power is first converted into DC power with converters and later converted back into three-phase AC power using variable voltage variable frequency drives to drive traction motors.
5. Traction Motors: The train is driven by three-phase AC synchronous traction motors.

Figure 1 depicts how the power flows from the overhead lines to traction motors when they are in motoring mode {Overhead supply → Pantograph → Transformer → Traction Converter/Inverter → Traction Motor}, as well as when they are in braking mode {Overhead supply ← Pantograph ← Transformer ← Traction Converter/Inverter ← Traction Motor}.

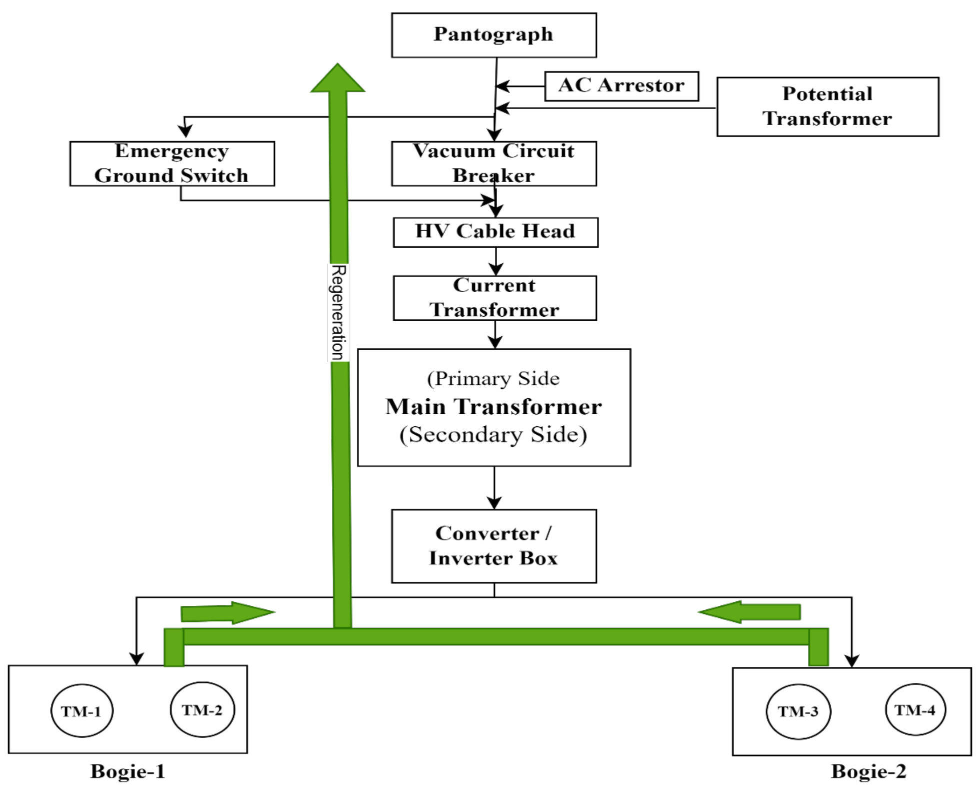

Fig 1. Power flow during motoring & braking (regeneration) of Rolling Stock

2.2 Speed–time characteristics of a Rolling Stock during various modes of operations

As shown in Figure 2, during motoring mode, the speed of the train increases (acceleration), requiring much energy, as the train uses tractive power to overcome resistance. While cruising, less power is required. During coasting, the train has no requirement for tractive power to move ahead and its speed is controlled by resistance and gravity. In the braking mode, the train comes to slow down by pneumatic brakes or regenerative braking systems.

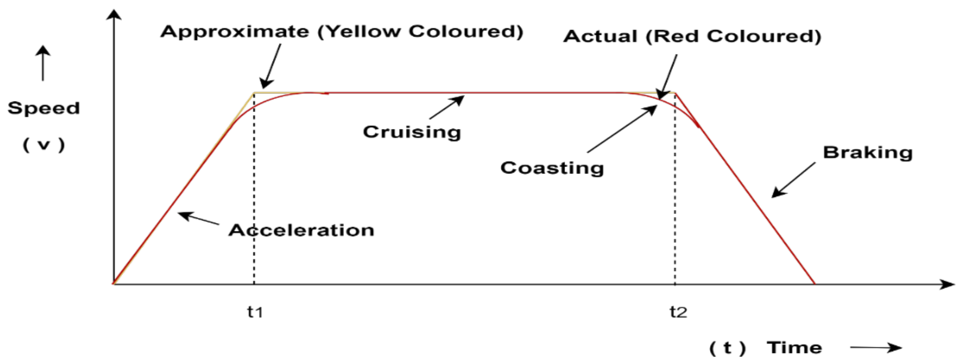

Fig 2. Speed–time characteristics of Rolling Stock during various modes of operations

2.3 Energy Consumption During Movement of a Rolling Stock (Mathematical Model)

2.3.1 Train Kinematics

For calculating the total energy consumption during the movement of rolling stock, we may calculate the traction and auxiliary energy consumption separately and then add them to have the total energy consumption. For calculating the traction force, we shall draw a free-body-diagram as shown in Figure 3, then apply Newton's laws of motion as per Equation (1).

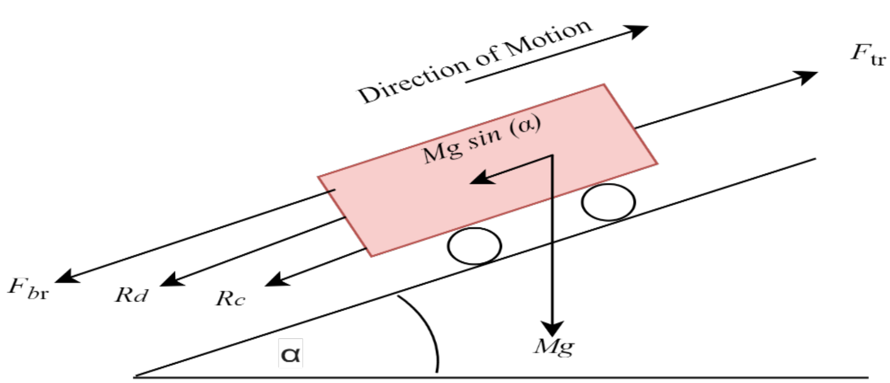

Fig 3. Various forces acting on a Rolling Stock during motion

According to Newton's laws of motion.[17]

$$F_{tr}(v) = F_{total} + F_{br}(v) + R_d(v) + R_c(v) + F_g \tag{1}$$

Where, $R_d = a + b\,v + c\,v^2 R_c(v) = \frac{600}{R_{ad}} Mg$

F_{total} = total force v = train speed;
F_{tr} = Tractive force F_{br} = braking effort
R_d = Dynamic resistance R_c = Curve resistance

2.3.2 Calculation of Traction Power Consumption

The energy required to produce the tactive effort is calculated using Equation (2) as follows:

$$P\,(t) = \ F_{tr} * v$$

Hence, the energy consumption over the time interval t_1 to t_2 is the integral of power over the operating cycle:

$E_{traction_{total}} = \int_{t_1}^{t_2} P(t)\,dt$

Now, the regenerative energy:

$E_{regen} = \eta_{regen} * \int_{t_1}^{t_2} P_{regen}(t)dt$

Hence,

net traction energy consumption

$$E_{tr_{net}} = E_{traction_{total}} - E_{regen} \tag{2}$$

2.3.3 Calculation of Auxiliary Power Consumption

The energy consumption at the tertiary winding is calculated using the Potential transformer and Current Transformer as shown in Equation (3). We know that Power consumption is written as

$$P_t = E_t * I_t$$

Auxiliary Power Supply (APS) calculates the energy consumption in every cycle and sends the value to the Train Control and Monitoring System (TCMS) in every communication cycle. TCMS collects and calculates the difference in the last count. TCMS integrates the calculated difference as the total energy consumption value.

So, the total auxiliary power consumption is calculated using the integral:

$P_{aux_{total}} = \int_{t_1}^{t_2} P(t)d\mathrm{t}$

Hence,

Total auxiliary energy consumption

$$(E_{aux_{total}}) = \frac{P_{aux_{total}}}{t} \tag{3}$$

2.3.4 Calculation of Total Energy Consumption

We know that the total energy consumed by a rolling stock is nothing but the combination of net traction energy used for providing the motion to the Rolling Stock and auxiliary energy which is used for train lighting, compressor, HVAC, battery charger, DC loads, etc. and may be calculated using Equation (4).

Total Energy consumption = Net traction energy consumption + Total auxiliary energy consumption

$$E_{total} = E_{tr_{net}} + E_{aux_{total}} \tag{4}$$

2.4 Optimization of Energy Involved in Rolling Stock

2.4.1 Time table Approach for synchronization of braking trains

The main objective of timetable optimization is to optimize energy consumed by maximizing the recovery of regenerative energy through a synchronized braking process for all trains operating within the same electrical section. The following constraints must be considered for optimization-

1. Terminal stations must ensure a minimum stopping time including the reversal time.
2. The dwell time for each train at each station must be fixed.
3. The total travel time for each train should not exceed the trip time in order to maintain the passenger service quality.

2.4.1.1 Design Flow for Optimal Time Table. The design of an optimized timetable (optimal timetable)[18] is based on the following points as shown in Figure 4.

1. Timetable in Force: The implemented and current operational timetable on the basis of which trains are running normally.
2. Timetable Adjustment: This step involves the adjustments in the current timetable which are necessary to achieve better synchronization.
3. Power Saving Model: A power-saving model is introduced to analyze the possibility of changing patterns of the current timetable which can minimize energy usage during train operations.
4. Optimal Timetable: The timetable adjustments and power-saving model are used to develop an optimal timetable that will optimize overall performance, including operational efficiency and energy consumption.

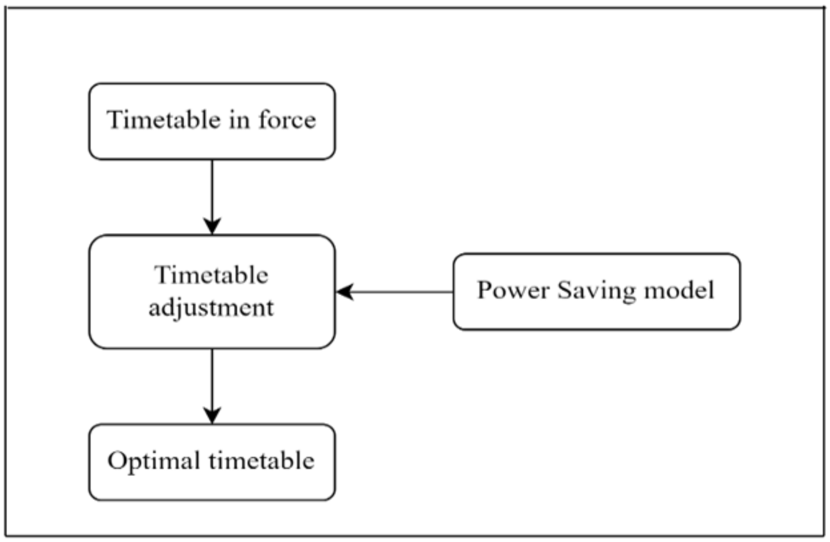

Fig 4. Design flow for optimal timetable

2.4.1.2 Placement and time synchronization of trains at stations. In the case of time-table optimization, to maximize the effect of the train's regenerative braking systems, a time-based braking synchronization strategy is employed for each station pair considering a scenario where one train arrives at a station simultaneously with another train departing from the same or a different station as shown in Figure 5.

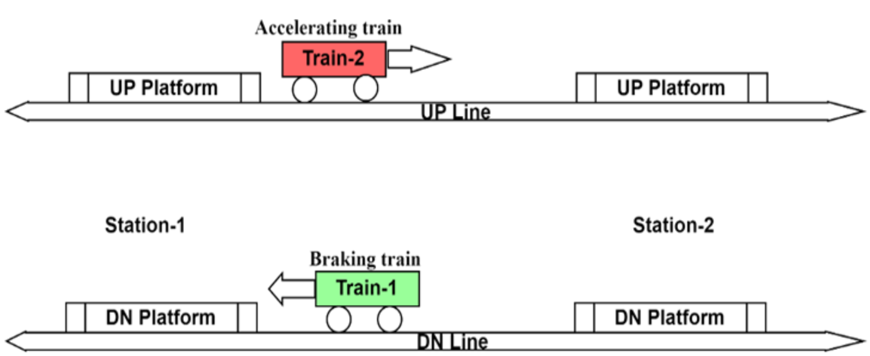

Fig 5. Arrangement of stations and placement of trains

If Train-1 is braking while Train-2 is motoring in the same electrical section, the regenerative power produced by Train-1 can be utilized by Train-2, thereby enhancing energy efficiency. This time synchronization is shown below in Figure 6.

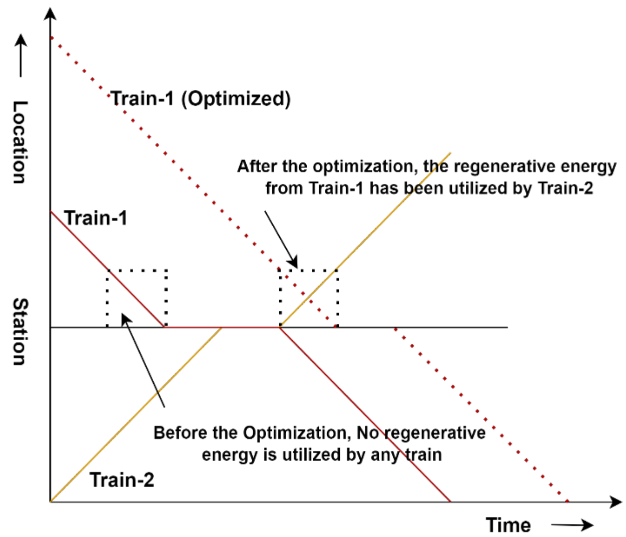

Fig 6. Regenerative braking for Train-1 & 2

2.4.1.3 Features of time table in force. The interstation journey time was measured as per the revenue service time table in force as shown below in Table 1.

Table 1. Time table in force

Station Number	Interstation Journey Time (Seconds)	
	UP Line	DN Line
1	0	119
2	119	126
3	126	119
4	119	136
5	136	155
6	155	124
7	124	183
8	183	136
9	136	123
10	123	153
11	153	161
12	161	116
13	116	169
14	169	116
15	116	169
16	169	125
17	125	169
18	169	0

Terminal Station-1: Dhanukarwadi Terminal Station-2: Aarey
Headway between two trains = 11 minutes 40 seconds. (Constant Headway)

Dwell time at platform = 30 seconds.
No of trains: Train-1, Train-2, Train-3, Train-4, Train-5, Train-6, Train-7, Train-8
Total Run Time = 2399 seconds = 39 min 59 sec.
Distance between terminal stations = 18.6 km.
Reversal Time = 2 min 01 sec.
Total Time for one round trip = 42 min.
Number of trips (Revenue services) = 77
Duration of revenue service = 16.0 hours (From 06:00 hrs. to 22:00 hrs.)

2.4.1.4 Time-table Optimization using Multi-Integer Linear Programming Model. For the optimization of the metro timetable to maximize the utilization of regenerative energy exchange at potential meeting points, an objective function is designed to minimize the total energy consumption[19] while coordinating train schedules.

1. Objective Function

The objective is to minimize the total energy consumption over all the interstation segments for both the UP and DN lines, considering energy savings from regenerative braking where possible:

$$Minimize\ E = \sum_{i}^{N} [E_{traction,\ UP}(i) + E_{traction,\ DN}(i) - E_{regen}(i)] \tag{5}$$

where:

$E_{traction,\ UP}(i)$ is the energy consumed by the train on the UP line between stations i and i+1
$E_{traction,\ DN}(i)$ is the energy consumed by the train on the DN line between stations i and i+1.
$E_{regen}(i)$ is the regenerative energy captured and utilized by a nearby accelerating train.

2. Constraints

i. Journey Time Constraints:

$$T_{min} \le T_{journey,\ UP}(i) \le T_{max} \,\&\, T_{min} \le T_{journey,\ DN}(i) \le T_{max} \tag{6}$$

ii. Synchronization for Regenerative Braking: To maximize energy exchange, the braking and accelerating phases of trains at potential meeting point stations (2, 3, 6, 9, 13, and 16) would be synchronized:

$$T_{UP}(i) - T_{DN}(i) \le \Delta T \tag{7}$$

where ΔT is a small tolerance that allows for effective energy exchange.

In this optimization (See Figure 7), time synchronization for braking on UP and DN lines is analyzed based on interstation journey times.

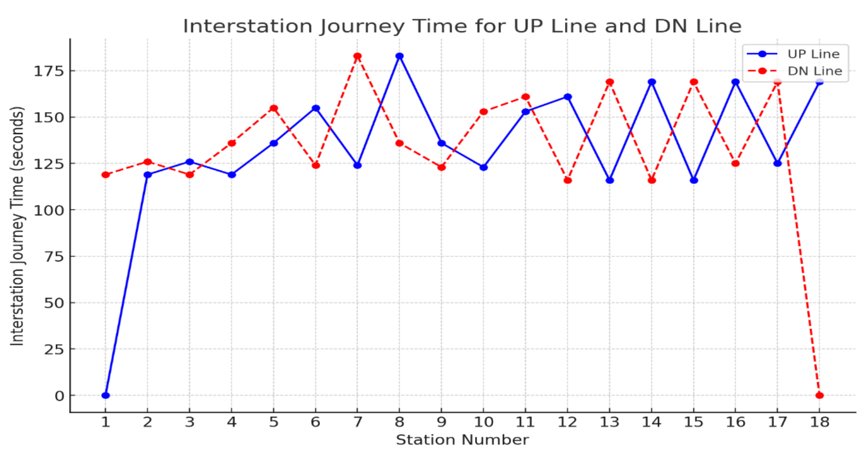

Fig 7. Graphical representation of interstation journey time

Therefore, to decide where trains on the UP and DN lines pass each other in an optimized timetable, we shall concentrate on those stations where the interstation travel time between successive stations on UP and DN lines is similar or aligned for efficient exchange of regenerative power. Train that covers the same distance, but over the same period, should provide the meeting points. The basis for a good timetable optimization with trains running on both lines optimally is given by the following points.

1. A graph is plotted for interstation journey time vs station number, where the journey times for running trains are similar or aligned, suggesting potential meeting points for the trains traveling in opposite directions.
2. By visually inspecting the optimal points where the UP and DN lines cross or have close values, we can locate the perfect stations on which the trains shall pass each other.
3. Finally, we graphically compare the train schedule and optimize the timetable in such a way that the trains meet at stations without any delay or any other operational conflict.

Potential train Passing Stations on UP & DN Line

1. Station 2 (UP Line: 119, DN Line: 126): The journey times are close, suggesting that trains could be passing near this station.
2. Station 3 (UP Line: 126, DN Line: 119): Similar journey times between stations 2 and 3 on both lines, which may align the train paths.
3. Station 6 (UP Line: 155, DN Line: 124): The journey times between station 5 and station 6 on the UP line and station 5 and station 6 on the DN line are close, possibly indicating another passing point.
4. Station 9 (UP Line: 136, DN Line: 123): Journey times between stations 8 and 9 are close on both lines, making it another candidate for a passing point.
5. Station 13 (UP Line: 116, DN Line: 116): Identical journey times, indicating a high likelihood of both trains passing near this station.
6. Station 16 (UP Line: 169, DN Line: 125): While there is a gap, it is possible they might pass each other either near station 16 or in the stretch between stations 15 and 16.

3 Results and Discussion

Energy data is downloaded from TCMS by using a portable test unit laptop and the same is plotted for total energy consumption, regeneration, and net energy consumption for time table in force and optimized time-table as shown below.

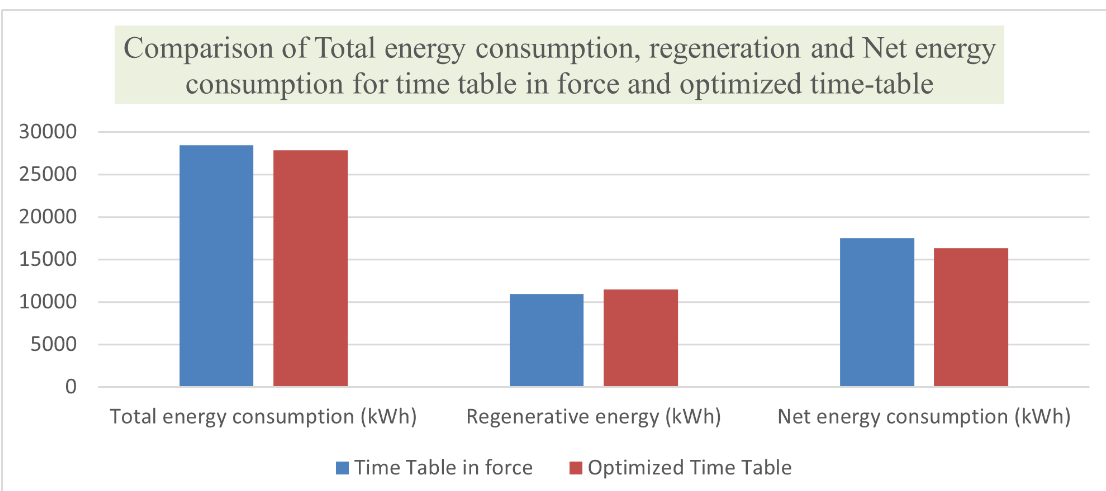

Fig 8. Comparison of total energy consumption, regeneration, and net energy consumption for the timetable in force and optimized time-table

Figure 8 shows that after implementing the optimized timetable, the whole day's journey time remains unchanged. The difference in the total energy consumption for the timetable in force and the optimized timetable is minimal, which indicates similar passenger flow on normal passenger service days and trial days. However, there is a 5.03% increase in regenerative braking energy utilization, from 10,939 kWh to 11,489 kWh, showing an improved energy efficiency. This reduces total energy consumption from 28463 kWh to 27837 kWh without affecting revenue service.

Several studies have delved into the potential of regenerative braking and timetable optimization to improve energy efficiency in rail systems:

Table 2. Comparison of the proposed method with existing literature

Study	Authors	Focus	Key Findings	Application
Timetable Optimization Model for Guangzhou Metro	(Zhao et al., 2022)	Timetable optimization to improve synchronization between braking and motoring trains	Achieved a 4.1 % utilization of regeneration [20] through a trial test of an optimized timetable	Real-world application in Guangzhou Metro
Multi-train Optimization	(Zhang et al., 2024)	Maximizing the use of regenerative braking energy in subway systems through multi-train optimization	Showed improvements in regenerative -energy [21] utilization, but confined to simulations only.	Simulation-based study
Improved differential evolution algorithm optimization	(Liu et.al. 2020)	Enhance the utilization of regeneration by optimizing dwell time, head-way, and bi-directional departure interval.	Showed improvements in regenerative -energy [22] utilization, but confined to simulations only.	Simulation-based study
Timetable Optimization for Mumbai Metro	Current Study (2024)	Real-world trial to optimize timetable for Mumbai Metro Line 2 & 7 for regenerative braking energy utilization	Conducted real-world trial on Mumbai metro Line 2 & 7 focusing on improving regenerative energy utilization by 5.03 %	Real-world trial for the Indian metro system

While the other available studies underscore the theoretical benefits of timetable optimization in improving the regenerative braking energy utilization which in turn saves the total energy consumption, most remain within the realm of simulations. This proposed research addresses this gap by conducting a real-world trial on Mumbai Metro's Lines 2 & 7, evaluating the practical impact of an optimized timetable on energy savings through regenerative braking utilization.

4 Conclusion

It is concluded that by making slight changes in the Mumbai Metro's timetable in force duly keeping the published schedules unchanged, has resulted in an approximately 5.03 % increase in regenerative energy utilization, which is better than the others compared with as shown in table-2. Regeneration of energy can be improved in the future if we consider dwell time to be variable in the optimization.

Acknowledgment

We would like to thank Mumbai Metro management for providing the necessary resources and facilities to conduct this research effectively.

Ethical Statement

Data used in this study is taken from the real-world data of Mumbai Metro Line-2 & 7 and proper ethical permission for the purpose of research study is already taken.

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