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On Radio k-Chromatic Number of Various Graphs

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Abstract

Objective: The main objective of the study is to reduce radio spectrums without any interference. This study aims to find the smallest span of radio k chromatic number of various graphs. The minimal number of colors essential to color a graph is called its span. **Methods:** In this paper, we address the issue of minimizing interference by modeling it as a radio k-coloring problem on graphs, when vertices and the connections between them represent the spectrum bandwidth indicated by edges. For a positive integer k, a radio k – coloring of simply connected graph T, is a function ϕ: V(T) $\rightarrow$\{$c_{0,}$ $c_{1,}$ $c_{2,}$ . . .\} such that $|\phi\ (u)\ -\ \phi\ (v)|\geq 1+k-\ d(u,v)$ for each pair of distinct vertices u and v of T, when diam(u,v) is the distance between u and v in T. The maximum color granted by ϕ is called span, which can be denoted by $rc_k(\phi)$. The radio k – chromatic number rc_k(T) of T is the minimal \{$rc_k(\phi)$\}, when ϕ is a radio k -coloring of T. **Findings:** This study presents the exact value of the radio k-chromatic number of some graphs like T_n, TC_n, A(T_n), $U_{n,n-1}$ for $2\leq k\leq$ diam(T), when diam(T) is the diameter of graph T. **Novelty:** By introducing an interference graph, the channel assigned gets transformed into a graph coloring issue, eventually confronting the channel assigned problem for radio spectrum.

Keywords: Radio k – Chromatic Number; Triangular Snake Graph; Triangular Cactus Graph; Alternative Triangular Snake Graph; Umbrella Graph

1 Introduction

Graph coloring is the technique of allocation of colors to each vertex of a graph T such that non-adjacent vertices get a uniform color. The main intent is to reduce the number of colors while coloring a graph T. The minimal number of colors essential to color a graph T is called its chromatic number of that graph T. Motivated by channel assigned problem, Chartand et al. [1] introduced radio k-coloring of graphs. For any graph T we denote the set of vertices V(T) and set of edges E(T) of T respectively. The distance d(u,v) between two vertices u and v of a graph, T is the length (number of edges) of the shortest path connecting u and v. The diameter diam(T) of a graph, T is the maximum d(u,v) taken over every pair of vertices u,v of T. For any positive integer $1\leq k\leq$ diam(T), radio k – coloring of T is a function ϕ: V (T) $\rightarrow\{C_i$:1 $\leq$ i $\leq$ n} satisfying the condition $|\phi$(u) – ϕ(v)$|\geq$ 1 + k – d (u,v), for all distinct u,v ε V(T). The radio k – chromatic number rc_k(T) of T is the minimum rc_k (ϕ) . The radio antipodal number, denoted by an(T), is the minimum span of a radio(diam(T)−1) coloring of T and the radio number, denoted by rn(T), is the minimum span of a radio diam(T) coloring of T. concentrate on finding a lower bound of rc_k(T). Orden, and Gimenez [2] give the Spectrum Graph Coloring and Applications to Wi-Fi Channel Assigned. Nirajan and Srinivasa Rao Kola [3] discussed the radio k-chromatic number of paths. The upper bound for radio k chromatic number was determined by the authors Laxman Saha, [4] and Elsayed, Badr [5]. The lower bound for radio k chromatic number was discussed by the authors Sandip Das, Sasthi [6], and Nirajan and Srinivasa Rao Kola [7]. In this paper, we determine the radio k-chromatic number for various graphs for any integer 2$\leq$ k $\leq$ diam(T). The rest of this paper is organized as follows. In section 2, some of the preliminary results and bounds are given briefly. In section 3, the radio k – chromatic number for various graphs for any integer 2 $\leq$ k $\leq$ diam(T). Finally, section 4 concludes the paper.

2 Preliminaries

Definition 1. [7] For any graph T we denote the set of vertices V(T) and set of edges E(T) of T respectively. The distance d(u,v) between two vertices u and v of a graph, T is the length (number of edges) of the shortest path connecting u and v. The diameter diam(T) of a graph, T is the maximum d(u,v) taken over every pair of vertices u,v of T. For any positive integer 1 $\leq$ k $\leq$ diam(T), radio k-coloring of T is a function ϕ : V (T) $\rightarrow\{C_i$:1 $\leq$ i $\leq$ n} satisfying the condition $|\phi$(u) – ϕ(v)$|$ $\geq$ 1 + k – d(u,v), for all distinct u,v ε V(T). The radio k- chromatic number rc_k(T) of T is the minimum rc_k (ϕ) .

Lemma 2. [7] For any graph T and 1 $\leq$ k $\leq$ diam(T), we have

$$rc_k(G) \geq \begin{cases} |D_k| - 2p + 2\sum_{i=0}^{p} L_i(p-i); \ if\, k = 2p+1 \\ |D_k| - 2p + 2\sum_{i=0}^{p} L_i(p-i) + 1; \ if\, k = 2p \end{cases}$$

Definition 3 . [8] A triangular snake graph T_n is obtained from a path s_1, s_2, ..., s_n by joining s_i and s_{i+1} to a new vertex t_i for 1 $\leq$ i $\leq$ n -1. That is, every edge of the path is replaced by a triangle C_3.

Definition 4 . [8] A triangular cactus TC_n is a linked graph that only consists of triangles a sits building blocks.

Definition 5. [8] An alternate triangular snake $A(T_n)$ is obtained from a path u_1,u_2,..., u_n by joining u_i and u_{i+1} (alternately) to a new vertex v_i. That is every alternate edge of path is replaced by C_3.

Definition 6 . [9] An umbrella graph $U_{n,n-1}$ is a graph that is formed by combining a path graph and the center vertex of a fan graph. The number of vertices in an umbrella graph is m+n and the number of edges is 2m+n-2.

3 Results and Discussion

In this section, we determine the radio k- chromatic number for various graphs such that Triangular Snake Graph T_n, Triangular Cactus Graph TC_n, Alternative Triangular Snake Graph $A(T_n)$, Umbrella Graph $U_{n,n-1}$ for any integer 2 $\leq$ k $\leq$ diam(T).

Theorem 3.1

For any positive integer n $\geq$ 3 and 2 $\leq$ k $\leq$ diam(T_n) then

$$rc_k(T_n) = \begin{cases} 6\,; & n = 3 \\ 5k - 3; & n \geq 4 \end{cases}$$

Proof:

Let the vertex set and edge set of triangular snake graph is defined by $V(T_n)$ = {v_i: 1 $\leq$ i $\leq$ n} $\cup$ {u_i 1 $\leq$ i $\leq$ n-1} and $E(T_n)$ = {v_i v_{i+1}: 1 $\leq$ i $\leq$ n-1} $\cup$ {v_i u_i: 1 $\leq$ i $\leq$ n-1} $\cup$ {v_{i+1} u_i: 1 $\leq$ i $\leq$ n-1} with $|V(T_n)|$ = 2n-1, $|E(T_n)|$ = 3n-3 and diam(T_n) = n-1

Case 1: when n = 3

From Lemma 2, it is observed that $rc_k\,(T_n) \geq |D_2| - 2 + 2\sum_{i=0}^{1} L_i\,(1-i) + 1$, we have $rc_k(T_n) \geq 6$

Consider the coloring function ϕ:V$(T_n)\rightarrow$\{C_0,C_1,C_2, ...,C_6\}, assign the colors for V(T_n) as follows;

$\phi(v_1)$ =C_0, $\phi(v_2)$ =C_2, $\phi(v_3)$ =C_4, $\phi(u_1)$ =C_5, $\phi(u_2)$ =$C_6$the condition of radio k-coloring, we have $rc_k(T_n) \leq 6$

Now let us apply the condition of radio k-coloring to the following pairs of vertices.

$|\phi(v_1)$ - $\phi(v_2)|$ $\geq$ 1 + k - d$(v_1\,,v_2)$

$|\phi(v_1)$ - $\phi(u_1)|$ $\geq$ 1 + k - d$(v_1\,,u_1)$

$|\phi(v_2)$ - $\phi(u_1)|$ $\geq$ 1 + k - d$(v_2\,,u_1)$

By a similar procedure, it can be verified that the remaining pair of nodes satisfies the radio k-coloring condition.
Therefore, we get $rc_k(T_n) = 6$

Case 2: when n≥4

From Lemma 2, it is observed that

$$rc_k(T_n) \geq \begin{cases} |D_k| - 2p + 2\sum_{i=0}^{p} L_i(p-i); \ if\, k = 2p+1 \\ |D_k| - 2p + 2\sum_{i=0}^{p} L_i(p-i) + 1; \ if\, k = 2p \end{cases}$$

We have $rc_k(TS_n) \geq 5k - 3$

Consider the coloring function ϕ:V(T_n) →{C_0,C_1,C_2, ...,C_{5k-3}}, assign the colors for V(T_n) as follows;

$$\phi(u_i : 1 \leq i \leq n-1) = \begin{cases} 5k-3; \ i = 0(mod\, 3) \\ 3k-1; \ i = 1(mod\, 3) \\ 4k-2; \ i = 2(mod\, 3) \end{cases}$$

$$\phi(v_i : 1 \leq i \leq n) = \begin{cases} 2k; \ i = 0(mod\, 3) \\ 0; \ i = 1(mod\, 3) \\ k; \ i = 2(mod\, 3) \end{cases}$$

Hence, the above assigned colors satisfy the condition of radio k-coloring, we have (T) $\leq$ 5k – 3

Now let us apply the condition of radio k-coloring to the following pairs of vertices.

$|\phi(v_i : 1 \leq i \leq n) - \phi(v_{i+1} : 1 \leq i \leq n-1)| \geq 1 + k - d(v_i : 1 \leq i \leq n, v_{i+1} : 1 \leq i \leq n-1)$
$|\phi(v_i : 1 \leq i \leq n) - \phi(u_{i+1} : 1 \leq i \leq n-1)| \geq 1 + k - d(v_i : 1 \leq i \leq n, u_{i+1} : 1 \leq i \leq n-1)$
$|\phi(v_{i+1} : 1 \leq i \leq n-1) - \phi(u_i : 1 \leq i \leq n-1)| \geq 1 + k - d(v_{i+1} : 1 \leq i \leq n-1, u_i : 1 \leq i \leq n-1)$

By a similar procedure, it can be verified that the remaining pair of nodes satisfies the radio k-coloring condition.
Therefore, we get $rc_k(T_n)$ = 5k– 3

Theorem 3.2

For any positive integer n $\geq$ 3 and 2 $\leq$ k $\leq$ diam(TC_n) then

$$rc_k(TC_n) = \begin{cases} nk - (n-2); \ n = 3,4 \\ 5k-3; \qquad n \geq 5 \end{cases}$$

Proof:

Let the vertex set and edge set of triangular cactus graph is defined by V(TC_n) = {v_i: 1 ≤ i ≤ n} ∪ {u_i: 1 ≤ i ≤ n-2} and E(TC_n) = {v_i v_{i+1}: 1 ≤ i ≤ n-1}∪{v_i u_i: 1 ≤ i ≤ n-2}∪{v_{i+1} u_i: 1 ≤ i ≤ n-2} with |V(TC_n) | = 2n-2, |E(TC_n) | = 3n-5 and diam(TC_n) = n-1

Case 1: when n=3 and 4

From Lemma 2, it is observed that

$$rc_k(T_n) \geq \begin{cases} |D_3| - 2 + 2\sum_{i=0}^{1} L_i(1-i); \ if\, k = 3 \\ |D_2| - 2 + 2\sum_{i=0}^{1} L_i(1-i) + 1; \ if\, k = 2 \end{cases}$$

We have $rc_k(TC_n) \geq nk - (n-2)$

Consider the coloring function ϕ:V(TC_n) →{C_0,C_1,C_2, ...,$C_{nk-(n-2)}$}, assign the colors for V(TC_n) as follows:

$\phi(u_1)=C_{nk-(n-3)}$ $\quad\phi(v_1)=\phi(v_4) = C_0$
$\phi(u_2)=C_{nk-(n-2)}$ $\quad\phi(v_2)=C_k$ $\quad\phi(v_3)=C_{2k}$

Hence, the above assigned colors satisfy the condition of radio k-coloring. So, we have $rc_k(TC_n) \leq nk - (n-2)$

Now let us apply the condition of radio k-coloring to the following pairs of vertices.

$|\phi(v_i : 1 \leq i \leq n) - \phi(v_{i+1} : 1 \leq i \leq n-1)| \geq 1 + k - d(v_i : 1 \leq i \leq n, v_{i+1} : 1 \leq i \leq n-1)$
$|\phi(v_i : 1 \leq i \leq n) - \phi(u_i : 1 \leq i \leq n-2)| \geq 1 + k - d(v_i : 1 \leq i \leq n, u_i : 1 \leq i \leq n-2)$
$|\phi(v_{i+1} : 1 \leq i \leq n-1) - \phi(u_i : 1 \leq i \leq n-2)| \geq 1 + k - d(v_{i+1} : 1 \leq i \leq n-1, u_i : 1 \leq i \leq n-2)$

By a similar procedure, it can be verified that the remaining pair of nodes satisfies the radio k-coloring condition.
Therefore, we get $rc_k(TC_n) = nk - (n-2)$

Case 2: when n$\geq$5
From Lemma 2, it is observed that

$$rc_k(TC_n) \geq \begin{cases} |D_k| - 2p + 2\sum_{i=0}^{p} L_i(p-i);\ if\, k = 2p+1 \\ |D_k| - 2p + 2\sum_{i=0}^{p} L_i(p-i) + 1;\ if\, k = 2p \end{cases}$$

We have $rc_k(TC_n) \geq 5k - 3$
Consider the coloring function ϕ:V(TC_n) $\rightarrow$\{C_0,C_1,C_2, ...,C_{5k-3}\}, assign the colors for V(TC_n) as follows;

$$\phi(u_i : 1 \leq i \leq n-2) = \begin{cases} 5k-3;\ i = 0(mod\ 3) \\ 3k-1;\ i = 1(mod\ 3) \\ 4k-2;\ i = 2(mod\ 3) \end{cases}$$

$$\phi(v_i : 1 \leq i \leq n) = \begin{cases} 2k;\ i = 0(mod\ 3) \\ 0;\ i = 1(mod\ 3) \\ k;\ i = 2(mod\ 3) \end{cases}$$

Hence, the above assigned colors satisfy the condition of radio k-coloring, we have $rc_k(TC_n) \geq 5k - 3$
Now let us apply the condition of radio k-coloring to the following pairs of vertices.
$|\phi(v_i : 1 \leq i \leq n) - \phi(v_{i+1} : 1 \leq i \leq n-1)| \geq 1 + k - d(v_i : 1 \leq i \leq n, v_{i+1} : 1 \leq i \leq n-1)$
$|\phi(v_i : 1 \leq i \leq n) - \phi(u_i : 1 \leq i \leq n-2)| \geq 1 + k - d(v_i : 1 \leq i \leq n, u_i : 1 \leq i \leq n-2)$
$|\phi(v_{i+1} : 1 \leq i \leq n-1) - \phi(u_i : 1 \leq i \leq n-2)| \geq 1 + k - d(v_{i+1} : 1 \leq i \leq n-1, u_i : 1 \leq i \leq n-2)$
By a similar procedure, it can be verified that the remaining pair of nodes satisfies the radio k-coloring condition.
Therefore, we get $rc_k(TC_n)$=5k – 3

Theorem 3.3

For any positive integer n $\geq$ 4 and 2 $\leq$ k $\leq$ diam(A(T_n)) then
$rc_k(A(T_n)) = \begin{cases} 6; & k = 2 \\ 5k - 5; & 3 \leq k \leq diam\,(A(T_n)) \end{cases}$

Proof:
Let the vertex of alternative triangular snake graph is defined by V(A(T_n))=\{u_i:1 $\leq$ i $\leq$ n\}$\cup$\{v_i:1 $\leq$ i $\leq$ $\lfloor \frac{n}{2} \rfloor$\} and |V(A($T_n$))|= $\lfloor \frac{n}{2} \rfloor$+n with diam(A($T_n$)) = n-1
Case 1: when k =2
From Lemma 2, it is observed that

$$rc_k(A(TC_n)) \geq \begin{cases} |D_k| - 2p + 2\sum_{i=0}^{p} L_i(p-i);\ if\, k = 2p+1 \\ |D_k| - 2p + 2\sum_{i=0}^{p} L_i(p-i) + 1;\ if\, k = 2p \end{cases}$$

We that $rc_k(A(T_n)) \geq 6$
Consider the coloring function ϕ:V(A(T_n)) $\rightarrow$\{C_0,C_1,C_2, ...,C_6\}, assign the colors for V(A(T_n)) as follows;

$$\phi(u_i : 1 \leq i \leq n) = \begin{cases} 2k; & i = 0(mod\ 3) \\ 0; & i = 1(mod\ 3) \\ k; & i = 2(mod\ 3) \end{cases}$$

$$\phi(v_i : 1 \leq i \leq \left\lfloor \frac{n}{2} \right\rfloor) = \begin{cases} 5; & i = 0, 2(mod\ 3) \\ 6; & i = 1(mod\ 3) \end{cases}$$

Hence, the above assigned colors satisfy the condition of radio k-coloring. So, we have that $rc_k(A(T_n)) \leq 6$
Now let us apply the condition of radio k-coloring to the following pairs of vertices.
$|\phi(v_1) - \phi(v_2)| \geq 1 + k - d(v_1, v_2)$
$|\phi(v_2) - \phi(v_3)| \geq 1 + k - d(v_2, v_3)$
$|\phi(v_1) - \phi(u_1)| \geq 1 + k - d(v_1, u_1)$

By a similar procedure, it can be verified that the remaining pair of nodes satisfies the radio k-coloring condition.

Therefore, we get $rc_k(A(T_n))=6$. From Lemma 2, it is observed that

Case 2: when $3 \leq k \leq diam(A(T_n))$

$$rc_k(A(T_n)) \geq \begin{cases} |D_k| - 2p + 2\sum_{i=0}^{p} L_i(p-i);\ if\, k = 2p+1 \\ |D_k| - 2p + 2\sum_{i=0}^{p} L_i(p-i)+1;\ if\, k = 2p \end{cases}$$

We have that $rc_k(A(T_n) \geq 5k - 5$

Consider the coloring function ϕ:V(A(T_n))→{C_0,C_1,C_2, ...,C_{5k-5}}, assign the colors for V(A(T_n) as follows;

$$\phi(u_i : 1 \leq i \leq n) = \begin{cases} 2k;\ i = 0(mod3) \\ 0;\ i = 1(mod3) \\ k;\ i = 2(mod3) \end{cases}$$

$\phi(v_i:1\leq i \leq \lfloor \frac{n}{2} \rfloor) = C_{(2k+1)+(k-2)i}$ when $1\leq i \leq \lfloor \frac{n}{2} \rfloor$ Hence, the above assigned colors satisfy the condition of radio k-coloring. We have (A(T)) $\leq$ 5k – 5.

Now let us apply the condition of radio k-coloring to the following pairs of vertices.

$|\phi(u_i : 1 \leq i \leq n) - \phi(u_{i+1} : 1 \leq i \leq n-1)| \geq 1 + k - d(u_i : 1 \leq i \leq n, u_{i+1} : 1 \leq i \leq n-1)$

$|\phi(v_{i+1} : 1 \leq i \leq n-2) - \phi(u_i : 1 \leq i \leq n)| \geq 1 + k - d(v_{i+1} : 1 \leq i \leq n-2, u_i : 1 \leq i \leq n)$

$|\phi(v_i : 1 \leq i \leq n/2) - \phi(u_i : 1 \leq i \leq n)| \geq 1 + k - d(v_i : 1 \leq i \leq n/2, u_i : 1 \leq i \leq n)$

By a similar procedure, it can be verified that the remaining pair of nodes satisfies the radio k-coloring condition.

Therefore, we get $rc_k(AT_n))$=5k – 5

Theorem 3.4

For any positive integer n $\geq$ 3 and 2 $\leq$ k $\leq$ diam($U_{n,n-1}$) then

$$rc_k(U_{n,n-1}) = \begin{cases} 6;\ n = 3 \\ (n+1)k - (n-1);\ n = 4 \\ (k-1)n + k;\ n \geq 5 \end{cases}$$

Proof:

Let the vertex set and edge set of umbrella graph is defined by V($U_{n,n-1}$) = {u_i:1 $\leq$ i $\leq$ n} ∪ {v_i:1 $\leq$ i $\leq$ n-1} and E($U_{n,n-1}$) = {u_i u_{i+1}:1 $\leq$ i $\leq$ n-1} ∪ {v_i v_{i+1}:1 $\leq$ i $\leq$ n-2}∪ {v_1 u_i:1 $\leq$ i $\leq$ n} with |V($U_{n,n-1}$)| = 2n-1, |E($U_{n,n-1}$) |= 3n-3 and diam($U_{n,n-1}$) = n-1.

Case 1: When n=3

From Lemma 2, it is observed that $rc_k\left(U_{n,\ n-1}\right) \geq |D_2| - 2 + 2\sum_{i=0}^{1} L_i\left(1-i\right) + 1$, we have $rc_k(U_{n,n-1}) \geq 6$

Consider the coloring function ϕ:V($U_{n,n-1}$)→{C_0,C_1,C_2, ...,C_6}, assign the colors for the vertices of ($U_{n,n-1}$) as follows;

$$\phi(v_i : 1 \leq i \leq n-1) = \begin{cases} 2k;\ i = 0(mod3) \\ 0;\ i = 1(mod3) \\ k;\ i = 2(mod3) \end{cases}$$

$\phi(u_2) = C_6$ $\quad \phi(u_{2i-1}) = C_{k+i}$ $\quad$ when 1 $\leq$ i $\leq$ n

Hence, the above assigned colors satisfy the condition of radio k-coloring. So, we have that $rc_k(U_{n,n-1}) \leq 6$

Now let us apply the condition of radio k-coloring to the following pairs of vertices.

$|\phi(u_1)$ - $\phi(u_2)| \geq$ 1 + k - d(u_1 ,u_2)

$|\phi(v_1)$ - $\phi(u_1)| \geq$ 1 + k - d(v_1 ,u_1)

$|\phi(v_1)$ - $\phi(v_2)| \geq$ 1 + k - d(v_1 ,v_2)

By a similar procedure, it can be verified that the remaining pair of nodes satisfies the radio k-coloring condition.

Therefore, we get $rc_k(U_{n,n-1})$ = 6

Case 2 : When n=4

From Lemma 2, it is observed that

$$rc_k(A(T_n)) \geq \begin{cases} |D_2| - 2 + 2\sum_{i=0}^{1} L_i(1-i);\ if\, k = 3 \\ |D_2| - 2 + 2\sum_{i=0}^{1} L_i(1-i)+1;\ if\, k = 2 \end{cases}$$

We have $rc_k(U_{n,n-1}) \geq$ (n+1)k-(n-1)

Consider the coloring function ϕ:V($U_{n,n-1}$)→{C_0,C_1,C_2, ...,$C_{(n+1)k-(n-1)}$}, assign the colors for the vertices of ($U_{n,n-1}$) as follows;

$\phi(v_1)$ =C_0 $\quad\phi(u_1)$ =C_{2k-1}
$\phi(v_2)$ =C_2 $\quad\phi(u_2)$ =C_{4k-2}
$\phi(v_3)$ =C_4 $\quad\phi(u_3)$ =C_{3k-2}
$\phi(u_4)$ =C_{5k-3}

Hence, the above assigned colors satisfy the condition of radio k-coloring. So, we have that $rc_k(U_{n,n-1}) \leq$ (n+1)k-(n-1)

Now let us apply the condition of radio k-coloring to the following pairs of vertices.

$|\phi(u_2)$ - $\phi(u_3)| \geq$ 1 + k – d(u_2 ,u_3)
$|\phi(u_3)$ - $\phi(u_4)| \geq$ 1 + k - d(u_3 ,u_4)
$|\phi(v_2)$ - $\phi(v_3)| \geq$ 1 + k - d(v_2 ,v_3)

By a similar procedure, it can be verified that the remaining pair of nodes satisfies the radio k-coloring condition.

Therefore, we get $rc_k(U_{n,n-1})$ = (n+1)k-(n-1)

Case 3: When n≥5

From Lemma 2, it is observed that

$$rc_k(U_{n,n-1}) \geq \begin{cases} |D_k| - 2p + 2\sum_{i=0}^{p} L_i(p-i);\ if\, k = 2p+1 \\ |D_k| - 2p + 2\sum_{i=0}^{p} L_i(p-i) + 1;\ if\, k = 2p \end{cases}$$

We have $rc_k(U_{n,n-1}) \geq$ (k-1) n + k

Consider the coloring function ϕ:V($U_{n,n-1}$)→{C_0,C_1,C_2, ...,$C_{(k-1)n+k}$}, assign the colors for the vertices of ($U_{n,n-1}$) as follows;

$$\phi(u_i : 1 \leq i \leq n) = \begin{cases} 2k;\ i = 0(mod3) \\ 0;\ i = 1(mod3) \\ k;\ i = 2(mod3) \end{cases}$$

$\phi(u_{2i-1})$ =$C_{k+(k-1)i}$ when 1≤ i ≤ $\lceil \frac{n}{2} \rceil$
$\phi(u_{2i})$ = $C_{(k-1)\lceil \frac{n}{2} \rceil + k+(k-1)i}$ when 1≤ i ≤ $\lfloor \frac{n}{2} \rfloor$

Hence, the above assigned colors satisfy the condition of radio k-coloring. So, we have that $rc_k(U_{n,n-1}) \leq$ (k -1) n+k

Now let us apply the condition of radio k-coloring to the following pairs of vertices.

$|\phi(v_i : 1 \leq i \leq n) - \phi(v_{i+1} : 1 \leq i \leq n-1)| \geq 1 + k - d(v_i : 1 \leq i \leq n, v_{i+1} : 1 \leq i \leq n-1)$
$|\phi(v_i : 1 \leq i \leq n) - \phi(u_{i+1} : 1 \leq i \leq n-1)| \geq 1 + k - d(v_i : 1 \leq i \leq n, u_{i+1} : 1 \leq i \leq n-1)$
$|\phi(v_i : 1 \leq i \leq n) - \phi(u_{i+1} : 1 \leq i \leq n-1)| \geq 1 + k - d(v_i : 1 \leq i \leq n, u_{i+1} : 1 \leq i \leq n-1)$

By a similar procedure, it can be verified that the remaining pair of nodes satisfies the radio k-coloring condition.

Therefore, we get $rc_k(U_{n,n-1})$ = (k -1) n+k

4 Conclusion

In this paper, we attain the exact value of the radio k- chromatic number for T_n, TC_n, A(T_n), $U_{n,n-1}$. Despite the few results in radio k-coloring we still experiment on radio k-chromatic numbers of various families of graphs.

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