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Exponential Type Ratio and Product Estimator of Finite Population Mean in Stratified Sampling

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Abstract

Objective: This article presents an exponential-type ratio and product estimator of the finite population mean within the stratification framework. A common technique in survey sampling is stratification, which divides the population into similar subsets to improve the estimator's accuracy. **Methods**: This paper tackles the problem of determining the expression for bias and Mean Square Error (MSE) for the population up to the 1st degree of the approximation when stratification is present. **Findings**: Here it is obtained the optimum value of the modified estimator. The modified estimator is more effective than the unbiased, ratio and product type estimators. We have used one data set through the stratified random sampling technique, and the relative efficiency of the estimators obtained from the population set is higher. **Novelty**: New proposed stratification estimators that improve the existing mixed ratio and product exponential types. The primary goal of this study is to investigate how well the suggested estimators perform compared to current estimators. Thorough simulation research to test the estimator's accuracy and efficiency. Both theoretical and practical research have shown that the proposed estimators outperform competing estimators.

Keywords: Finite population mean; Stratification; Bias; Mean square error; Percent relative efficiency

1 Introduction

The frequently used method in the sampling survey is stratification, which seeks to enhance the accuracy and precision of estimates for population parameters by separating the population into separate and distinct subgroups or strata. Each stratum is then sampled independently, and estimates for each stratum are combined to obtain an overall estimate for the entire population. Various estimators have been developed for population parameters under stratification, each with strengths and weaknesses.

In sampling theory, the most influential textbooks were penned by a wide variety of well-known authors, such as Ahmad et al. [1] works onSimulation Study: Population Distribution Function Estimation Using Dual Auxiliary Information Under Stratified Sampling Scheme. Bhushan et al. [2] study on Efficient Estimation of the Population

Mean Under Stratified Ranked Set Sampling. Semary et al.[3] developed a new comprehensive class of estimators for population proportion using auxiliary attributes: Simulation and an application. Pandey et al.[4] study on Improved Estimation of Population Variance in Stratified Successive Sampling Using Calibrated Weights Under Non-Response. Sabir, Sanaullah, and S. Gupta[5] study on Estimation of Mean Considering the Joint Influence of Measurement Errors and Non-Response in Two-Phase Sampling Designs. Sharma et al.[6] proposed enhanced Estimators and Effective Estimation Procedures for Population Variance Under Missing Random Data in Successive Sampling. Singh et al.[7] proposed a Ratio Cum Product-Type Exponential Estimator in Double Sampling for Stratification of Finite Population Mean. Ullah et al.[8] work on estimating Finite Population Mean in Simple and Stratified Random Sampling by Utilizing the Auxiliary, Ranks, and Square of the Auxiliary Information. Yadav et al.[9] developed a modified Exponential Family for Improved Searls Estimation of Finite Population Mean. Ahmad and Shabbir et al.[10] works on proportion Estimation Using an Enhanced Class of Estimators Under Simple Random Sampling: Application with Real Data Sets and Simulation. Measurement. Kumar and Tiwari et al.[11] developed a composite class of ratio estimators for the mean of a finite population in simple random sampling. Advances and Applications in Mathematical Sciences. Khan et al.[12] proposed an enhanced exponential ratio-cum-ratio estimator in ranked set sampling using transformed auxiliary information. Hussain and Sajad et al.[13] proposed a proportion Based Dual Unbiased Exponential Type Estimators of Population Mean. Seminary, Ahmad et al.[14] proposed an improved novel estimation for estimating population distribution function using auxiliary information under a stratified sampling strategy. Iftikhar et al.[15] proposed a Novel and Improved Logarithmic Ratio-Product Type Estimator of Mean in Stratified Random Sampling. Mathematical Problems in Engineering. Muneer, Siraj, Alamgir Khalil, and Javid Shabbir[16] explained A Parent-Generalized Family of Chain Ratio Exponential Estimators in Stratified Random Sampling Using Supplementary Variables. It is noticed that, across all datasets, the theoretical and empirical MSEs declined as the sample size rose gradually. Also, chain exponential type estimators work better in low correlation situations, while chain ratio type estimators work better in high correlation. Within stratified simple random sampling, Bhushan et al.[17] proposed several practical classes of combined and independent estimators for determining the population mean Y. To back up their theoretical findings; they ran a simulation study on simulated symmetric and symmetric populations. Nikita, Malik and Sumiti[18] define A New Ratio and Product Type Estimator for Mean Under Stratified Random Sampling. Ahmad and Masood et al.[19] proposed an improved modified estimator for the estimation of the median using auxiliary information under simple random sampling.

By gathering and analysing data on an auxiliary variable for use in stratified random sampling, this work primarily seeks to improve estimations of the mean of limited populations.

The manuscript's main points are as follows: Section 1 includes a literature review of ”Some existing estimators” along with notes and procedures; Section 2 presents a new combined ratio exponential type estimator based on a stratified random sampling strategy and discusses optimum variance; Section 3 discusses the comparative analysis of the proposed enhanced ratio exponential type estimator's efficiency $\widehat{\widehat{\overline{Y}}}{}^{(\alpha)}_{STR}$ with $\widehat{\widehat{\overline{Y}}}_{PS}$ and $\widehat{\widehat{\overline{Y}}}_{RC}$; Section 4 presents a new combined product exponential type estimator based on a stratified random sampling strategy and discusses optimum variance; Section 5 discusses Comparisons of the proposed enhanced product exponential type estimator's efficiency $\widehat{\widehat{\overline{Y}}}{}^{(\beta)}_{STP}$ with $\widehat{\widehat{\overline{Y}}}_{PS}$ and $\widehat{\widehat{\overline{Y}}}_{PC}$; Section 6 discusses the case study on population data and Section 7 ”Conclusion,” offers a numerical evaluation to highlight the paper's contribution

Advantages and Real life uses of Stratified Sampling

Stratified sampling might be convenient from an administrative standpoint. To choose a few towns from a big state, for example, it is easier to manage if you think of the districts as levels. This is because the administrative set up at the local scale may be used because of this goal if the districts are considered strata. Important characteristics of surveys conducted at the national level include administrative conveniences like these, as well as the ease with which fieldwork may be organized. It's possible that there are distinct sampling issues in various portions of the population. In this example, persons who live in houses, hotels, hospitals, prisons, and so on are all going to be handled differently so that the spending patterns of households may be studied.

Stratified sampling is useful when the sampling frame for sub-populations is more easily accessible than the population sampling frame. When studying a large population, it's more feasible to sample subgroups than the full population. Through the use of stratified sampling, one is able to gather some piece of information of the population. In the SRS, it is feasible that some important or useful part of the population might end up being under- or unrepresented. Using the sampling technique named as stratification, one is able to find a sample that is representative of the population to any required extent, with different parts of the sample reflecting different parts of the population. It is also feasible to achieve the level of representation of some particular population segments that is desired. If it is derived the strata in such a way that, a substantial improvement in efficiency can be attained.

1.1. Notations

Let us define a population of finite size say $P = (P_1, P_2, \ldots .P_N)$ of size N, further splits into M strata of size $N_j (j = 1, 2, \ldots, M)$. Let y represent the research variable and x represent auxiliary variables. The values of y_{jk}, x_{jk} and $z_{jk}(j = 1, 2, \ldots, M; k = 1, 2, \ldots, N_j)$ are observed on the k^{th} unit of the j^{th} stratum. Here, the x variable exhibits a positive correlation with y which is defined as study variable , but the variable z shows a negative correlation with y. A n_j sized sample is selected from every stratum, resulting in a sample of size $\sum_{j=1}^{M} n_j$.

$$\overline{Z}_j = \frac{1}{N_j} \sum_{k=1}^{N_j} z_{jk} : j^{th}\, stratum\, mean\, of\, the\, study\, variate "z"$$

$$\overline{Y}_j = \frac{1}{N_j} \sum_{k=1}^{N_j} y_{jk} : j^{th}\, stratum\, mean\, of\, the\, study\, variate "y"$$

$$\overline{X}_j = \frac{1}{N_j} \sum_{k=1}^{N_j} x_{jk} : j^{th}\, stratum\, mean\, of\, the\, auxiliary\, variate "x"$$

$$\overline{Y} = \frac{1}{N} \sum_{j=1}^{L} \sum_{k=1}^{N_j} y_{jk} = \sum_{j=1}^{L} W_j \overline{Y}_j : Population\, mean\, of\, the\, study\, variate\, "y"$$

$$\overline{X} = \frac{1}{N} \sum_{j=1}^{L} \sum_{k=1}^{N_j} x_{jk} = \sum_{j=1}^{L} W_j \overline{X}_j : Population\, mean\, of\, the\, auxiliary\, variate\, "x"\, and$$

$$\overline{Z} = \frac{1}{N} \sum_{j=1}^{L} \sum_{k=1}^{N_j} z_{jk} = \sum_{j=1}^{L} W_j Z_j : "Population\, mean\, of\, the\, auxiliary\, variate\, "z".$$

From Stephen (1945) [15] the typical unbiased estimator $\overline{Y}$ in stratification event is defined as;

$$\overline{Y}_{PS} = \sum_{j=1}^{M} W_j \overline{y}_j \tag{1.1}$$

Where

$W_j = \frac{N_j}{N}$ is the weight of the j^{th} stratum and $\overline{Y}_j = \frac{1}{n_j} \sum_{k=1}^{nj} y_{jk}$ is defined as mean of sample of the n_j sample units that exist in the j^{th} stratum.

To the 1st degree of approximation the variance for i $\overline{Y}_{PS}$ is determined as follows using the findings from Stephen (1945):

$$Var\left(\overline{Y}_{PS}\right) = \sum_{j=1}^{M} W_j^2 \gamma_j S_{yj}^2 \tag{1.2}$$

Where, $S_{yj}^2 = \left(\frac{1}{N_j - 1}\right) \sum_{k=1}^{N_j} \left(y_{jk} - \overline{Y}_j\right)^2$

For the stratification event, the estimators of population mean $\overline{Y}$ named as separate ratio and product type estimator is

$$\widehat{\overline{Y}}_{RC} = \sum_{j=1}^{M} W_j \overline{y}_j \left(\frac{\overline{X}_j}{\overline{x}_j}\right) \tag{1.3}$$

And

$$\widehat{\overline{Y}}_{PC} = \sum_{j=1}^{M} W_j \overline{y}_j \left(\frac{\overline{z}_j}{\overline{Z}_j}\right) \tag{1.4}$$

For $\widehat{\overline{Y}}_{RC}$and $\widehat{\overline{Y}}_{PC}$biases and mean squared errors are calculated as follows, upto the 1st degree of approximation:

$$B\left(\widehat{\overline{Y}}_{RC}\right) = \overline{Y}\sum_{j=1}^{M} W_j^2\gamma_j\left(C_{xj}^2 - \rho_{yxj}C_{xj}C_{yj}\right) \tag{1.5}$$

$$MSE\left(\widehat{\overline{Y}}_{RC}\right) = \overline{Y}^2\left[\sum_{j=1}^{M} W_j^2\gamma_j\left(C_{yj}^2 + C_{xj}^2 - 2\rho_{yxj}C_{xj}C_{yj}\right)\right] \tag{1.6}$$

$$B\left(\widehat{\overline{Y}}_{PC}\right) = \overline{Y}\sum_{j=1}^{M} W_j^2\gamma_j C_{yj}C_{zj}\rho_{yzj} \tag{1.7}$$

And

$$MSE\left(\widehat{\overline{Y}}_{PC}\right) = \overline{Y}^2\left[\sum_{j=1}^{M} W_j^2\gamma_j\left(C_{yj}^2 + C_{xj}^2 + 2\rho_{yxj}C_{xj}C_{yj}\right)\right] \tag{1.8}$$

2 Suggested combined ratio exponential type estimator

By the event of stratification, let propose the enhanced combined ratio exponential type estimator for $\overline{Y}$(population mean) as

$$\widehat{\overline{Y}}_{STR}^{(\propto)} = \sum_{j=1}^{M} W_j\overline{Y}_j\left(\frac{\overline{X}_j}{\overline{x}_j}\right)^{\propto} exp\left(\frac{\overline{X}_j - \overline{x}_j}{(\propto+1)\overline{X}_j + (1-\propto)\overline{x}_j}\right) \tag{2.1}$$

Where, $\propto$ is a real constant.

For getting bias and mean square error of the suggested estimator $\widehat{\overline{Y}}_{STR}^{(\propto)}$, use

$$\overline{y_j} = \overline{Y}_j\left(1+e_{0j}\right), \overline{x}_j = \overline{X}_j\left(1+e_{1j}\right) and\ \overline{z}_j = \overline{Z}_j\left(1+e_{2j}\right)\ such\ that$$
$$E\left(e_{0j}\right) = E\left(e_{1j}\right) = E\left(e_{2j}\right) = 0$$
$$E\left(e_{0j}^2\right) = \gamma_j C_{yj}^2$$
$$E\left(e_{1j}^2\right) = \gamma_j C_{xj}^2$$
$$E\left(e_{0j}e_{1j}\right) = \gamma_j\rho_{yxj}C_{yj}C_{xj}$$

$$E\left(e_{0j}e_{2j}\right) = \gamma_j\rho_{yzj}C_{yj}C_{zj}$$

$$E\left(e_{1j}e_{2j}\right) = \gamma_j\rho_{xzj}C_{xj}C_{zj}$$

$$E\left(e_{2j}^2\right) = \gamma_j C_{zj}^2$$

Expressing Equation (2.1) in the terms of e_{hj} $((h = 0,1,2)$ and by increasing the exponential function to the right, obtained with

$$\widehat{\overline{Y}}_{STR}^{(\propto)} = \sum_{j=1}^{M} W_j\overline{Y}_j\left(1+e_{0j}\right)\left(1+e_{1j}\right)^{-\propto} exp\left[-\frac{e_{1j}}{2}\left\{1+\left(\frac{1-\propto}{2}\right)e_{1j}\right\}^{-1}\right]$$

And

$$\widehat{\overline{Y}}_{STR}^{(\propto)} = \sum_{j=1}^{M} W_j\overline{Y}_j\left[1+e_{0j} - \left(\propto+\frac{1}{2}\right)e_{1j} + \frac{(4\propto^2+6\propto+3)e_{1j}^2}{8} - \left(\propto+\frac{1}{2}\right)e_{0j}e_{1j}\right]$$

$$(\widehat{\overline{Y}}_{STR}^{(\propto)}-\overline{Y})=\sum_{j=1}^{M}W_j\overline{Y}_j\left[e_{0j}-\left(\propto+\frac{1}{2}\right)e_{1j}+\frac{(4\propto^2+6\propto+3)e_{1j}^2}{8}-\left(\propto+\frac{1}{2}\right)e_{0j}e_{1j}\right] \tag{2.2}$$

By computing the expected value of both sides of Equation (2.2), it is determine the bias of the proposed estimator $\widehat{\overline{Y}}_{STR}^{(\propto)}$is acquired at the first level of approximation.

$$B\left(\widehat{\overline{Y}}_{STR}^{\propto}\right)=\sum_{j=1}^{M}W_j^2\gamma_j\overline{Y}\left[\frac{(4\propto^2+6\propto+3)\,C_{xj}^2}{8}-\left(\propto+\frac{1}{2}\right)\rho_{yxj}C_{yj}C_{xj}\right] \tag{2.3}$$

By calculating the expectation and squaring sides of Equation (2.2), it is obtained the Mean Squared Error (MSE) of the proposed estimator. $\widehat{\overline{Y}}_{STR}^{(\propto)}$to the 1st order approximation

$$MSE\left(\widehat{\overline{Y}}_{STR}^{\propto}\right)=\overline{Y}^2\sum_{j=1}^{M}W_j\gamma_j\left(C_{yj}^2+\left(\propto+\frac{1}{2}\right)^2C_{xj}^2-2\left(\propto+\frac{1}{2}\right)\rho_{yxj}C_{yj}C_{xj}\right) \tag{2.4}$$

Which is minimized for

$$\propto=\left(\frac{C_{yj}\rho_{yxj}}{C_{xj}}-\frac{1}{2}\right) \tag{2.5}$$

Up to the first degree of approximation by using Equation (2.5) in Equation (2.4) and having the least Mean Squared Error (min.MSE) of the estimator $\widehat{\overline{Y}}_{STR}^{(\propto)}$as

$$min.MSE\left(\widehat{\overline{Y}}_{STR}^{(\propto)}\right)=\sum_{j=1}^{M}W_j^2\gamma_jC_{yj}^2\left(1-\rho_{yxj}^2\right) \tag{2.6}$$

Where, $\rho_{yxj}=\frac{C_{yxj}}{C_{yj}C_{xj}}$.

3 Comparative analysis of the proposed enhanced ratio exponential type estimator's efficiency $\widehat{\overline{Y}}_{STR}^{(\alpha)}$with $\widehat{\overline{Y}}_{PS}$ and $\widehat{\overline{Y}}_{RC}$

It can be seen by observing Equations (1.2), (1.6) and (2.4) that the proposed estimator $\widehat{\overline{Y}}_{STR}^{(\propto)}$is better in efficiency than;

(i) Typically, $\widehat{\overline{Y}}_{PS}$(unbiased estimator) is if

$$\sum_{j=1}^{M}W_j\gamma_j\left[\left(\alpha+\frac{1}{2}\right)^2C_{xj}^2-2\left(\alpha+\frac{1}{2}\right)\rho_{yxj}C_{yj}C_{xj}\right]<0 \tag{3.1}$$

(ii) The conventional independent ratio estimator $\widehat{\overline{Y}}_{RPS}$ if

$$\sum_{j=1}^{M}W_j\gamma_j\left\{\left[\left(\alpha+\frac{1}{2}\right)^2-1\right]C_{xj}^2-2\left(\alpha-\frac{1}{2}\right)\rho_{yxj}C_{yj}C_{xj}\right\}<0 \tag{3.2}$$

4 An enhanced estimator of the combined product exponential type

By the event of stratification, an enhanced combined product exponential type estimator for $\overline{Y}$(population mean) is being proposed as

$$\widehat{\overline{Y}}_{STP}^{(\beta)}=\sum_{j=1}^{M}W_j\overline{y}_j\left(\frac{\overline{X}_j}{\overline{x}_j}\right)\,exp\,\left(\frac{\overline{z}_j-\overline{z}_j}{(\beta+1)\,\overline{X}_j+(1-\beta)\,\overline{x}_j}\right) \tag{4.1}$$

Where, β is a real constant.

The estimator $\widehat{\overline{Y}}_{STP}^{(\beta)}$ in Equation (4.1) expressing in terms of $e_{hj}(h=0,1,2)$ and by increasing the exponential function to the right, obtained with

$$\widehat{\overline{Y}}_{STP}^{(\beta)} = \sum_{j=1}^{M} W_j \overline{Y}_j \left(1+e_{0j}\right)\left(1+e_{1j}\right)^{\beta} \; exp\left[\frac{e_{1j}}{2}\left\{1+\left(\frac{1-\beta}{2}\right)e_{1j}\right\}^{-1}\right] \tag{4.2}$$

And

$$\widehat{\overline{Y}}_{STP}^{(\beta)} = \sum_{j=1}^{M} W_j \overline{Y}_j \left[1+e_{0j}-\left(\beta-\frac{1}{2}\right)e_{1j}+\frac{(4\beta^2+2\beta-1)e_{1j}^2}{8}-\left(\beta-\frac{1}{2}\right)e_{0j}e_{1j}\right]$$

$$(\widehat{\overline{Y}}_{STP}^{(\beta)} - \overline{Y}) = \sum_{j=1}^{M} W_j \overline{Y}_j \left[e_{0j}-\left(\beta-\frac{1}{2}\right)e_{1j}+\frac{(4\beta^2+2\beta-1)e_{1j}^2}{8}-\left(\beta-\frac{1}{2}\right)e_{0j}e_{1j}\right]$$

By following the conventional method, here it can be calculate the bias as well as mean squared error of the proposed estimator $\widehat{\overline{Y}}_{STP}^{(\beta)}$

$$B\left(\widehat{\overline{Y}}_{STP}^{(\beta)}\right) = \sum_{j=1}^{M} W_j^2 \gamma_j \overline{Y}\left[\frac{\left(4\beta^2+2\beta-1\right)C_{xj}^2}{8}-\left(\beta-\frac{1}{2}\right)\rho_{yxj}C_{yj}C_{xj}\right] \tag{4.3}$$

And

$$MSE\left(\widehat{\overline{Y}}_{STP}^{\beta}\right) = \overline{Y}^2 \sum_{j=1}^{M} W_j \gamma_j \left(C_{yj}^2+\left(\beta-\frac{1}{2}\right)^2 C_{xj}^2-2\left(\beta-\frac{1}{2}\right)\rho_{yxj}C_{yj}C_{xj}\right) \tag{4.4}$$

Which is minimized for

$$\beta = \left(\frac{C_{yj}\rho_{yxj}}{C_{xj}}+\frac{1}{2}\right) \tag{4.5}$$

By substituting the value Equation (4.5) into the Equation (4.4) and obtain with the least MSE of the estimator $\widehat{\overline{Y}}_{STP}^{(\beta)}$ upto the 1st degree of approximation

$$min.MSE\left(\widehat{\overline{Y}}_{STP}^{(\beta)}\right) = \sum_{j=1}^{M} W_j^2 \gamma_j C_{yj}^2\left(1-\rho_{yxj}^2\right) \tag{4.6}$$

5 Comparisons of the proposed enhanced product exponential type estimator's efficiency $\widehat{\overline{Y}}_{STP}^{(\boldsymbol{\beta})}$ with $\widehat{\overline{Y}}_{PS}$ and $\widehat{\overline{Y}}_{PC}$

Based on Equations (1.2), (1.8) and (4.4), it may be inferred that the proposed estimator $\widehat{\overline{Y}}_{STP}^{(\beta)}$ is better in efficiency than

(i) Typically, the conventional unbiased estimator $\widehat{\overline{Y}}_{PS}$ if

$$\sum_{j=1}^{M} W_j \gamma_j \left(\left(\beta-\frac{1}{2}\right)^2 C_{xj}^2-2\left(\beta-\frac{1}{2}\right)\rho_{yxj}C_{yj}C_{xj}\right) < 0 \tag{5.1}$$

(ii) The conventional independent ratio estimator $\widehat{\overline{Y}}_{PC}$ if

$$\sum_{j=1}^{M} W_j \gamma_j \left\{\left[\left(\beta-\frac{1}{2}\right)^2-1\right]C_{xj}^2-2\left(\beta+\frac{1}{2}\right)\rho_{xyj}C_{yj}C_{xj}\right\} < 0 \tag{5.2}$$

6 Numerical Study

Two natural population data sets are being considered in order to evaluate the performance of the proposed estimators; the population descriptions are provided below:

Population - [Source: Koyuncu & Kadilar (2009) [20]]

Y : Numbers of Teachers.

X: Numbers of Students.

Table 1.

Constant	Stratum	Constant	Stratum	Constant	Stratum
N	923	n_2	21	$\bar{Y}_5$	267.03
n	180	n_3	29	$\bar{Y}_6$	393.84
N_1	127	n_4	38	$\bar{X}_1$	20804.59
N_2	117	n_5	22	$\bar{X}_2$	9211.79
N_3	103	n_6	39	$\bar{X}_3$	14309.30
N_4	170	$\bar{Y}_1$	703.74	$\bar{X}_4$	9478.85
N_5	205	$\bar{Y}_2$	413	$\bar{X}_5$	5569.95
N_6	201	$\bar{Y}_3$	573.17	$\bar{X}_6$	12997.59
n_1	31	$\bar{Y}_4$	424.46	γ_1	0.0243840

Table 2.

Constant	Stratum	Constant	Stratum	Constant	Stratum
γ_2	0.03907204	C_{y5}	1.511643	ρ_{yx4}	0.983
γ_3	0.02477402	C_{y6}	1.807137	ρ_{yx5}	0.989
γ_4	0.02043344	C_{x1}	1.465386	ρ_{yx6}	0.965
γ_5	0.0405765	C_{x2}	1.647971	W_1	0.1375948
γ_6	0.0206659	C_{x3}	1.9253	W_2	0.126761
$\bar{X}$	11440.49848	C_{x4}	1.922061	W_3	0.111593
$\bar{Y}$	436.4330288	C_{x5}	1.525647	W_4	0.184182
C_{y1}	1.263016	C_{x6}	1.776802	W_5	0.2221002
C_{y2}	1.561554	ρ_{yx1}	0.936	W_6	0.217768
C_{y3}	1.803072	ρ_{yx2}	0.996		
C_{y4}	1.908786	ρ_{yx3}	0.994		

Table 3. Percent relative efficiencies of $\overline{Y}_{PS}$, $\widehat{\overline{Y}}_{RC}$ and $\widehat{\overline{Y}}_{STR}^{(\propto)}$ w.r.t $\overline{Y}_{PS}$

Estimators	PRE's			
	$\propto=-0.239$	$\propto=-0.240$	$\propto=-0.2427$	$\propto=-0.243$
$\overline{Y}_{PS}$	100.00	100.00	100.00	100.00
$\widehat{\overline{Y}}_{RC}$	2368.02185	2368.02185	2368.02185	2368.02185
$\widehat{\overline{Y}}_{STR}^{(\propto)}$	**3752.343144**	**3321.36025**	**2534.499561**	**2469.437219**

Table 4. Percent relative efficiencies of $\overline{Y}_{PS}$, $\widehat{\overline{Y}}_{PC}$ and $\widehat{\overline{Y}}_{STP}^{(\beta)}$ w.r.t $\overline{Y}_{PS}$

Continued on next page

Table 4 continued

Estimators	PRE's			
	β=**0.19**	β=**0.205**	β=**0.210**	β=**0.225**
$\overline{Y}_{PS}$	100.00	100.00	100.00	100.00
$\widehat{\overline{Y}}_{PC}$	24.72945101	24.72945101	24.72945101	24.72945101
$\widehat{\overline{Y}}^{(\beta)}_{STP}$	**360.5409069**	**447.3916541**	**486.2054781**	**655.922987**

7 Conclusion

The use of supplementary data to enhance the estimator's accuracy has been the subject of several writers' efforts. Here, SRSWOR is used to develop a new estimator for this research. This paper studies the bias and MSE equations of the suggested estimators. Theoretically compared the recommended estimators' MSEs to those of the standard $\overline{Y}_{PS}$, ratio $\overline{Y}_{RC}$ and $\overline{Y}_{PC}$ product and determined the requirements for obtaining the minimal MSE. It makes theoretical and numerical comparisons with other estimators to ascertain their optimality. Table 3 and Table 4 show the relative effectiveness of several estimators with respect to $\overline{Y}_{PS}$. Compared to other estimators, the proposed ones have proven more effective in some cases and demonstrate that the proposed estimators have greater relative efficiencies than the conventional $\overline{Y}_{PS}$, ratio $\overline{Y}_{RC}$ and product $\overline{Y}_{PC}$. We get better results when the conditions obtained in sections 3 and 5 are satisfied on the suggested estimators in practice.

The comparison between the suggested estimators against the efficacy of other estimators us $\widehat{\overline{Y}}^{(\beta)}_{ST}$ing two population data sets is done. Regarding numerical efficiency, the proposed estimators outperform existing estimators across all population sets, including the conventional sample mean, ratio, and product. In many practical situations, the given estimators $\widehat{\overline{Y}}^{(\propto)}_{ST}$and $\widehat{\overline{Y}}^{(\beta)}_{ST}$. One can add post-stratification to the recommended estimator for more efficient estimate results.

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