INDIAN JOURNAL OF SCIENCE AND TECHNOLOGY

RESEARCH ARTICLE

Check for updates

OPEN ACCESS
Received: 10-09-2024
Accepted: 30-09-2024
Published: 17-10-2024

Citation: @Sasikala CM, Nivetha AA (2024) Exploring Mean Cordial Labeling Possible Combination Position of Vertices in Graphs: A Computational Approach. Indian Journal of Science and Technology 17(39): 4111-4118. https://doi.org/10.17485/IJST/v17i39.2915

* **Corresponding author**.

kalasasispkc@gmail.com

Funding: None

Competing Interests: None

Published By Indian Society for Education and Environment (iSee)

ISSN
Print: 0974-6846
Electronic: 0974-5645

Exploring Mean Cordial Labeling Possible Combination Position of Vertices in Graphs: A Computational Approach

C Muthulakshmi @Sasikala[1]*, A Akil Nivetha[2]

1 Associate Professor, Department of Mathematics, Sri Paramakalyani College, Alwarkurichi, Affiliated to Manonmaniam Sundaranar University, Tirunelveli, 627412, Tamil Nadu, India
2 Research Scholar, Department of Mathematics, Sri Paramakalyani College, Alwarkurichi, Affiliated to Manonmaniam Sundaranar University, Tirunelveli, 627412, Tamil Nadu, India

Abstract

Objectives: We have developed the Python module of mean cordial labeling for the different types of graphs, like Path, Cycle, and Subdivision of Star graphs. We aim to identify the number of combination positions of vertices that satisfy the mean cordial labeling rules. We have found mathematical equations that describe the labeling behavior in these graphs. **Methods:** In this paper, a Python program was developed to find the mean cordial labeling of the Path, Cycle, and Subdivision of the Star graph. We have obtained the number of combination positions of vertex obeying the mean cordial labeling condition using this Python module. We have drawn the graph between the number of vertices and the number of combination positions of the vertex. **Findings:** We have found that the different graph types like Path, Cycle, and Subdivision of Star graph satisfy the mean cordial labeling condition. The result indicates that the number of combination positions of the Path and cycle graph fulfill the power equation $Y=a*x^b$ and the Subdivision of the Star graph meets the exponential growth equation $Y=a*b^x$. **Novelty:** In this paper, a new method has been developed to find the number of combination position of varies graphs using the Python module. This computational approach will be useful to analyze the biological networking and protein-protein interaction study.

Keywords: Mean Cordial; Path; Cycle; Subdivision of Star graph; Computational graph theory

1 Introduction

The number of vertices and edges of G is denoted by |V(G)| and |E(G)|, correspondingly. Graph Theory and cordial labeling in the context of cordial number, graph labeling is a branch of research that reaches out to many forms of node identification [1]. Cordial labeling, a later version of graceful and harmonic labeling [2] many methods were introduced. It talks about the basic ideas of a pleasant label and describes its properties on diverse graphs [3]. Mean cordial labeling, is a type of assignment of labels to graph vertices such that the mean label value over pairs of adjacent vertices is constrained. It[4] delves into mean cordial labeling of some graph classes and points out more useful applications in network theory and other indirect fields.

The discovery of cordial labeling and mean square cordial labeling for some graph structures like Star of Bistar Graphs describes recent network topologies[5–7]. New algorithms for fair graph coloring, developed by one of the researchers, promise faster computation and sculpt ability to large graphs. Graph labeling is an active area in network analysis research[8,9]. In most research literature, cordial labeling has been viewed as an algorithmic question to be solved using computational approaches[10–13]. Cordial labeling like prime and Geometric mean cordial labeling analysis was done by many researchers[14–17]. In this paper, not only the finding of mean cordial labeling but also a computational approach for mean cordial labeling and the vertex combination position was discussed.

2 Methodology

The mean cordial labeling formula ($\lceil$f(u)+f(v)/2$\rceil$. $|v_{f(i)-}v_{f(j)}|\leq 1$ $|e_{f(i)}$-$e_{f(j)}|\leq 1$) has been used for our computational mean cordial labeling analysis[4].

Definition 2.1

Let f be a map from V(G) to {0,1,2}. For each edge uv assign the label $\lceil$f(u)+f(v)/2$\rceil$, f is called a mean cordial labeling if $|v_{f(i)-}v_{f(j)}|\leq 1$ $|e_{f(i)}$-$e_{f(j)}|\leq 1$, where $v_{f(x)}$ and $e_{f(x)}$ denote the number of vertices and edges respectively labeled with x (x=0,1,2). A graph with the mean cordial labeling is called a mean cordial graph[4].

Definition 2.2

Let W=$e_1e_2\ldots e_k$ (e_i=u_iu_{i+1}) be a walk.

Is closed if u1=uk+1

Is a path if ui$\neq$uj for all i$\neq$j

Is a cycle, if it is closed, and ui$\neq$uj for i$\neq$j except that u1=uk+1[1]

Definition 2.3

A star graph $K_{1,m}$ is a complete bipartite graph with one internal node and m-leaves. A star having three edges is called as claw[5].

Definition 2.4

Let G be graph. The subdivision graph S(G) is obtained from G by subdividing each edge of G with a vertex[6].

Definition 2.5

The equation y=a*x^b is known as a power equation. It describes a functional relationship where one quantity varies as a power of another. This type of equation is often used to model scaling laws and phenomena where the rate of change follows a multiplicative pattern.

Definition 2.6

The equation y=a*b^x describes a form of exponential growth where y is the value of the quantity at a given x. a is a constant that represents the initial value or the coefficient. x is the independent variable and b is the exponent that determines the growth rate.

In section 3, we have discussed the Python module and the number of vertex position combinations in the mean cordial labeling behavior of the Path, Cycle, and Subdivision of the Star graph.

3 Results and Discussion

In this paper, a novel approach has been presented to analyze mean cordial labeling for graphs with a large number of vertices up to 50 compared to previous research papers[4]. From the constant values of **a** and **b,** we will find a very large number of vertices.

We developed the new Python module Figure 1 to find the number of combination positions of vertices. This model allowed finding the number of combination positions of a large number of vertices. Manually very difficult to analyze the combination position which satisfies the mean cordial labeling conditions, but very fast to find the combination position using this Python module. Till now there is no computational method for this mean cordial labeling analysis and cordial labeling analysis.

```
import itertools
for x in map(''.join, itertools.product('012', repeat=n)):
for i in range(n):
if x[i]=='0':
s1=s1+1
elif x[i]=='1':
s2=s2+1
elif x[i]=='2':
s3=s3+1
for l in range(0,n-1):
y=math.ceil(int(((int(x[l])+int(x[l+1]))/2.0))
b= math.ceil(int(((int(x[0])+int(x[n-1]))/2.0))
if y==0:
s4=s4+1
if y==1:
s5=s5+1
if y==2:
s6=s6+1
if b==0:
s4=s4+1
if b==1:
s5=s5+1
if b==2:
s6=s6+1
if s1==m1 and s2==s3==m2:
if s4==m2 and s5==s6==m2:
if s2==m1 and s1==s3==m2:
if s4==m2 and s5==s6==m2:
if s3==m1 and s2==s1==m2:
if s4==m2 and s5==s6==2:
print(x,s1,s2,s3,s4,s5,s6)
```

Fig 1. Python module to compute the possible combination of vertex in mean cordial labeling for Path P_n, Cycle C_n and Star graph.

Theorem 3.1:

Any path P_n is mean cordial [4]. Given a graph with mean cordial labeling of path P_n, the vertex position combinations exhibit a predictable pattern that follows a power equation ($Y=a*x^b$) distribution.

Let P_n be a path graph with n vertices. Denote by $V(P_n)$ the number of vertex position combinations that satisfy the mean cordial labeling for P_n.

Then, there exist constants **a** and **b** such that the number of vertex position combinations in P_n and satisfy the following power equation:

$V(P_n) = a * n^b$

where a and b are constants that depend on the structure of the Path graphs P_n. n be the number of vertex in the path graph

Proof:

Python module to compute the possible combination of the vertex in mean cordial labeling for Path P_n. The itertools module in Python is a powerful library that provides a collection of functions for creating iterators. These functions are especially useful for handling tasks like looping, generating combinations, and permutations, and performing other complex iteration-related operations. Here we are using Itertools is a module in Python to generate iteration values of the repetitions 0, 1, and 2.

In this for loop represent the combination of 0, 1, 2 (vertex labeling) in n times.

Example:

If n=9 then we get 3^9 =19683 Combinations.

000000000, 000000001, 000000002,……… 222222222.

s1, s2 and s3 denote the number of vertices of 0, 1, and 2 respectively

We have computed edge labeling using the mean cordial labeling formula

$$\lceil (f(u)+f(v))/2 \rceil$$

Here for loop is a control flow statement used in programming to repeatedly execute a block of code a specified number of times or iterate over a collection of elements.

The values s4, s5, s6 and it represent the number of edge labeling values as 0, 1, and 2 respectively.

In P_n graph if n=9 we have substituted m1=m2=3 in the programming. We determined whether the graph is mean cordial and identified all possible vertex combinations that satisfy the mean cordial labeling for the P_n graph. In the end, we calculated the total number of valid combinations Table 1. We have observed up to n=50. We also found the constant value **a** and **b**. Next, we plotted Figure 2 the relationship between the number of vertices and the number of vertex position combinations for the Path graph. This graphical analysis revealed a power equation of the form $Y=a*x^b$ for the Path graph. We found three different results in n (n=3,6,9…, n=4,7,10… and n=5,8,11..). These findings are valuable for research in network theory.

Table 1. The number of possible combinations of vertex that obey the mean cordial labeling of P_n graph

n	Number of possible combinations of vertex	n	Number of possible combinations of vertex	n	Number of possible combinations of vertex
4	12	5	22	6	54
7	54	8	100	9	192
10	192	11	364	12	525
13	525	14	1120	15	1188
16	1188	17	1506	18	2216
19	2216	20	2649	21	3689
22	3689	23	4305	24	5753
25	5753	26	6593	27	8529
28	8529	29	9636	30	12149
31	12149	32	13566	33	16748
34	16748	35	18523	36	22469
37	22469	38	24651	39	29462
40	29462	41	32102	42	37880
43	37880	44	41031	45	47886
46	47886	47	51602	48	59644
49	59644	50	63982		

Theorem 3.2:

Any Cycle C_n is mean cordial [4]. Given a graph with mean cordial labeling of C_n, the vertex position combinations exhibit a predictable pattern that follows a power law ($Y=a*x^b$) distribution.

Let C_n be a cycle graph with n vertices. Denote by $V(C_n)$ the number of vertex position combinations that satisfy the mean cordial labeling for C_n.

Then, there exist constants a and b such that the number of vertex position combinations in C_n and satisfy the following power equation:

$V(C_n) = a * n^b$

where a and b are constants that depend on the structure of the Cycle graphs Cn. n be the number of vertex in the Cycle graph

Proof:

A Python module was developed to calculate the possible vertex combinations in mean cordial labeling for a Cycle graph. The itertools module, a versatile library in Python, plays a crucial role in this process by offering functions that create iterators. These functions are particularly useful for tasks such as looping, generating combinations, and permutations, and performing various complex iteration-based operations.

Itertools is a module in Python to generate iteration values of the repetitions 0,1 and 2.

In this program for loop represents the combination of 0,1,2 (vertex labeling) in n times.

The values s1,s2,s3 represents the number of vertex labeling values as 0,1,2 respectively

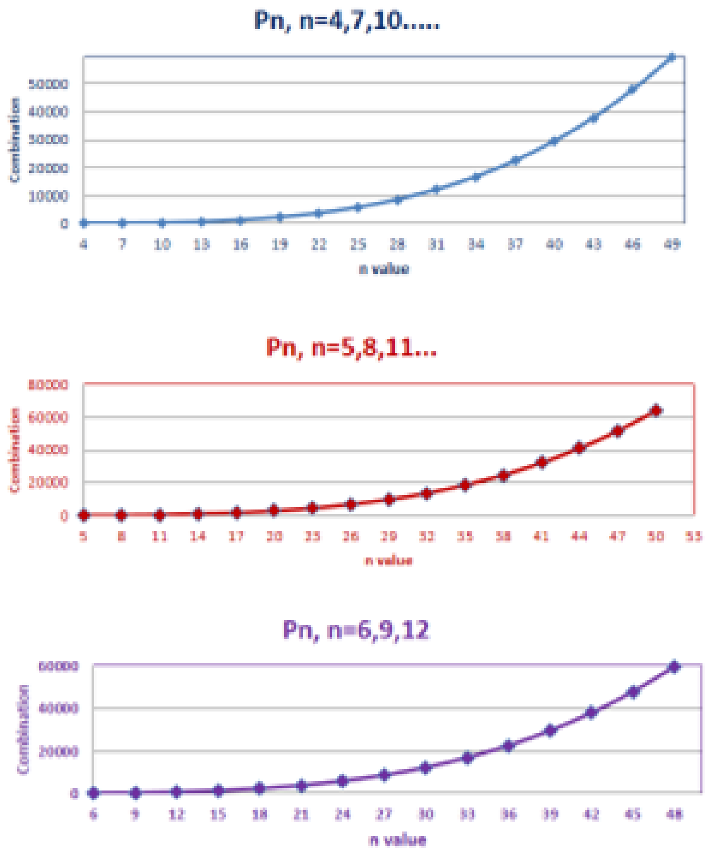

Fig 2. The number of vertex vs the number of possible combinations of vertex in mean cordial labeling for path P_n

If n=5 then, we get 3^5 =243 Combinations.
(00000,00001,00002,00010 ….. 22222)
We have computed edge labeling using the mean cordial labeling formula

$$\lceil (f(u)+f(v))/2 \rceil$$

Here for loop is a control flow statement used in programming to repeatedly execute a block of code a specified number of times or iterate over a collection of elements.

The values s4, s5, s6 represent the number of edge labeling values as 0, 1, 2 respectively.

In C_n graph if n=5 we have substituted m1=1 and m2=2 in the programming. We determined whether the graph is mean cordial and identified all possible vertex combinations that satisfy the mean cordial labeling for the C_n graph. In the end, we calculated the total number of valid combinations. We have observed up to n=50 Table 2. We also found the constant value **a** and **b**. Next, we have plotted the graph Figure 3 between the number of vertices and the number of vertex position combinations for the Cycle graph. This graphical analysis revealed a power equation of the form $Y=a*x^b$ for the Cycle graph.

We found three different results in n (if n=3,6,9… there is no vertex position composition in the output because it is not a mean cordial labeling but n=4,7,10… and n=5,8,11.. is a mean cordial graph). These findings are valuable for research in molecular structural studies.

Table 2. Number of possible combinations of vertex and edges that obey the mean cordial labeling of C_n graph

n	Number of possible combinations of vertex
4	8
5	20

Continued on next page

Table 2 continued

7	42
8	80
10	160
11	253
13	455
14	644
16	893
17	1099
19	1608
20	1916
22	2654
23	3090
25	4110
26	4700
28	6055
29	6827
31	8576
32	9560
34	11762
35	12988
37	15707
38	17206
40	20506
41	22312
43	26259
44	28407
46	33071
47	35595
49	41047
50	43983

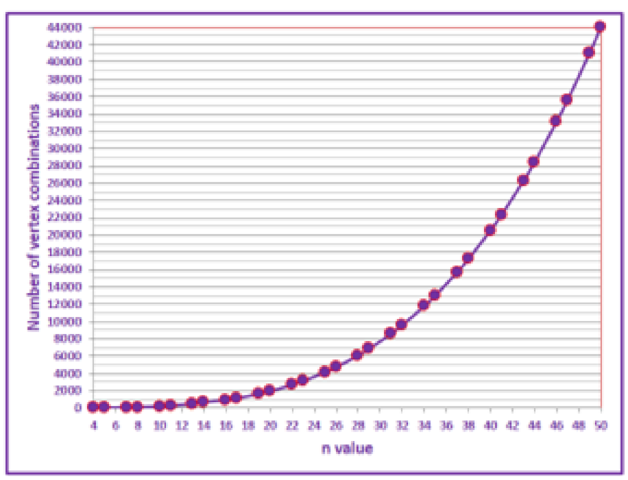

Fig 3. The number of vertex vs the number of possible combinations of vertex in mean cordial labeling for Cycle C_n

Theorem 3.3:

Any Subdivision of Star graph $S(K_{1,n})$ is mean cordial [4]. Given a graph with mean cordial labeling of $S(K_{1,n})$ graph, the vertex position combinations exhibit a predictable pattern that follows an exponential growth law ($Y=a*b^x$) distribution. Let $S(K_{1,n})$ be a Star graph with n outer vertices and one central vertex. Denote by $V(S(K_{1,n}))$ the number of vertex position combinations that satisfy the mean cordial labeling for $S(K_{1,n})$. Then, the number of valid vertex position combinations grows exponentially with respect to n, following the equation:

$V(S(K_{1,n}))=a*b^n$

where a and b are constants determined by the specific properties of the Subdivision of the Star graph.

Proof:

We are using Itertools to generate iteration values of the repetitions 0,1 and 2.

In this for loop represent the combination of 0,1,2 (vertex labeling) in m times.

In $S(K_{1,6})$ graph n=6 number of vertices m=13

We get 3^n Combinations if m=13

Hence 3^{13} = 1594323 combinations.

0000000000000, 0000000000001, 0000000000002,……… 2222222222222.

s1, s2 and s3 denotes the number of vertices of 0,1,2 respectively

We have computed edge labeling using mean cordial labeling formula

$$\lceil (f(u)+f(v))/2 \rceil$$

We determined whether the graph is mean cordial and identified all possible vertex combinations that satisfy the mean cordial labeling for the $S(K_{1,n})$ graph. In the end, we calculated the total number of valid combinations. We have plotted the values between the number of vertices and the number of vertex position combinations for the $S(K_{1,n})$ graph. This graphical analysis Figure 4 revealed an exponential growth equation of the form $Y=a*b^x$ for the $S(K_{1,n})$ graph. We have calculated the number of possible combinations up to n=15. The result clearly indicates that the number of possible combinations (Three different values of n) of the $S(K_{1,n})$ graph obeys the exponential growth equation $Y=a*b^x$. These findings are valuable for research in complex networks.

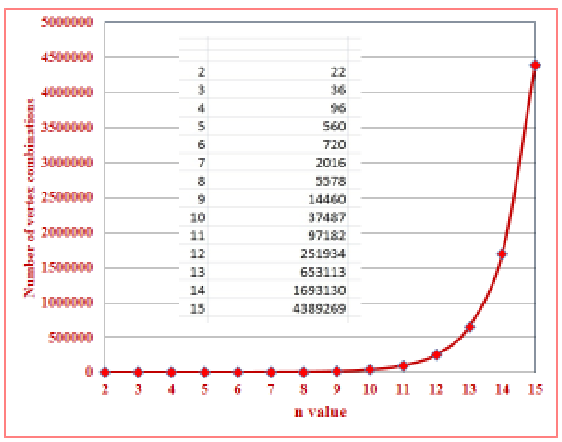

Fig 4. The number of vertex vs number of possible combinations of mean cordial labeling for $S(K_{1,n})$

4 Conclusions

A new Python module was developed to find the mean cordial labeling for any given number of vertices, as well as to determine the number of possible vertex position combinations that satisfy the mean cordial labeling conditions. In mean cordial research, it has not yet been determined how many combinations of vertices are possible using a computer program and creating a Python module, making this a novel approach to research. In contrast, previous research only explored mean cordial labeling for the specific number of vertices only (number of vertices up to 10). From the above analysis, we have designed the Python module and found a number of combination positions of vertices in Paths, Cycles, and Stars graphs. We got two different results the Power equation obeys ($y=a*x^b$) mean cordial labeling of the Path and Cycle graph and the Exponential growth equation ($y=a*b^x$) satisfies the mean cordial labeling of the Subdivision of the Star graph. These results play an important role in the deepest understanding of mean cordial labeling and its behaviors. Compared to other research our methods are novel and computational method. Mean cordial labeling of complex graphs and graphs containing a large number of vertices is easy to analyze and predict. This Computational approach leads to analysis the molecular structure and network mapping.

References

1) Harary F. Graph Theory. vol. 55 of 3. New Delhi. Addison Wesley. 1969;p. 338–338. Available from: https://doi.org/10.1201/9780429493768.
2) Gallian JA. A Dynamic Survey of Graph Labeling. *Electronic Journal of Combinatorics*. 2011;18:1–219. Available from: https://doi.org/10.37236/92.
3) Cahit I. Cordial Graphs: A Weaker Version of Graceful and Harmonious Graphs. *Ars Combinatoria*. 1987;23(3):201–207. Available from: https://www.academia.edu/54739299/Cordial_Graphs_A_Weaker_Version_of_Graceful_and_Harmonious_Graphs.
4) Ponraj R, Sivakumar M, Sundaram M. Mean Cordial Labeling of Graphs. *Open Journal of Discrete Mathematics*. 2012;2:145–148. Available from: http://dx.doi.org/10.4236/ojdm.2012.24029.
5) Gupta P, Gupta S, Srivastav S, Ganesan G. Cordial Labeling for Star of Bistar Graph. *CEUR Workshop Proceedings CEUR- WSorg 2023*;3244:66–73. Available from: https://doi.org/10.5281/zenodo.7378465.
6) Dhanalakshmi S, Parvathi N. Mean Square Cordial Labeling on Star Related Graphs. *Journal of Physics: Conference Series*. 2019;1377. Available from: https://doi.org/10.1088/1742-6596/1377/1/012027.
7) Lourdusamy A, Veronisha E, Beaula FJ. Group S4 Difference Cordial Labeling. *Indian Journal of Science and Technology*. 2022;15(32):1561–1568. Available from: https://doi.org/10.17485/IJST/v15i32.614.
8) Brandes U, Erlebach T. Network Analysis: Methodological Foundations. Springer. 2019;p. 341–372. Available from: https://doi.org/10.1007/978-3-662-04683-4.
9) Batagelj V, Doreian P, Ferligoj A, Kejžar N. Understanding Large Temporal Networks and Spatial Networks: Exploration, Pattern Searching, Visualization, and Network Evolution. John Wiley & Sons. 2014;p. 18–68. Available from: https://doi.org/10.1002/9781118915536.
10) Ghosh S, Pal A. Algorithmic Approach of Cordial Labeling on Cartesian Product Between Balanced Bipartite Graph and Path. *Communications Faculty of Sciences University of Ankara Series A1 Mathematics and Statistics*. 2019;68(2):2064–2078. Available from: https://doi.org/10.31801/cfsuasmas.558582.
11) Kleinberg J, Tardos E. Algorithm Design. Pearson Education. 2014;p. 73–97.
12) Chakraborty S, Fischer F, Schmidt M. On Algorithms for Finding and Enumerating Homomorphisms. *ACM Transactions on Algorithms (TALG)*. 2016;13(1):1–25. Available from: https://doi.org/10.1145/2980768.
13) Demaine ED, Hajiaghayi MT. Graph Theoretic Approaches to Algorithms in Data Streams and Beyond. *Theoretical Computer Science*. 2021;851:114–125. Available from: https://doi.org/10.1016/j.tcs.2020.08.018.
14) Elrokh A, Al-Shamiri MMA, Almazah MMA, El-Hay AA. A Novel Problem for Solving Permuted Cordial Labeling of Graphs. *Symmetry*. 2023;15:825–825. Available from: https://doi.org/10.3390/sym15040825.
15) Vennila L, Vidhyarani P. 4-Total Geometric Mean Cordial Labeling Of Graphs. Educational Administration. *Theory and Practice*. 2024;30:14203–14206. Available from: https://doi.org/10.53555/kuey.v30i5.6425.
16) Mitra S, Bhoumik S. Tribonacci Cordial Labeling of Graphs. *Journal of Applied Mathematics and Physics*. 2022;10:1394–1402. Available from: https://doi.org/10.4236/jamp.2022.104098.
17) Nathiya A, Manimaekalai K. A study on prime cordial labeling and graceful graphs. *Malaya Journal of Matematik*. 2022;S(2):3966–3970. Available from: https://doi.org/10.26637/MJM0S20/1024.