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RESEARCH ARTICLE

Generalized Fuzzy Technique and its Consistent Assessment in Multicriteria Decision Making of Medical Decisions

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Abstract

Objectives: The purpose of this study is to develop a decision-making expert system to assist the diagnostic decisions effectively. Generalized fuzzy sets (Vague sets) are used to model the uncertainty that exists in the process. **Method:** Initially, concepts of centroid and signed distance are generalized for Trapezoidal Vague Numbers (TVNs). This model introduces some desirable properties for the proposed distance measure. A multicriteria decision-making (MCDM) is introduced using trapezoidal vague numbers (TVNs) and other intuitions in medical diagnostics. In this MCDM, firstly, each symptom is classified into some criteria. Patients' states and weights for the existing symptoms are assumed as vague point entries. As a mathematical tool, the present model used vague relations for illustrating different associations like symptom criteria and diseases and the state of the patients. The present model also discusses confidence interval-based statistical analysis for the construction of TVNs, which are the prominent component of the study. **Findings:** A numerical computation is illustrated with the whole procedure of the decision-making. To show the capability and distinctness of the proposed methodology, a comparative discussion for outcomes is given. The present method coincides with the existing fuzzy-based method for the initial diagnosis, while possibility degrees of diagnoses differences for patient P_1 and patient P_2 differ. It ranges from 1% to 16.3% in the method based on the existing traditionally fuzzy approach, and in the present approach, these differences range from 2.15% to 7.46%. **Novelty:** This research contributes to presenting a novel decision expert system that is more capable of making diagnosis efficient. This study provides a practical and visual tool to assess potential outcomes of the proposed technique through a numerical example.

Keywords: Vague set; Lower and upper membership function; Vague point; Trapezoidal vague number; Medical diagnosis

1 Introduction

Healthcare is an area in which the applicability of fuzzy sets was recognized quite early due to the uncertainties involved in the process of diagnosis. Multiple diseases in a single patient might disrupt the typical symptom pattern, and a single symptom may signal more than one condition. The inability to properly assign relative weights to the facts in order to convey their degree of relevance in the decision-making process adds to the severity of the diagnostic problem. The fuzzy set theory provides an effective way for utterance of the decision information dealing with uncertainty-based information. This theory has been applied extensively to many research problems. Attur and Harikumar utilize the fuzzy inference system for Alzheimer disease[(1)]. Navin and Mukesh Krishnan develop a clinical decision support system using fuzzy logic[(2)]. Dhiman and Sharma provide a fuzzy inference system to identify COVID-19, and they also suggested a fuzzy-based approach to prevent the disease[(3)]. Gupta et al. examine fuzzy logic-based systems in medical diagnosis[(4)]. Kumar and Pandey predicted the waiting time targets for outpatients in healthcare units using fuzzy linear programming[(5)]. Hosseinzadeh et al. described the determination of fuzzy weights in multicriteria decision-making[(6)]. Turkarslan et al. created the cosine similarity measure and used it in medical diagnosis[(7)]. Shoaip et al. attempt to integrate ontology semantics reasoning and fuzzy inference and implement it on Alzheimer's disease[(8)]. A vague set takes into account the favorable and unfavorable evidence separately, providing a lower bound and an upper bound within which the membership grade may lie. The notion of vague sets is similar to intuitionistic fuzzy sets. But it assumes interval estimation for the membership to deal with the hesitation part effectively. Cui et al. presented a decision-making approach based on vague sets; they utilized vague sets and evidence theory to finalize the decision-making[(9)]. Multicriteria group decision-making using intuitionistic fuzzy numbers was used in COVID-19 by Dutta and Borah[(10)]. Kumar and Pandey found inconsistencies in the switching between type-2 fuzzy sets and intuitionistic fuzzy sets and improved it[(11)]. A weighted distance measure of a linguistic intuitionistic fuzzy set was proposed by Kumar and Chen[(12)]. Kumar et al. provided a brief note on vague set theory[(13)]. They applied vague set theory in career determination using assumed data. Aung et al. applied multicriteria decision-making to analyze medical waste management systems[(14)]. Recently, numerous researchers have addressed the multicriteria decision-making in medical decisions. Anzilli and Giove introduce a multicriteria approach that is based on non-additive measures and the Choquet-integral, applied in medical diagnosis[(15)]. Wei et al.'s approach was utilized with multicriteria decision-making using intuitionistic normal cloud and cloud distance entropy[(16)]. Sayan et al. use multicriteria decision-making techniques to determine the capacity evaluation of diagnostic tests[(17)]. Fu et al. develop a data-driven MCDM framework in the context of the evidential reasoning approach, utilize it to model the diagnosis of thyroid cancer, and generate data-driven diagnostic results[(18)]. Panattoni et al. identify the multicriteria decision-making for the feasibility of measuring preferences for chemotherapy among early-stage breast cancer survivors using a direct rank ordering[(19)]. Uzoka et al. propose an analytic hierarchy-process-based decision support system in the diagnosis of typhoid fever[(20)]. Very recently, Garg et al. proposed a multicriteria decision-making technique that ranks the four alternatives[(21)]. Momena et al. use the dual-hesitant hexagonal fuzzy multicriteria decision-making technique to diagnose the disease[(22)]. Mustafa et al. present two types of expert systems for medical diagnosis and compare achieved results with those of human experts[(23)]. Sirocchi et al. proposed a medical diagnostic decision model using machine learning[(24)]. Göndöcs and Dörfler utilize artificial intelligence in medical decisions[(25)]. Kumar develops a rule base for early diagnosis of the pneumonia disease[(26)].

Due to the inherent uncertainty available, the medical diagnostic procedure requires a more advanced uncertainty handling technique. Also, all existing strategies examine pertinent multicriteria decision models and concentrate on rating the choices, i.e., analytic hierarchy processes. As per our best knowledge, no method for categorizing the symptoms into criteria exists. In response to previous studies and the requirement for the diagnostic system's capacities to manage further uncertainty reduction, the current method presents a novel multicriteria-based generalized fuzzy strategy by categorizing symptoms into criteria. Furthermore, vague relations are utilized to estimate the occurrence of diseases. The basic objective of the technique entails the following key points:

- To create a signed distance and centroid for trapezoidal fuzzy numbers.
- To propose statistical analysis for non-membership estimates as well as for TVN estimates.
- To extend the methodology for a novel multicriteria-based generalized fuzzy technique.

Finally, we illustrated the usefulness of the proposed technique.

The rest of the research article is structured as follows: Section 2 gives basic concepts of vague sets, the signed distance between two TVNs, and the centroid of a trapezoidal vague number. The architecture of the decision-making using TVNs and numerical computation are also proposed in the same section. Finally, comparative result analysis and conclusion of the work are given in Sections 3 and 4, respectively.

2 Methodology

2.1 Vague set[27]

A vague set U is characterized by a membership function $\mu_{\widetilde{A}} : U \rightarrow [0,1]$and a non-membership or false membership function, $v_{\widetilde{A}} : U \rightarrow [0,1]$bounded by $\mu_{\widetilde{A}}(u)$, and $1 - v_{\widetilde{A}}(u) = \mu'_{\widetilde{A}}(u)$. Expressions given below can be used to represent a vague set $\widetilde{A}$ for discrete and continuous universes of discourse U, respectively.

$\widetilde{A} = \sum_{n=1}^{m} [\mu_{\widetilde{A}} \quad (u_n), \quad 1 - v_{\widetilde{A}} \quad (u_n)] / u_n,\ u_n \in U\ for\, n = 1, 2, \ldots, m;\ \widetilde{A} = \int_{u \in U} [\mu_{\widetilde{A}} \quad (u), \quad 1 - v_{\widetilde{A}} \quad (u)] / u.$

Vague set in continuous universe of discourse is represented pictorially as follows:

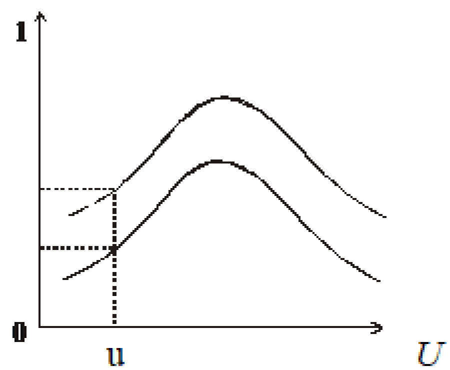

Fig 1. Vague set

The lower curve shows the $\mu_{\widetilde{A}}(u)$ – lower bound of association, and the upper curve shows $1 - v_{\widetilde{A}}(u) = \mu'_{\widetilde{A}}(u)$ the upper bound of association.

2.2 Convex vague set[27]

Let $\tilde{A}$ be a vague set of the universe of discourse U with $\mu_{\tilde{A}}$ and $v_{\tilde{A}}$ as its membership and non-membership functions, respectively. The vague set is convex if and only if for every $u_1 u_2 \in U$, $\mu_{\widetilde{A}} \quad (\lambda u_1 + (1-\lambda) u_2) \geq min \quad (\mu_{\widetilde{A}} \quad (u_1), \quad \mu_{\widetilde{A}} \quad (u_2))$, and $1 - \quad v_{\widetilde{A}} \quad \lambda u_1 \quad + (1-\lambda) u_2) \geq min(1 - \quad v_{\widetilde{A}} \quad (u_1) \ , 1 - \quad v_{\widetilde{A}} \quad (u_2)$ where $\lambda \in$ [0,1]

2.3 Normal vague set[27]

A vague set $\tilde{A}$ in the universe of discourse U is called normal, if $\exists u_i \in U$ such that $1 - \quad v_{\widetilde{A}} \quad (u_i) = 1$

Vague number[27]: A vague number is a vague set in the universe of the discourse U that is both convex and normal.

2.4 Trapezoidal vague number[27]

A Trapezoidal vague number Ã is characterized by a pair of membership functions: a lower membership function and an upper membership function, respectively:

$$\mu_{\widetilde{A}} \quad (u) = \begin{cases} \dfrac{k(u-a)}{b-a}, & a \leq u \leq b \\ k, & b \leq u \leq c \\ \dfrac{k(d-u)}{d-c}, & c \leq u \leq d \end{cases}, \qquad \mu'_{\widetilde{A}} \quad (u) = \begin{cases} \dfrac{(u-a)}{b-a}, & a \leq u \leq b \\ 1, & b \leq u \leq c \\ \dfrac{(d-u)}{d-c}, & c \leq u \leq d \end{cases} \tag{1}$$

where $1 - v_{\widetilde{A}}(u) = \mu'_{\widetilde{A}}(u)$ and $k \in \ [0,1]$

It is denoted by using the notation: [(a, b, c, d); k; 1]. When k = 1, the trapezoidal vague number reduces to a trapezoidal fuzzy number. In a trapezoidal vague number (TVN), $\widetilde{A}$= [(a, b, c, d); k; 1], if a=b=c=d, say, then $\widetilde{A}$= [(b, b, b, b); k; 1] = $\widetilde{b}_k$ is said to be a vague point.

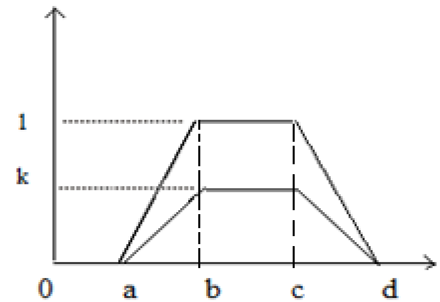

Fig 2. Trapezoidal vague number

2.5 Centroid of a TVN

The centroid of a trapezoidal number [(a, b, c, d)**;** k; 1] is defined as:

$C_{\widetilde{A}} = \frac{\int_{-\infty}^{\infty} u[\mu_{\widetilde{A}}(u) - \mu'_{\widetilde{A}}(u)]du}{\int_{-\infty}^{\infty} [(\mu_{\widetilde{A}}(u) - \mu'_{\widetilde{A}}(u)]du}$, for$k \in [0,1]$;

$C_{\widetilde{A}} = \frac{\int_{-\infty}^{\infty} u\mu_{\widetilde{A}}(u)du}{\int_{-\infty}^{\infty} \mu_{\widetilde{A}}(u)du}$, for k = 1.

$\mu_{\widetilde{A}}(u)$ and $\mu'_{\widetilde{A}}(u)$are used from Equation (1) and computing the values of the integrals, the centroid of a TVN is

$$C_{\widetilde{A}} = \frac{(a^3 - 3ab^2 + 2b^3)(d-c) + 3(c^2 - b^2)(b-a)(d-c) + (d^3 - 3dc^2 + 2c^3)(b-a)}{3(b-a)(d-c)[(b-a) + 2(c-b) + (d-c)]} \tag{2}$$

where a $\neq b \neq c \neq d$. If $a \neq b = c \neq d$ then $C_{\widetilde{A}} = \frac{1}{3}(a+b+d)$for every value of k. Let

$$\Upsilon_N(k) = \{[(a,b,c,d);k;1] : a < b < c < d,\ a,b,c,d\ \in R,$$

and $\Upsilon_P(k) = \{[(b,b,b,b);k;1] : b\ \in R, k \in [0,1]\}$ be the sets of TVNs, and vague points, respectively. It is obvious that $\Upsilon_P(k) \subset \Upsilon_N(k)$. The operations $\oplus$ *and* $\otimes$ can be defined for TVNs as follows:

$$[(a_1,b_1,c_1,d_1);k_1;1] \oplus [(a_2,b_2,c_2,d_2);k_2;1] = \left[(a_1+a_2, b_1+b_2, c_1+c_2, d_1+d_2); min(k_1,k_2);1\right] \tag{3}$$

$$[(a_1,b_1,c_1,d_1);k_1;1] \otimes [(a_2,b_2,c_2,d_2);k_2;1] = \left[(a_1a_2, b_1b_2, c_1c_2, d_1d_2); min(k_1,k_2);1\right] \tag{4}$$

Property 1: If $\widetilde{A},\widetilde{B} \in \Upsilon_N(k)$ and $\tilde{l}_k, \widetilde{m}_k,\ and\ \tilde{n}_k \in \Upsilon_P(k)$, then

a) $\tilde{l}_k \oplus \widetilde{m}_k = \widetilde{l+m}_k, \tilde{l}_k \otimes \widetilde{m}_k = \widetilde{lm}_k$

b) If $\widetilde{A} = [(a_1,b_1,c_1,d_1);k;1]$ and $\widetilde{B}$= $[(a_2,b_2,c_2,d_2);k;1]$, then $\widetilde{A} \oplus \widetilde{B}$= $[(a_1+a_2, b_1+b_2, c_1+c_2, d_1+d_2);k;1]$

c) If $\widetilde{A} = [(a_1,b_1,c_1,d_1);k;1]$, then $\tilde{l}_k \otimes \widetilde{A} = [(la_1, lb_1, lc_1, ld_1);k;1]$ for l > 0 and = [(0, 0, 0, 0); k; 1]$\in \Upsilon_P(k)$ for l = 0.

Proof (a): Let $\tilde{l}_k = [(l,\ l,\ l,\ l);k;\ 1]$ and $\widetilde{m}_k$=$[(m,\ m,\ m,\ m);k;\ 1]$. Using Equation (3)

$$[(a_1,b_1,c_1,d_1);k_1;1] \oplus [(a_2,b_2,c_2,d_2);k_2;1] = \left[(a_1+a_2, b_1+b_2, c_1+c_2, d_1+d_2); min(k_1,k_2);1\right]$$

So, $\tilde{l}_k \oplus \widetilde{m}_k$= $[(l+m, l+m, l+m, l+m); min(k,k);1]$, $\tilde{l}_k \oplus \widetilde{m}_k$=$[(l+m, l+m, l+m, l+m);k;1]$
$\tilde{l}_k \oplus \widetilde{m}_k$= $\widetilde{l+m}_k$, Also $\tilde{l}_k \otimes \widetilde{m}_k$= $[(lm,lm,lm,lm); min(k,k);1]$, $\tilde{l}_k \otimes \widetilde{m}_k$= $[(lm,lm,lm,lm);k;1]$ = $\widetilde{lm}_k$

Proof (b) &(c): Similar to (a), the proof is quite obvious.

A TVN $\widetilde{A}$ has two membership functions-a lower function,$\mu_{\widetilde{A}}(u)$ and an upper one, $\mu'_{\widetilde{A}}(u)$. Correspondingly, it has α-cuts,

$$^{\alpha}\mu_{\widetilde{A}} \ = \ \left[a + \frac{\alpha}{k}(b-a),\ d - \frac{\alpha}{k}(d-c)\right] = \left[{}^{\alpha}_{l}\mu_{\widetilde{A}},\ \ {}^{\alpha}_{r}\mu_{\widetilde{A}}\right],\ \ {}^{\alpha}\mu'_{\widetilde{A}} \ = [a + \alpha(b-a),\ d - \alpha(d-c)] = \left[{}^{\alpha}_{l}\mu'_{\widetilde{A}},\ \ {}^{\alpha}_{r}\mu'_{\widetilde{A}}\right]. \tag{5}$$

2.6 Signed distance between two TVNs

The concept of the signed distance of triangular vague numbers is advanced to TVNs in this article. The signed distance between two TVNs $\widetilde{A}=[(a_1,b_1,c_1,d_1);k_1;1]$and $\widetilde{B}=[(a_2,b_2,c_2,d_2);k_2;1]$ is defined as:

$$d\left(\widetilde{A},\widetilde{B}\right)=\frac{1}{2}\left[\int_0^1\left(\frac{{}_l^{\alpha}\mu'_{\widetilde{A}}\;+{}_r^{\alpha}\mu'_{\widetilde{A}}}{2}-\frac{{}_l^{\alpha}\mu'_{B}\;+\;{}_r^{\alpha}\mu'_{\widetilde{B}}}{2}\right)d\alpha+\frac{1}{k_1}\int_0^{k_1}\frac{{}_l^{\alpha}\mu_{\widetilde{A}}\;+\;{}_r^{\alpha}\mu_{\widetilde{A}}}{2}d\alpha\right] -\frac{1}{k_2}\int_0^{k_2}\frac{{}_l^{\alpha}\mu_{\widetilde{B}}\;+\;{}_r^{\alpha}\mu_{\widetilde{B}}}{2}d\alpha \tag{6}$$

Using Equations (5) and (6) we get

$$d\left(\widetilde{A},\widetilde{B}\right)\;=\;\frac{1}{4}\left[(a_1-a_2)+(b_1-b_2)+(c_1-c_2)+(d_1-d_2)\right] \tag{7}$$

Property 2: If $\widetilde{A}=[(a_1,b_1,c_1,d_1);k;1]$,$\widetilde{B}=[(a_2,b_2,c_2,d_2);k;1]\in\Upsilon_N(k)$, and [(0, 0, 0, 0); k; 1]$\in\Upsilon_P(k)$ then

$$a)\;d\left(\widetilde{A},\tilde{0}_k\right)=\frac{1}{4}(a_1+b_1+c_1+d_1) \tag{8}$$

$$b)\;d\left(\widetilde{A},\widetilde{B}\right)=d\left(\widetilde{A},\tilde{0}_k\right)-d\left(\widetilde{B},\tilde{0}_k\right) \tag{9}$$

Proof of (a): Let $\widetilde{A}=[(a_1,b_1,c_1,d_1);k_1;1]$ and $\widetilde{B}=\tilde{0}_k$= [(0, 0, 0, 0); k; 1]. Using Equation (7)

$d\left(\widetilde{A},\widetilde{B}\right)=\frac{1}{4}\left[(a_1-a_2)+(b_1-b_2)+(c_1-c_2)+(d_1-d_2)\right]$

$d\left(\widetilde{A},\widetilde{B}\right)=\frac{1}{4}\left[(a_1-0)+(b_1-0)+(c_1-0)+(d_1-0)\right]$, $d\left(\widetilde{A},\tilde{0}_k\right)=\frac{1}{4}(a_1+b_1+c_1+d_1)$.

Proof of (b): Using Equations (8) and (9), proof is trivial.

2.7 Architecture of the Decision-making using TVNs

Let S=$\{S_1,S_2,\ldots,S_n\}$, P=$\{P_1,P_2,\ldots,P_l\}$, and D=$\{D_1,D_2,\ldots,D_m\}$be sets of symptoms, patients, and diseases. For each $p\in\{1,2,\ldots,n\}$, $S_p=\left\{S_{p1},S_{p2},\ldots,S_{pn(p)}\right\}$be the set of symptom characters. $\mu_{\widetilde{R}}{}^p$: represents the lower bound of membership relation between symptoms criteria and diseases and $\upsilon_{\widetilde{R}}{}^p$: represents non- memberships which contributes the upper bound of the same relation for different values of p.

$$\mu^p{}_{\widetilde{R}}\quad(S_p,D)=\begin{bmatrix} r_{p11} & r_{p12} & \cdot\;\cdot & r_{p1m}\\ r_{p21} & r_{p22} & \cdot\;\cdot & r_{p2m}\\ \cdot & \cdot\;\cdot\;\cdot & & \cdot\\ \cdot & \cdot\;\cdot\;\cdot & & \cdot\\ r_{pn(p)1} & r_{pn(p)2} & \cdot\;\cdot & r_{pn(p)m}\end{bmatrix},\quad \upsilon^p{}_{\widetilde{R}}\quad(S_p,D)=\begin{bmatrix} c_{p11} & c_{p12} & \cdot\;\cdot & c_{p1m}\\ c_{p21} & c_{p22} & \cdot\;\cdot & c_{p2m}\\ \cdot & \cdot\;\cdot\;\cdot & & \cdot\\ \cdot & \cdot\;\cdot\;\cdot & & \cdot\\ c_{pn(p)1} & c_{pn(p)2} & \cdot\;\cdot & c_{pn(p)m}\end{bmatrix} \tag{10}$$

Where $0\le r_{pij}\le 1, 0\le c_{pij}\le 1$and…,i=1,2,…,n(p)j=1,2,…,m,. Substituting, $r'_{pij}=1-c_{pij}$, we get the upper bounds of the membership relation (upper membership relation):

$$\mu'^p{}_{\widetilde{R}}\quad(S_p,D)\;=\begin{bmatrix} r'_{p11} & r'_{p12} & \cdot\;\cdot & r'_{p1m}\\ r'_{p21} & r'_{p22} & \cdot\;\cdot & r'_{p2m}\\ \cdot & \cdot\;\cdot\;\cdot & & \cdot\\ \cdot & \cdot\;\cdot\;\cdot & & \cdot\\ r'_{pn(p)1} & r'_{pn(p)2} & \cdot\;\cdot & r'_{pn(p)m}\end{bmatrix} \tag{11}$$

The knowledge of the relations $\mu_{\widetilde{R}}{}^p\left(S_p,D\right)$ *and* $\mu'_{\widetilde{R}}{}^p(S_p,D)$ is based on previous existing medical experience. Vague membership relations between patients and symptom criteria (characters), i.e., $\mu_{\widetilde{A}}{}^p(P,S_p)$ and $\mu'_{\widetilde{A}}{}^p\left(P,S_p\right)$ are given as follows:

$$\mu'_{\widetilde{A}^p}\ (P,\, S_p) = \begin{bmatrix} a_{p11} & a_{p12} & . & . & a_{p\ln(p)} \\ a_{p21} & a_{p22} & . & . & a_{p\ln(p)} \\ . & . & . & . & . \\ . & . & . & . & . \\ a_{pl1} & a_{pl2} & . & . & a_{p\ln(p)} \end{bmatrix},\quad \mu'^{p}{}_{\widetilde{A}}\ (P, S_p) = \begin{bmatrix} a'_{p11} & a'_{p12} & . & . & a'_{pln(p)} \\ a'_{p21} & a'_{p22} & . & . & a'_{pln(p)} \\ . & . & . & . & . \\ . & . & . & . & . \\ a'_{pl1} & a'_{pl2} & . & . & a'_{pln(p)} \end{bmatrix} \tag{12}$$

2.7.1 Determination of weight

The weight vector $\widetilde{A} = (a_1, a_2, \ldots, a_n)$ for $(S_1, S_2, \ldots, S_n)$ is taken by the formula: $a_p = \frac{N_p}{N}$, p=1,2,…,n. $N = \sum_{p=1}^{n} N_p$, where N_p be the number of past patients for each p. This knowledge also depends upon previous medical experience.

2.7.2 Determination the grades r$_{pij}$, r'$_{pij}$

This information is based on previous medical data. To estimate the membership grade for each S_{pi}, 2,…, n(p), let

N_{pij}: Number of cases that have thesymptom character S_{pi} with diagnosis d_j;

N'_{pij}: Number of cases having symptom characters S_{pi} but not having the diagnosis d_j;

N_{pi}: Number of the total cases having symptom character S_{pi};

N'_{pi}: Number of the total cases not having symptom character S_{pi}, (i=1,2,..,.n(p); j=1,2,…,m). $\sum_{j=1}^{m} N_{pij} = N_{pi}$ for each i and p; $\sum_{j=1}^{m} N'_{pij} = N'_{pi}$ for each i and p; r_{pij} and r'_{pij} can now be determined by the following formulae:

$r_{pij} = \frac{N_{pij}}{N_{pi}}$ and $r'_{pij} = 1 - c_{pij}$ where $c_{pij} = \frac{N'_{pij}}{N'_{pi}}$, also $\sum_{j=1}^{m} r_{pij} = 1$,$\sum_{j=1}^{m} c_{pij} = 1$ for each $i = 1, 2, \ldots, n(p)$. Point estimates, r_{pij} and r'_{pij} are being transformed into interval estimates using statistical approach, 99% confidence interval as follows:

$$\left[r^{(1)}_{pij}, r^{(2)}_{pij}\right] = \left[r_{pij} - 2.575\sqrt{\frac{r_{pij}(1-r_{pij})}{N_{pi}}}, r_{pij} + 2.575\sqrt{\frac{r_{pij}(1-r_{pij})}{N_{pi}}}\right] \tag{13}$$

$$\left[c^{(1)}_{pij}, c^{(2)}_{pij}\right] = \left[c_{pij} - 2.575\sqrt{\frac{c_{pij}(1-c_{pij})}{N_{pi}}}, c_{pij} + 2.575\sqrt{\frac{c_{pij}(1-c_{pij})}{N_{pi}}}\right], \tag{14}$$

and $\left[r^{(1)}_{pij}, r^{(2)}_{pij}\right] = \left[1 - c^{(1)}_{pij}, 1 - c^{(2)}_{pij}\right]$. Using Equations (13) and (14), TVNs, $\tilde{r}^T_{pij}$ and $\tilde{r}'_{pij}{}^T$ (representing lower and upper memberships) are taken as follows:

$$\tilde{r}^T_{pij} = \left[r^{(1)}_{pij}, r^l_{pij}, r^u_{pij}, r^{(2)}_{pij}; k_{pij}; 1\right],\ \tilde{r}'_{pij}{}^T = \left[r^{(1)'}_{pij}, r^{l'}_{pij}, r^{u'}_{pij}, r^{(2)'}_{pij}; k'_{pij}; 1\right] \tag{15}$$

Property 3: For TVNs, $\tilde{r}^T_{pij}$, $\tilde{r}'_{pij}{}^T$ as given in Equation (15), distance properties can be stated as follows:

a) $d(\tilde{r}^T_{pij},\, 0_k) = \frac{1}{4}[r^{(1)}{}_{pij} + r^l{}_{pij} + r^u{}_{pij} + r^{(2)}{}_{pij}]$

b) $d(\tilde{r}'_{pij}{}^T,\, 0_k) = \frac{1}{4}[r^{(1)'}{}_{pij} + r^{l'}{}_{pij} + r^{u'}{}_{pij} + r^{(2)'}{}_{pij}]$

Proof: Using Equation (8), both properties can be proved easily.

For deriving a method of diagnostic decision based on TVNs, matrices, $\mu_{\widetilde{R}}{}^p(S_p, D)$ and $\mu'_{\widetilde{R}}{}^p(S_p, D)$given in Equations (11) and (12) are advanced using TVNs as the entries of the relations. Let $\mu_{\widetilde{R}*}{}^p\left(S_p, D\right) = \mu_{\widetilde{R}*}{}^p$ and $\mu'_{\widetilde{R}*}{}^p\left(S_p, D\right) = \mu'_{\widetilde{R}*}{}^p$ denote the advanced matrices, respectively as follows where the entries $\tilde{r}^T_{pij}$ and $\tilde{r}'_{pij}{}^T$ are given by Equation (15).

$$\mu^p_{\widetilde{R}*} = \begin{bmatrix} \tilde{r}^T_{p11} & \tilde{r}^T_{p12} & . & . & \tilde{r}^T_{p1m} \\ \tilde{r}^T_{p21} & \tilde{r}^T_{p22} & . & . & \tilde{r}^T_{p2m} \\ . & . & . & . & . \\ . & . & . & . & . \\ \tilde{r}^T_{pn(p)1} & \tilde{r}^T_{pn(p)2} & . & . & \tilde{r}^T_{pn(p)m} \end{bmatrix},\ \mu'^p_{\widetilde{R}*} = \begin{bmatrix} \tilde{r}^T_{p11} & \tilde{r}^T_{p12} & . & . & \tilde{r}^T_{p1m} \\ \tilde{r}^T_{p21} & \tilde{r}^T_{p22} & . & . & \tilde{r}^T_{p2m} \\ . & . & . & . & . \\ . & . & . & . & . \\ \tilde{r}^T_{pn(p)1} & \tilde{r}^T_{pn(p)2} & . & . & \tilde{r}^T_{pn(p)m} \end{bmatrix} \tag{16}$$

$\mu_{\widetilde{A}*}{}^{p}\left(P,S_{p}\right)=\mu_{\widetilde{A}*}{}^{p}$ and $\mu'_{\widetilde{A}*}{}^{p}\left(P,S_{p}\right)=\mu'_{\widetilde{A}*}{}^{p}$ represent matrices which are obtained by replacing entries of $\mu_{\widetilde{A}}{}^{p}(P,S_p)$ and $\mu'_{\widetilde{A}}{}^{p}(P,S_p)$ by vague points $\tilde{a}_{pijk}=[(a_{pij},a_{pij},a_{pij},a_{pij});k;1]$,
$\tilde{a}'_{pijk}=[(a'_{pij},a'_{pij},a'_{pij},a'_{pij});k;1]$.

$$\mu^{p}_{\widetilde{A}*}=\begin{bmatrix}\tilde{a}_{p11k} & \tilde{a}_{p12k} & \cdot & \cdot & \tilde{a}_{p1n(p)k}\\ \tilde{a}_{p21k} & \tilde{a}_{p22k} & \cdot & \cdot & \tilde{a}_{p2n(p)k}\\ \cdot & \cdot & \cdot & \cdot & \cdot\\ \cdot & \cdot & \cdot & \cdot & \cdot\\ \tilde{a}_{pl1k} & \tilde{a}_{pl2k} & \cdot & \cdot & \tilde{a}_{pln(p)k}\end{bmatrix},\ \mu'_{\widetilde{A}*}{}^{p}=\begin{bmatrix}\tilde{a}'_{p11k} & \tilde{a}'_{p12k} & \cdot & \cdot & \tilde{a}'_{p1n(p)k}\\ \tilde{a}'_{p21k} & \tilde{a}'_{p22k} & \cdot & \cdot & \tilde{a}'_{p2n(p)k}\\ \cdot & \cdot & \cdot & \cdot & \cdot\\ \cdot & \cdot & \cdot & \cdot & \cdot\\ \tilde{a}'_{pl1k} & \tilde{a}'_{pl2k} & \cdot & \cdot & \tilde{a}'_{pln(p)k}\end{bmatrix} \tag{17}$$

$where\ \ p=1,\ 2\ldots n$.Using the following composition rules to determine two relations $\mu_{\widetilde{T}*}{}^{p}(P,D)$ and $\mu'_{\widetilde{T}*}{}^{p}(P,D)$.

$$\tilde{t}^{T}_{pij}=\left[\left(\tilde{a}_{pi1k}\otimes\tilde{r}^{T}_{p1j}\right)\oplus\left(\tilde{a}_{pi2k}\otimes\tilde{r}^{T}_{p2j}\right)\oplus\ldots\oplus\left(\tilde{a}_{pin(p)k}\otimes\tilde{r}^{T}_{pn(p)j}\right)\right] \tag{18}$$

$$\tilde{t}'_{pij}{}^{T}=\left[\left(\tilde{a}'_{pi1k}\otimes\tilde{r}'_{p1j}{}^{T}\right)\oplus\left(\tilde{a}'_{pi2k}\otimes\tilde{r}'_{p2j}{}^{T}\right)\oplus\ldots\oplus\left(\tilde{a}'_{pin(p)k}\otimes\tilde{r}'_{pn(p)j}{}^{T}\right)\right] \tag{19}$$

For p=1,2,…,n, i=1,2,…,l and j=1,2,…,m. Therefore,

$$\mu^{p}_{\widetilde{T}*}(P,D)=\begin{bmatrix}\tilde{t}^{T}_{p11} & \tilde{t}^{T}_{p12} & \cdot & \cdot & \tilde{t}^{T}_{p1m}\\ \tilde{t}^{T}_{p21} & \tilde{t}^{T}_{p22} & \cdot & \cdot & \tilde{t}^{T}_{p2m}\\ \cdot & \cdot & \cdot & \cdot & \cdot\\ \cdot & \cdot & \cdot & \cdot & \cdot\\ \tilde{t}^{T}_{pl1} & \tilde{t}^{T}_{pl2} & \cdot & \cdot & \tilde{t}^{T}_{plm}\end{bmatrix},\ \mu'_{\widetilde{T}*}{}^{p}(P,D)=\begin{bmatrix}\tilde{t}'_{p11}{}^{T} & \tilde{t}'_{p12}{}^{T} & \cdot & \cdot & \tilde{t}'_{p1m}{}^{T}\\ \tilde{t}'_{p21}{}^{T} & \tilde{t}'_{p22}{}^{T} & \cdot & \cdot & \tilde{t}'_{p2m}{}^{T}\\ \cdot & \cdot & \cdot & \cdot & \cdot\\ \cdot & \cdot & \cdot & \cdot & \cdot\\ \tilde{t}'_{pl1}{}^{T} & \tilde{t}'_{pl2}{}^{T} & \cdot & \cdot & \tilde{t}'_{plm}{}^{T}\end{bmatrix} \tag{20}$$

Relations $\mu_{\widetilde{B}*}{}^{i}(D)$ and $\mu'_{\widetilde{B}*}{}^{i}(D)$ are obtained as follows:

$$\mu^{i}_{\widetilde{B}*}(D)=[\tilde{a}_{1k},\tilde{a}_{2k},\ldots,\tilde{a}_{nk}]\,o\begin{bmatrix}\tilde{t}^{T}_{1i1} & \tilde{t}^{T}_{1i2} & \cdot & \cdot & \tilde{t}^{T}_{1im}\\ \tilde{t}^{T}_{2i1} & \tilde{t}^{T}_{2i2} & \cdot & \cdot & \tilde{t}^{T}_{2im}\\ \cdot & \cdot & \cdot & \cdot & \cdot\\ \cdot & \cdot & \cdot & \cdot & \cdot\\ \tilde{t}^{T}_{ni1} & \tilde{t}^{T}_{ni2} & \cdot & \cdot & \tilde{t}^{T}_{nim}\end{bmatrix}=\begin{bmatrix}\tilde{b}^{T}_{i1} & \tilde{b}^{T}_{i2} & \cdot & \cdot & \tilde{b}^{T}_{im}\end{bmatrix} \tag{21}$$

$$\mu'_{\widetilde{B}*}{}^{i}(D)=[\tilde{a}_{1k},\tilde{a}_{2k},\ldots,\tilde{a}_{nk}]\,o\begin{bmatrix}\widetilde{t'}^{T}_{1i1} & \widetilde{t'}^{T}_{1i2} & \cdot & \cdot & \widetilde{t'}^{T}_{1im}\\ \widetilde{t'}^{T}_{2i1} & \widetilde{t'}^{T}_{2i2} & \cdot & \cdot & \widetilde{t'}^{T}_{2im}\\ \cdot & \cdot & \cdot & \cdot & \cdot\\ \cdot & \cdot & \cdot & \cdot & \cdot\\ \widetilde{t'}^{T}_{ni1} & \widetilde{t'}^{T}_{ni2} & \cdot & \cdot & \widetilde{t'}^{T}_{nim}\end{bmatrix}=\begin{bmatrix}\widetilde{b'}^{T}_{i1} & \widetilde{b'}^{T}_{i2} & \cdot & \cdot & \widetilde{b'}^{T}_{im}\end{bmatrix} \tag{22}$$

Each element of $\mu_{\widetilde{B}*}{}^{i}(D)$ and $\mu'_{\widetilde{B}*}{}^{i}(D)$ are obtained by the following composition rules:

$$\tilde{b}^{T}_{ij}=\left[\left(\left(\tilde{a}_{1k}\otimes\tilde{t}^{T}_{1ij}\right)\oplus\left(\tilde{a}_{2k}\otimes\tilde{t}^{T}_{2ij}\right)\oplus\ldots\oplus\left(\tilde{a}_{nk}\otimes\tilde{t}^{T}_{nij}\right)\right);min(k,mink_{pij});1\right] \tag{23}$$

$$\widetilde{b'}^{T}_{ij}=\left[(\tilde{a}_{1k}\otimes\widetilde{t'}^{T}_{1ij})\oplus(\tilde{a}_{2_k}\otimes\widetilde{t'}^{T}_{2ij})\oplus\ldots\oplus(\tilde{a}_{nk}\otimes\widetilde{t'}^{T}_{nij}));min(k,mink'_{pij});1\right] \tag{24}$$

where p =1,2,…,n; i is any fixed number up to j =1,2,…,m. More elaborately,

$\tilde{b}_{ij}^{T}$=$\left[\sum_{p=1}^{n} a_p \sum_{h=1}^{n(p)} a_{pih} r_{pij}^{(1)}, \sum_{p=1}^{n} a_p \sum_{h=1}^{n(p)} a_{pih} r_{pij}^{l}, \sum_{p=1}^{n} a_p \sum_{h=1}^{n(p)} a_{pih} r_{pij}^{u}, \sum_{p=1}^{n} a_p \sum_{h=1}^{n(p)} a_{pih} r_{pij}^{(2)}\right]$,
$\widetilde{b'}_{ij}^{T}$=$\left[\sum_{p=1}^{n} a_p \sum_{h=1}^{n(p)} a'_{pih} r_{pij}^{(1)'}, \sum_{p=1}^{n} a_p \sum_{h=1}^{n(p)} a'_{pih} r_{pij}^{l'}, \sum_{p=1}^{n} a_p \sum_{h=1}^{n(p)} a'_{pih} r_{pij}^{u'}, \sum_{p=1}^{n} a'_p \sum_{h=1}^{n(p)} a'_{pih} r_{pij}^{(2)'}\right]$

Property 4: For $\tilde{b}_{ij}^{T}$and $\widetilde{b'}_{ij}^{T}$ distances, $d\left(\tilde{b}_{ij}^{T}, \tilde{0}_k\right)$, $d\left(\widetilde{b'}_{ij}^{T}, \tilde{0}_k\right)$ are as follows:

a) $\frac{1}{4}\left[\sum_{p=1}^{n} a_p \sum_{h=1}^{n(p)} a_{pih} r_{pij}^{(1)} + \sum_{p=1}^{n} a_p \sum_{h=1}^{n(p)} a_{pih} r_{pij}^{l} + \sum_{p=1}^{n} a_p \sum_{h=1}^{n(p)} a_{pih} r_{pij}^{u} + \sum_{p=1}^{n} a_p \sum_{h=1}^{n(p)} a_{pih} r_{pij}^{(2)}\right]$

b) $\frac{1}{4}\left[\sum_{p=1}^{n} a_p \sum_{h=1}^{n(p)} a'_{pih} r_{pij}^{(1)'} + \sum_{p=1}^{n} a_p \sum_{h=1}^{n(p)} a'_{pih} r_{pij}^{l'} + \sum_{p=1}^{n} a_p \sum_{h=1}^{n(p)} a'_{pih} r_{pij}^{u'} + \sum_{p=1}^{n} a'_p \sum_{h=1}^{n(p)} a'_{pih} r_{pij}^{(2)'}\right]$

Proof: Proof is obvious using the concept of signed distance defined in Equation (9).

Using properties (23) and (24) [Equations (23) and (24)] and property 4 [Equation (4)], matrices $\mu_{\widetilde{B}*}{}^{i}(D)$ and $\mu'_{\widetilde{B}*}{}^{i}(D)$ are formed. After aggregating and normalizing the matrices, $\mu_{\widetilde{B}o*}{}^{i}(D)$ is obtained.

Property 5: In $\mu_{\widetilde{B}o*}{}^{i}(D)$= $[d_{ih}^{*}]$, if $\max_{1\le h\le m} d_{ih}^{*}$= d_{ij}^{*}, then patient $P_i's$ diagnosis can be given as dj.

2.7.3 Numerical computation

The methodology discussed in Subsection-2.7 is illustrated through an example. S = {S_1(headache), S_2 (fever), S_3 (phlegm)}; S_{11} (minor), S_{12} (median), S_{13} (strong)};S_{21} (low), S_{22} (medium), S_{23} (high)}; = {S_{31} (light), S_{32} (thick)} are the symptom characters associated to corresponding symptoms.

D = {D_1(cold), D_2 (pulmonary tuberculosis), D_3 (pertussis)}, P = {P_1,P_2}.

Grades in relations between patients and symptom characters are input with the help of the physician's knowledge. These relations are denoted by$\mu_{\widetilde{A}}{}^{1}(P,S_1)$, $\mu_{\widetilde{A}}{}^{2}(P,S_2)$, *and* $\mu_{\widetilde{A}}{}^{2}(P,S_2)$for symptoms headache, fever, and phlegm. Matrix forms of these relations are computed as follows:

$$\mu^{1}{}_{\widetilde{A}}\ (P,S_1) = \begin{bmatrix}0.9 & 0.1 & 0.0\\ 0.0 & 0.3 & 0.7\end{bmatrix},\ \mu_{\widetilde{A}}^{2}\ (P,S_2) = \begin{bmatrix}1.0 & 0.0 & 0.0\\ 0.0 & 0.2 & 0.8\end{bmatrix},\ \mu_{\widetilde{A}}^{3}\ (P,S_3) = \begin{bmatrix}0.8 & 0.2\\ 0.1 & 0.9\end{bmatrix}.$$

Similarly, non-membership relations between patients and symptom characters are given as follows:

$$\upsilon_{\widetilde{R}}^{1}\ (S_1,D) = \begin{bmatrix}0.1 & 0.8 & 0.9\\ 0.9 & 0.7 & 0.2\end{bmatrix},\ \upsilon_{\widetilde{R}}^{2}\ (S_2,D) = \begin{bmatrix}0.0 & 0.9 & 1.0\\ 0.9 & 0.7 & 0.1\end{bmatrix},\ \upsilon_{\widetilde{R}}^{3}\ (S_3,D) = \begin{bmatrix}0.1 & 0.7\\ 0.8 & 0.1\end{bmatrix}.$$

From non-membership relations, following upper membership relations are obtained:

$$\mu_{\widetilde{R}}^{\prime 1}\ (S_1,D) = \begin{bmatrix}0.9 & 0.2 & 0.1\\ 0.1 & 0.3 & 0.8\end{bmatrix},\ \mu_{\widetilde{R}}^{\prime 2}\ (S_2,D) = \begin{bmatrix}1.0 & 0.1 & 0.0\\ 0.1 & 0.3 & 0.9\end{bmatrix},\ \mu_{\widetilde{R}}^{\prime 3}\ (S_3,D) = \begin{bmatrix}0.9 & 0.3\\ 0.2 & 0.9\end{bmatrix}.$$

Using statistical data analysis namely "determination of "r_{pij} *and* r'_{pij}" of Section-2, membership relations and non-membership relations are obtained as follows:

$$\mu_{\widetilde{R}}^{1}\ (S_1,D) = \begin{bmatrix}0.2 & 0.1 & 0.7\\ 0.3 & 0.4 & 0.3\\ 0.1 & 0.5 & 0.4\end{bmatrix},\ \mu_{\widetilde{R}}^{2}\ (S_2,D) = \begin{bmatrix}0.8 & 0.1 & 0.1\\ 0.5 & 0.3 & 0.2\\ 0.6 & 0.2 & 0.2\end{bmatrix},\ \mu_{\widetilde{R}}^{2}\ (S_2,D) = \begin{bmatrix}0.3 & 0.2 & 0.5\\ 0.1 & 0.3 & 0.6\end{bmatrix}$$

$$\upsilon_{\widetilde{R}}^{1}\ (S_1,D) = \begin{bmatrix}0.4 & 0.45 & 0.15\\ 0.35 & 0.3 & 0.35\\ 0.45 & 0.25 & 0.3\end{bmatrix},\ \upsilon_{\widetilde{R}}^{2}\ (S_2,D) = \begin{bmatrix}0.1 & 0.45 & 0.45\\ 0.25 & 0.35 & 0.40\\ 0.2 & 0.40 & 0.40\end{bmatrix},$$

$$\upsilon_{\widetilde{R}}^{3}\ (S_3,D) = \begin{bmatrix}0.35 & 0.4 & 0.25\\ 0.45 & 0.35 & 0.2\end{bmatrix}$$

This knowledge is gathered from the existing medical data. Now $\upsilon_{\widetilde{R}}{}^{1}(S_1,D)$, $\upsilon_{\widetilde{R}}{}^{2}(S_2,D)$and $\upsilon_{\widetilde{R}}{}^{3}(S_3,D)$ *contribute* to find the upper membership relations:

$$\mu'^{1}_{\widetilde{R}}\ (S_1,D) = \begin{bmatrix}0.60 & 0.55 & 0.85\\ 0.65 & 0.70 & 0.65\\ 0.55 & 0.75 & 0.70\end{bmatrix},\ \mu'^{2}_{\widetilde{R}}\ (S_2,D) = \begin{bmatrix}0.90 & 0.55 & 0.55\\ 0.75 & 0.65 & 0.60\\ 0.80 & 0.60 & 0.60\end{bmatrix},$$

$$\mu_{\widetilde{R}}^{\prime 3}\ (S_3,D) = \begin{bmatrix}0.65 & 0.60 & 0.75\\ 0.5 & 0.65 & 0.80\end{bmatrix}$$

Using Equations (18), (19) and (20), we get $\mu_{\widetilde{T}}{}^{*p}(P,D)$, $\mu'_{\widetilde{T}}{}^{*p}(P,D)$, respectively, for p=1,2,3; l=1,2; m=1,2,3 as follows:

Using Equations (21), (22), (23) and (24) and property 5 [Equation (5)], the expected percentage of presence of disease in the two patients can be estimated as: : P_1:D_1 is 37.10%; D_2 is 23.64%; is 39.25%

P_2: D_1 is 33.29%; D_2 is 33.09%; D_3 is 40.75%

It is obvious from (25) that the diagnosis for the patient P_1 is D_3 for P_2 it is D_3.

${\mu_{\widetilde{T}_*}}^1$	[(-0.024,0.11,0.31,0.447);k;1] [(-0.005,0.06,0.26,0.325);k;1]	[(-0.054,0.03,0.23,0.314);k;1] [(0.243,0.37,0.57,0.697);k;1]	[(0.396,0.56,0.76,0.924);k;1] [(0.151,0.27,0.47,0.589);k;1]
${\mu_{\widetilde{T}_*}}^2$	[(0.612,0.7,0.9,0.988);k;1] [(0.318,0.48,0.68,0.842);k;1]	[(-0.041,0,0.2,0.241);k;1] [(0.002,0.12,0.32,0.438);k;1]	[(-0.041, 0, 0.2, 0.241);k;1] [(-0.013, 0.1, 0.3, 0.413);k;1]
${\mu_{\widetilde{T}_*}}^3$	[(0.021,0.16,0.36,0.499);k;1] [(-0.033,0.02,0.22,0.273);k;1]	[(-0.007,0.12,0.32,0.447);k;1] [(0.073,0.19,0.39,0.507);k;1]	[(0.244,0.42,0.62,0.796);k;1] [(0.354,0.49,0.69,0.826);k;1]

${\mu'_{\widetilde{T}_*}}^1$	[(0.203,0.505,0.705,1.007);k;1] [(0.166,0.480,0.680,0.994);k;1]	[(0.322,0.465,0.665,0.808);k;1] [(0.413,0.635,0.835,1.057);k;1]	[(0.631,0.730,0.930,1.029);k;1] [(0.433,0.585,0.785,0.937);k;1]
${\mu'_{\widetilde{T}_*}}^2$	[(0.773,0.80,1.000,1.027);k;1] [(0.635,0.690,0.890,0.945);k;1]	[(0.256,0.450,0.650,0.844);k;1] [(0.279,0.510,0.710,0.941);k;1]	[(0.208,0.450,0.650,0.892);k;1] [(0.225,0.500,0.700,0.975);k;1]
${\mu'_{\widetilde{T}_*}}^3$	[(-0.042,0.530,0.730,1.302);k;1] [(0.018,0.460,0.660,1.102);k;1]	[(0.151,0.510,0.710,1.069);k;1] [(0.050,0.545,0.745,1.240);k;1]	[(0.485,0.660,0.860,1.035);k;1] [(0.476,0.695,0.895,1.114);k;1]

3 Results and Discussions

After comparing the findings of the proposed approach and the conventional approach provided by the following table, it can be said that both ways support the initial diagnosis for patients P1 and P2. However, all illness possibilities are changed. This demonstrates how the hesitation portion must be taken into account when drawing conclusions from any decision-making problem, something that the conventional fuzzy approach did not address. It's also important to consider the potential for changes to the differences between the highest and second-highest scores.

Table 1. Comparison of present approach with conventional fuzzy set approach

	D_1	D_2	D_3	Result
Patient (P_1)	41.20%	16.60%	42.20%	Conventional fuzzy set approach [28]
Patient (P_2)	26.60%	30.50%	42.90%	
Patient (P_1)	37.10%	23.64%	39.25%	Proposed Approach
Patient (P_2)	33.29%	33.09%	40.75%	

By using the existing method, which is based on the conventional fuzzy set approach, the possibility differences for patients P1 and P2 vary from 1% to 16.3%, but in our approach, these differences range from 2.15% to 7.46%. Since differences are narrowed, the present model advises the doctor that the primary treatment of the patient should be according to the disease with the highest probability, and at the same time the response of the patient should be noted. The prominent reason for this is that it is necessary to keep in mind the possibilities of the disease with the second highest probability. The introduction of an uncertainty-handling tool that manages the blur condition far better than point or interval estimate is the consideration of a trapezoidal ambiguous number. Using the statistical measure based on the confidence interval technique, a good range of membership is determined. To the best of our knowledge, no research has been done on the classification of symptoms by multicriteria analysis, based on our review of pertinent literature. However, our methodology is effective in doing so.

Remark: Our approach, which is based on the use of vague sets, distinguishes the study from fuzzy based approaches currently in use. Because no one is using vague relations with multicriteria of symptoms to reach a final diagnosis. Additionally, articles based on multicriteria decision-making in medical diagnosis differ with our method because in our study symptoms are linguistically classified into criteria. It is different intuition to handle symptom-disease relationships and states of patients.

4 Conclusion

To cover up the non-specificity in the symptom-disease relationship data and vagueness in the patient's symptom observation, a good decision model must take into account the favourable and unfavourable evidence separately. It must also be ready to handle some hidden or unobserved uncertainties. So, it may be useful to identify meaningful lower and upper bounds for the membership grades to make the diagnostic decision more credible. In the present study, symptoms are classified into criteria to judge the grade of association between symptoms and diseases precisely. The proposed expert system will be appealing while dealing with uncertain information. This work advanced the notion of signed distance and centroid for trapezoidal vague numbers and provided decision-making in medical diagnosis. The result of the proposed methodology has proven its utility.

The present approach may require slightly more computational demand, but it handles the medical decision-making problem with effective realistic and credible results. The use of computer programs that follow the steps of the method will definitely remove the disadvantage of complexity. The focus of the advancement of this study is to extend the methodology to interval-valued vague sets by considering membership and non-membership itself as interval-valued functions. The suggested method's application to actual data to investigate disease diagnosis may need to be implemented as a further step.

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