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RESEARCH ARTICLE

Complementary Nil Equitable Domination in Fuzzy Graphs

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Abstract

Objectives: To demonstrate an inventive showcase prominent as complementary nil equitable dominating set, complementary nil equitable domination value in fuzzy graphs (FGs), and pertinent innovations. **Methods:** Strong arc in FGs ideas are used to perceive complementary nil equitable domination value. **Findings:** Modern concepts of complementary nil equitable dominance in FGs will lead to a number of applications, including the creation of cell phone towers in remote places, traffic light signals, and Google Maps. **Novelty:** Employing the ideas of equitable and strong arc in FGs, the complementary nil equitable dominating set and complementary nil equitable domination value in FGs are obtained. Some points are observed in this concept and some theorem related to the complementary nil equitable dominant set, its numbers are proved.
AMS Subject Classification: 05C72.

Keywords: Fuzzy graph; Equitable dominating set; Equitable domination number; Complementary nil equitable dominating set; Complementary nil equitable domination number

1 Introduction

Ramya S, and Lavanya S.[1] stated independent domination in FG in 2020. Complementary nil g-eccentric domination fuzzy graph concepts were introduced by Mohamed Ismayil and Muthupandiyan[2] in 2020. S. Muthupandiyan, and A. Mohamed Ismayil[3] addressed perfect g-eccentric domination in FG in 2021. Rao Y, Kosari S, et al.[4] initiated equitable domination in vague graphs with application in medical sciences in 2021. Mallinath KS, Kulkarni LN[5] introduced inverse equitable domination in graphs in 2021. Kumaran, Narayanan et al.[6] introduced the application of fuzzy networks using efficient domination in 2023. Janofer K, S. Firthous Fatima[7] stated connected end anti-fuzzy equitable dominating set in anti-FG in 2023. The concept of equitable domination in the neutrosophic graph using a strong arc was introduced by Meenakshi and Senbagamalar in 2023[8]. Akula, N.K., S, S.B., Tarakaramu, N. et al.[9] initiated an intuitionistic FG variation coefficient measure with application to selecting a reliable alliance partner in 2024. Upper g-eccentric domination in fuzzy graphs concepts introduced by S. Muthupandiyan and A. Mohamed Ismayil[10] in 2024. Catherine Grace John,[11] et al. initiated divisor 2-equitable domination in FG in 2024.

Basic Definitions

Definition 1.1[3]: A FG $G = (\sigma,\ \mu)$ is portrayed with 2 relations ρ on V and μ on $E \subseteq V \times V$, $\sigma : V \to [0,1]$ and $\mu : E \to [0,1]$, $\mu(x,y) \leq \rho(x) \wedge \rho(y)\ \forall x,y \in V$. It is anticipated that V is both finite and non-empty, whereas μ is both reflexive and symmetric. We indicate the crisp graph $G^* = (\sigma^*, \mu^*)$ of the FG $G(\sigma,\ \mu)$, $\sigma^* = \{x \in V : \rho(x) > 0\}$ and $\mu^* = \{(x,y) \in E : \mu(x,y) > 0\}$. The FG $G = (\sigma,\ \mu)$ is represented trivial in this case $|\rho *| = 1$.

Definition 1.2[2]: A path P of length n is a sequence of distinct nodes $u_0, u_1, \ldots, u_n$ such that $\mu(u_{i-1}, u_i) > 0, i = 1, 2, \ldots, n$ and the degree of membership of a weakest arc is defined as its strength.

Definition 1.3[2]: An edge is said to be strong if its weight is equal to the strength of connectedness of its end nodes. Symbolically, $\mu(u,v) \geq CONN_{G-(u,v)}(u,v)$.

Definition 1.4[2]: The order and size of a fuzzy graph $G(\sigma,\mu)$ are defined by $p = \sum_{u \in V} \sigma(u)$ and $q = \sum_{uv \in E} \mu(u,v)$ respectively.

Definition 1.5[3] : Let $G(\sigma,\mu)$ be a fuzzy graph. The strong degree of a vertex $v \in \sigma^*$ is defined as the sum of membership values of all strong arcs incident at v and it is denoted by $d_s(v)$. Also, it is described by $d_s(v) = \sum_{u \in N_s(v)} \mu(u,v)$ where $N_s(v)$ mentioned the family of all strong neighbors of v.

Definition 1.6[2]: A fuzzy graph $G(\sigma,\ \mu)$ is connected if $CONNG(u,v) > 0$ where $CONNG(u,v)$ is the strength of connectedness between two vertices $u,\ v$ in $G(\sigma,\ \mu)$.

Definition 1.7[2]In a fuzzy graph $G(\sigma,\ \mu)$, strength of connectedness between two vertices $u,\ v \in V(G)$ is maximum strength of all paths between $u,\ v$ in $V(G)$.

Definition 1.8[2]: A subset D of V is said to be a dominating set (DS) in G if for each $v \notin D,\ u \in D$ such that u dominates v.

Definition 1.9[5]: A sub set $D \subseteq V(G)$ of a fuzzy graphs $G(\sigma,\ \mu)$ is said to be an equitable dominating set (EDS) if each $v \in V - D$, there exists a vertex $u \in D$ such that $uv \in E(G)$ and $|deg_s(u) - deg_s(v)| \leq 1$.

Definition 1.10[2]: An ensemble $D \subseteq V(G)$ of a FG $G = (\sigma,\ \mu)$ is said to be a complementary nil dominating ensemble (CNDS) if D is dominating set (DS) and its complement $V - D$ is not a DS.

2 Methodology

The concept of strong arc in FG was used by Sunitha and Sunil Mathew S. Complementary nil domination in FG was introduced by Mohamed Ismayil and Ismayil Mohideen. ED in FG was introduced by Dharmalingam and Rani. These ideas motivated us to develop the concept of complementary nil equitable domination in FG.

3 Results and Discussion

This segment, complementary nil equitable dominating set, its number obtained. Bounds and few theorems pertinent to complementary nil equitable dominating set and number stated and proved.

Complementary nil equitable domination in fuzzy graphs

Definition 3.1:

An ensemble $D \subseteq V(G)$ of a fuzzy graph $G = (\sigma,\ \mu)$ is said to be a complementary nil equitable DS (Simply CNEDS) if D is EDS and its complement $V - D$ is not an EDS. A CNEDS D of a FG $G = (\sigma, \mu)$ is said to be a minimal CNEDS if there is no CNEDS $D^{'}$ such that $D^{'} \subset D$. The lowest scalar value taken over all minimal CNEDS is said to be a complementary nil equitable domination value and is mentioned by the notion γ_{cneqd}, the analogous lowest CNEDS is represented by γ_{cneqd}-set. The highest scalar value taken over all minimal CNEDS is known as the Upper complementary nil equitable dominating value and is represented by the notion Γ_{cneqd} .

Example 3.1:

From the FG figure (Figure 1), the following are observed.

(i) The dominating set (ensemble) is $D = \{v_1,\ v_3\}$ and $\gamma(G) = 0.5$.

(ii) Equitable dominating ensemble is $D_1 = \{v_1,\ v_4\}$ and $\gamma_{eqd} G) = 0.7$.

(iii) Complementary nil equitable dominating set is $D_2 = \{v_1,\ v_2,\ v_3\}$ and $\gamma_{cneqd}(G) = 0.9$.

(iv) Upper complementary nil equitable dominating set is $D_3 = \{v_1, v_4, v_5\}$ and
(v) $\Gamma_{cneqd}(G) = 1.1$
(vi) $\gamma(G) \le \gamma_{eqd}(G) \le \gamma_{cneq}(G) \le \Gamma_{cneqd}(G)$.

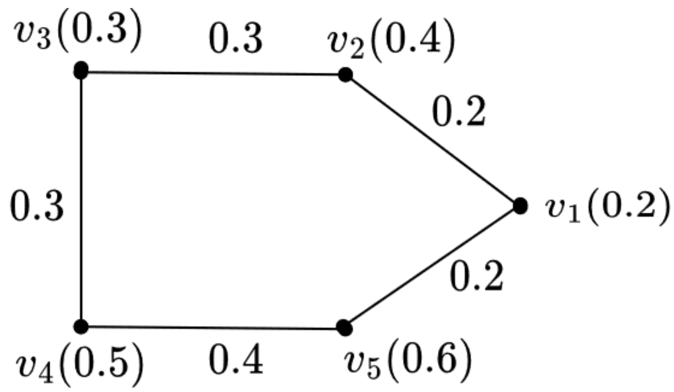

Fig 1. Complementary nil equitable domination in a FG

Observation 3.1:

For any FG $G = (\sigma, \mu)$,

1. Every super ensemble of a CNEDS is also a CNEDS.
2. Complement of a CNEDS is not a CNEDS.
3. Complement of a γ-set is not a CNEDS.
4. γ_{cneqd} -set need not be unique.
5. $\gamma_{cneqd} \le \Gamma_{cneqd}$.

Definition 3.2:

Let $G = (\sigma, \mu)$ be a FG, $S \subseteq V$. A node $u \in S$ is notorious as an enclave of S if $\mu(u, v) < \sigma(u) \wedge \sigma(v) \forall\ v \in V - S$ that is $N_s[u] \subseteq S$.

Theorem 3.1:

An equitable dominating ensemble D is a CNEDS if and only if it carries a minimum of one enclave.

Proof. Let D is FG's $G = (\sigma, \mu)$ CNEDS. Then $V - S$ isn't an equitable dominating ensemble which implies that $\exists$ a node $u \in D$, $\mu(u, v) < \sigma(u) \wedge \sigma(v)\ \forall v \in V - S$. $\therefore u$ be an enclave of D . So, D contains a minimum of one enclave. Contrariwise, Imagine an equitable dominating ensemble D carry an enclaves. Let us consider u be S's enclave without losing generality.

That is $\mu(u, v) < \sigma(u) \wedge \sigma(v)$, for all $v \in V - S$. Hence $V - S$ isn't a DS. So, an EDS D is a CNEDS.

Theorem 3.2:

If D is a CNEDS of a FG $G = (\sigma, \mu)$, then $\exists$ a node $u \in D$, $D - \{u\}$ is an equitable dominating ensemble.

Proof. Let D is a complementary nil equitable dominating ensemble of a FG $G = (\sigma, \mu)$. According to theorem 3.1, each CNEDS has minimum one enclave in D. Let $u \in D$ is an enclave of D . Then $\mu(u, v) < \sigma(u) \wedge \sigma(v) \forall v \in V - S$. Because G is a connected FG, $\exists$ minimum a node $w \in D$, $\mu(u, w) = \sigma(u) \wedge \sigma(w)$. $\therefore D - \{u\}$ is an equitable dominating ensemble.

Theorem 3.3:

A complementary nil equitable dominating ensemble in a FG $G = (\sigma, \mu)$ isn't singleton.

Proof. Let D be a complementary nil equitable dominating set of a FG $G = (\sigma, \mu)$. According to theorem 3.1, each complementary nil equitable dominating ensemble has a minimum one enclave in D. Let $u \in D$ is an enclave of D. Then $(u, v) < \sigma(u) \wedge \sigma(v) \forall\ v \in V - D$.....(1). Imagine D contains only one node u , (1) pageant it must be isolated in G, it is a discrepancy in connectedness. Hence a complementary nil DS has one more node.

Theorem 3.4:

Let $G = (\sigma,\ \mu)$ is a FG, D is a γ_{cneqd}-ensemble of G. If u, v are 2 enclaves of D, subsequently

(1) $N\_s\,[u] \cap N\_s\,[v] \neq \phi$ and

(2) $\mu(u,v) \leq \sigma(u) \wedge \sigma(v),$ that is u and v are proximal.

Proof. Let D be the lowest CNEDS of a FG $G = (\sigma,\mu)$ and let u, v are 2 enclaves of D.

- Imagine $N_s[u] \cap N_s[v] = \phi$. Then u is an enclave of $D - N_s(v)$, $V - (D - N_s(v))$ isn't an equitable dominating ensemble. $\therefore D - N_s(v)$ be a CNEDS of G also $|D - N_s(v)| < |D| = \gamma_{cneqd}(G)$. Which is contrary to the lowest possible of D. $\therefore N_s[u] \cap N_s[v] \neq \phi$.
- Deems $\mu(u,v) \leq \sigma(u) \wedge \sigma(v)$, u, v are non-proximity. Then $u \notin N(v)$ and so u be an enclave of $D - \{v\}$, $V - (D - \{v\})$ is not an EDS. Hence $D - \{v\}$ is a CNEDS, which is contrary to the minimality of D. For the reason that, u and v are proximity.

Theorem 3.5:

A CNEDS of a FG $G = (\sigma,\mu)$ isn't independent.

Proof. Let $G = (\sigma,\mu)$ is a FG. Imagine a CNEDS D of G is independent. Then D is a minimal equitable DS, $V - D$ is an equitable dominating ensemble. Since, S isn't a CNEDS, which is a conflict.

Theorem 3.6:

A CNEDS D of a FG $G = (\sigma,\mu)$ is minimal $\Longleftrightarrow$ for every $u \in D$ minimum one of the below-mentioned points is satisfied(i) there exists $v \in V - D$ such that $N_s(v) \cap S = \{u\}$.(ii) $V - (D - \{u\})$ is an equitable dominating ensemble of G.

Proof. If D be a minimal CNEDS of a FG $G = (\sigma,\mu)$. Imagine, if $\exists u \in D$, u doesn't satisfy both the given rules (i) and (ii). Later, according to a theorem 3.1, D isn't a minimal equitable dominating ensemble. So, the proper subset $D_1 = D - \{u\}$ isn't an equitable dominating ensemble. By our hypothesis on (ii) $V - (D - \{u\})$ is an equitable DS. So, $D_1 = D - \{u\}$ is a CNEDS. Which is contrary to the minimality of the CNEDS D.

Conversely, let D is a CNEDS, for every $u \in S$ minimum one of the 2 points hold. Now we prove that D is minimal CNEDS of D. Assume D isn't minimal, then $\exists$ a node $u \in D$, $D - \{u\}$ is a CNEDS. As $D - \{u\}$ is a CNEDS, u is proximity to a minimum one node $v \in D - \{u\}$. Also $D - \{u\}$ is a DS, each node $v \in V - D$ is proximity to a minimum of one node in $D - \{u\}$. That is $N_s(v) \cap D \neq \{u\}$, points (i) does not occur. As $D - \{u\}$ is a CNEDS, $V - (D - \{u\})$ not an equitable DS, condition (ii) does not occur, this is a contrariwise to our imagination. So D is a minimal EDS of G.

Theorem 3.7:

For any FG $G = (\sigma,\mu)$ each γ_{cneqd} -set of G halve with each γ-ensemble of G.

Proof. Let D is a γ_{cned}-ensemble and D is a γ-ensemble of $G = (\sigma,\mu)$. Assume $D \cap D' = \phi$, then $D' \subseteq V - D, V - D$ has a DS D . $\therefore V - D$, a super ensemble of D', is a DS. This is a contrariwise to our hypothesis. So, $D \cap D' \neq \phi$.

Corollary 3.1:

For any FG $G = (\sigma,\mu)$ any $2\gamma_{cneqd}$-ensembles are halves.

Observation 3.2:

- For any FG $G = (\sigma,\mu)\,, \gamma \leq \gamma_{cneqd} \ \leq p$.
- $\gamma_{cneqd}\,(K_\sigma - e) \leq p - \sigma_0$, where $\sigma_0 = min_{u \in V}\,\sigma(u)$.
- $\gamma_{cneqd}\,(K_\sigma - e) = p - \sigma(u)$, where $\sigma(u)$ is obtained from $\mu\,(e) \leq \sigma(u) \wedge \sigma(v) = \sigma(v)$.
- For any FG $G = (\sigma,\mu)\,, 2\sigma_0 \leq \gamma_{cneqd} \leq p - \sigma_0$.
- For a complete bipartite FG, $\gamma_{cneqd}\left(K_{\sigma_1,\sigma_2}\right) \leq min\,(|\sigma_1|\,, |\sigma_2|) + \sigma_n$, where $\sigma_n = max_{u \in V}\,\sigma(u)$
- For a complete bipartite FG

$$\gamma_{cneqd}\left(K_{\sigma_1,\sigma_2}\right) = \begin{cases} |\sigma_1| + \sigma_{20} & , if\ |\sigma_1| < |\sigma_2| \\ |\sigma_2| + \sigma_{10} & , if\ |\sigma_1| > |\sigma_2| \\ |\sigma_1| + \sigma_0 & , if\ |\sigma_1| = |\sigma_2| \end{cases}$$

where σ_{10} and σ_{20} are the lowest value of a node in σ_1-set, σ_2-set respectively.

- Let T_σ be a tree in an FG $G=(\sigma,\mu)$, Then $\gamma_{cneqd}(T_\sigma) \le \gamma(T_\sigma)+\sigma_n$.

Theorem 3.8:

For any FG $G=(\sigma,\mu)$ if $\gamma=\frac{p}{2}$, then $\gamma_{cneqd}(G)=\frac{p}{2}+\sigma_0$.

Proof. Let $G=(\sigma,\mu)$ be an FG, let D be γ-ensemble of G with $\gamma(G)=\frac{p}{2}$. Hence $V-D$ is a minimum EDS with $|V-D|=\frac{p}{2}$. Select a node $u\in V$, $\sigma(u)=\sigma_0$. Now either $u\in D$ or $u\in V-D$. Since $[(V-D)\cup\{u\}]$ or $D-\{u\}$ is a CNEDS. Then either $D'\cup\{u\}$ or $(V-D)-\{u\}$ is not an equitable DS. $\therefore$ Either $D'\cup\{u\}$ or $[(V-D)-\{u\}]$ is a CNEDS. Since $\gamma_{cneqd}=\frac{p}{2}+\sigma_0$.

Theorem 3.9

For any FG $G=(\sigma,\mu),\Gamma+\gamma_{cneqd}\le p+\sigma_n$.

Proof. Let D is a Γ-set of G, then $\exists$ a $u\in D$, $D-\{v\}$ is not an equitable DS of G. Since S is a minimal equitable DS. Next, $V-D$ is an equitable DS and $[(V-D)\cup\{u\}]$ is also an equitable DS. Accompaniment of $[(V-D)\cup\{u\}]$ is $D-\{u\}$, but $D-\{u\}$ isn't an equitable DS. $\therefore(V-D)\cup\{u\}$ is a CNEDS. Since $\gamma_{cned}\le|(V-S)\cup\{u\}|=p-\Gamma+\sigma_n$. Thus $\Gamma+\gamma_{cneqd}\le p+\sigma_n$.

Theorem 3.10:

Consider the fuzzy tree T_σ, next $\gamma_{cned}(T_\sigma)\le p-r+\sigma_{p_0}$, where r is the scalar value of the family of all hanging nodes in T_σ and σ_{p_0} is the lowest membership value of a hanging node.

Proof. Consider the fuzzy tree T_σ, then the family of all non-hanging nodes together with a hanging node form a CNEDS. So, $\gamma_{cneqd}(T_\sigma)\le r+\sigma_{p_0}$, r is the scalar value of the family of all hanging nodes in T_σ and σ_{p_0} is the lowest membership value of a hanging node.

4 Conclusion

In this research, complementary nil equitable dominating set, and its number in FGs are obtained. Theorems related to complementary nil dominating sets and numbers are stated and proved. Bounds and some points are observed and discussed.

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