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RESEARCH ARTICLE

A Glimpse on Integral Solutions of Ternary Uniform Second Degree Equation $5\left(\mathbf{x}^{2}+\mathbf{y}^{2}\right)-3xy=2000z^{2}$

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Abstract

Objectives: Many integer solutions to ternary homogeneous quadratic given by $5\left(x^{2}+y^{2}\right)-3xy=2000z^{2}$ are determined. Various choices of integer solutions are secure from beginning to end employing linear modifications and used to simplify expressions. **Methods:** Different transformations are utilized to determine many solutions in integers for the considered polynomial equation. **Findings:** Five distinct transformations are applied to obtain choices of integral solutions for the considered second-degree equation having three unknowns. **Novelty:** The equation in the title has been reduced to a solvable ternary quadratic equation by employing suitable transformation. **Mathematics Subject Classification: 11D09**

Keywords: Quadratic having three unknowns; Homogeneous quadratic; Integer Solutions; Ternary Quadratic Equation; Pair of equations

1 Introduction

There are plenty of motivating and attractive problems in the area of quadratic algebraic equations. The uniform or non-uniform second-degree equations having three unknowns are rich in variety and have attracted many mathematicians. S. Vidhyalakshmi, and T. Mahalakshmi [(1)] initiated to obtain an integer solution for a study on the homogeneous cone in 2019. S. Vidhyalakshmi, M.A. Gopalan [(2)] introduced finding the integral solutions for homogeneous quadratic equations with three unknowns in 2021. P. Sangeetha, and T. Divyapriya [(3)] discussed to solutions in integers for the ternary second-degree polynomial equation in 2023. J.Shanthi, T.Mahalakshmi,S. Vidhyalakshmi, M.A. Gopalan [(4)] searched to obtain an integral solution on integer solutions to the uniform second-degree equation with three unknowns in 2023. T. Mahalakshmi, and E. Shalini [(5)] analyzed to invention of an integer solution to the homogeneous ternary quadratic Diophantine equation in 2023. J. Shanthi, M.A. Gopalan [(6)]stated the solutions in integer for the ternary quadratic diophantine equation in 2021. M.A. Gopalan, S. Vidhyalakshmi, J. Shanthi, and V. Anbuvalli [(7)] discussed to finding the integer Solutions of ternary quadratic diophantine equation in 2022. G. Dhanalakshmi, and M.A. Gopalan [(8)] analyzed integer solutions to uniform second-degree equations having three unknowns in 2022.

N. Thiruniraiselvi and M.A Gopalan[9] stated to treasure solutions in integer to delineation of integral solutions to uniform fourth degree equation having four unknowns in 2023. Shreemathi[10] discussed getting a solution on a fourth-degree equation having four unknowns given by $7xy+3z^2 = 3w^4$. Though there are many second-degree Diophantine equations (homogeneous or non-homogeneous), still it is an important topic of current research. The above problems motivated us to search various choices of non-trivial integer solutions to the uniform cone represented by the ternary quadratic diophantine equation $5\left(x^2+y^2\right)-3xy=2000z^2$. Different sets of solutions in integers are obtained by applying the method of factorization and also, through employing the linear transformations.

2 Methodology

The uniform second-degree polynomial equation having three unknowns is

$$5\left(x^2+y^2\right)-3xy=2000z^2 \tag{2.1}$$

The process to determine different patterns of integral solutions for Equation (2.1) is illustrated below:

3 Results and Discussion

Choice 3.1

Substituting

$$x=60\left(3\alpha+3\beta+10V\right), y=600\beta, z=30\alpha+9V \tag{3.1}$$

in Equation (2.1) , it reduces to the Pythagorean equation

$$\alpha^2=\beta^2+V^2 \tag{3.2}$$

which is satisfied by

$$V=2pq, \beta=p^2-q^2, \alpha=p^2+q^2, p>q>0 \tag{3.3}$$

In view of Equation (3.1), we have

$$x=60\left(20pq+6p^2\right), y=600\left(p^2-q^2\right), z=30\left(p^2+q^2\right)+18pq \tag{3.4}$$

Thus, Equation (3.4) gives the solutions in integers to Equation (2.1).

Note 3.1.1

However, apart from Equation (3.3), one may consider the solutions to Equation (3.2) as

$$V=p^2-q^2, \beta=2pq, \alpha=p^2+q^2$$

leading to a different set of integer solutions to Equation (2.1).

Choice 3.2

Choosing

$$x=60(A+B+10T), y=200(A+10T), z=10A+91T \tag{3.5}$$

in Equation (2.1), it reduces to

$$A^2=B^2+91T^2 \tag{3.6}$$

It is seen that Equation (3.6) is satisfied by

$$T=2rs, B=91r^2-s^2, A=91r^2+s^2$$

From Equation (3.5), we have

$$x=60\left(182r^2+20rs\right), y=200\left(91r^2+s^2+20rs\right), z=10\left(91r^2+s^2\right)+182rs$$

Note 3.2.1

Consider Equation (3.6) as double equations as given below in Table 1.

Table 1. Pair of connection

Pair	I	II	III	IV	V	VI
$A+B$	T^2	$7T^2$	$13T^2$	$91T^2$	$13\,T$	$91\,T$
$A-B$	91	13	7	1	$7\,T$	T

Solving each of the above pair of equations, one obtains values of $A,\ B,\ T$.

Taking into consideration Equation (3.5), the respective solutions in integer for Equation (2.1) are obtained and they are shown as follows:

Solutions to Equation (2.1) from pair I:

$$x=60\left(4k^2+24k+11\right),y=200\left(2k^2+22k+56\right),z=10\left(2k^2+2k+46\right)+91(2k+1)$$

Solutions to Equation (2.1) from pair II:

$$x=60\left(28k^2+48k+17\right),y=200\left(14k^2+34k+20\right),z=10\left(14k^2+14k+10\right)+91(2k+1)$$

Solutions to Equation (2.1) from pair III:

$$x=60\left(52k^2+72k+23\right),y=200\left(26k^2+46k+20\right),z=10\left(26k^2+26k+10\right)+91(2k+1)$$

Solutions to Equation (2.1) from pair IV:

$$x=60\left(364k^2+384k+101\right),y=200\left(182k^2+202k+56\right)$$

$$,z=10\left(182k^2+182k+46\right)+91(2k+1)$$

Solutions to Equation (2.1) from pair V:

$$x=60*23k,y=200*20k,z=191k$$

Solutions to Equation (2.1) from pair VI:

$$x=60*101k,y=200*56k,z=551k$$

Choice 3.3

Write Equation (3.6) as

$$B^2+91T^2=A^2*1 \tag{3.7}$$

Assume

$$A=100\left(a^2+91b^2\right) \tag{3.8}$$

Consider 1 as

$$1=\frac{(3+i\sqrt{91})(3-i\sqrt{91})}{100} \tag{3.9}$$

Substituting Equation (3.8) & Equation (3.9) in Equation (3.7) and applying factorization, consider

$$B+i\sqrt{91}T=10(3+i\sqrt{91})(a+i\sqrt{91}b)^2$$

giving

$$B=10\left[3\left(a^2-91b^2\right)-182ab\right],T=10\left[\left(a^2-91b^2\right)+6ab\right]$$

From Equation (3.5), the integer solutions to Equation (2.1) are found to be

$$x=600\left(23a^2-273b^2-122ab\right),y=2000\left(20a^2+60ab\right),z=1910a^2+90*91b^2+5460ab$$

Note 3.3.1

Apart from Equation (3.9), the integer 1 is given by

$$1=\frac{\left(91r^2-s^2+i\sqrt{91}2rs\right)\left(91r^2-s^2-i\sqrt{91}2rs\right)}{\left(91r^2+s^2\right)^2}$$

This choice leads to different set of integer solutions to Equation (2.1).

Choice 3.4

Rewrite Equation (3.6) as

$$A^2-91T^2=B^2*1 \tag{3.10}$$

Assume

$$B=9\left(a^2-91b^2\right) \tag{3.11}$$

Consider 1 as

$$1=\frac{(10+\sqrt{91})(10-\sqrt{91})}{9} \tag{3.12}$$

Substituting Equation (3.11) & Equation (3.12) in Equation (3.10) and using factorization, consider

$$A+\sqrt{91}T=3(10+\sqrt{91})(a+\sqrt{91}b)^2$$

from which we get

$$\left[A=30a^2+30*91b^2+546ab\right],T=3a^2+3^*91b^2+60ab$$

From Equation (3.5), the integer solutions to Equation (2.1) are found to be

$$x=60\left(69a^2+51*91b^2+1146ab\right),y=200\left(60a^2+60*91b^2+1146ab\right)$$
$$z=573a^2+573*91b^2+10920ab$$

Note 3.4.1

Apart from Equation (3.12), the integer 1 is given by

$$1=\frac{\left(91r^2+s^2+\sqrt{91}2rs\right)\left(91r^2+s^2-\sqrt{91}2rs\right)}{\left(91r^2-s^2\right)^2}$$

This choice leads to a different set of integer solutions to Equation (2.1).

Choice 3. 5

Express Equation (3.6) in the ratio form as

$$\frac{A+B}{7T}=\frac{13T}{A-B}=\frac{P}{Q},Q\neq 0 \tag{3.13}$$

Solving the above system of double equations and using the buttery method, the values of

$$A=7P^2+13Q^2,B=7P^2-13Q^2,T=2PQ$$

In view of Equation (3.5), the solutions in integers to Equation (2.1) are given by

$$x=60\left(14P^2+20PQ\right),y=200\left(7P^2+13Q^2+20PQ\right),z=10\left(7P^2+13Q^2\right)+182PQ$$

4 Conclusion

An exertion has been made to get various sets of integral solutions for the uniform cone given by$5\left(x^2+y^2\right)-3xy=2000z^2$. The transformation technique and factorization method are utilized to obtain integer solutions to the given uniform cone. The significance of the above techniques is to simplify the given equation to an elegant solvable equation. As the ternary quadratic equations are plenty, the readers and researchers may search for other forms of ternary quadratic equations to determine their integer solutions.

References

1) Lakshmi SV, Mahalakshmi T. A Study on The Homogeneous Cone x^2+7y^2=23z^2. *International Research Journal of Engineering and Technology (IRJET)*. 2019;6(3):5000–5007. Available from: https://www.irjet.net/archives/V6/i3/IRJET-V6I31277.pdf.
2) Vidhyalakshmi S, Gopalan MA. Homogeneous Quadratic Equation With Three Unknowns z2 =6x2-2y2. *International Journal of Enhanced Research in Science*. 2021;10(12):31–37. Available from: https://erpublications.com/uploaded_files/download/s-vidhyalakshmi-m-a-gopalan_hfzLH.pdf.
3) Sangeetha P, Divyapriya T. On the Ternary Quadratic Diophantine Equation x2+ 14 xy + y2 = z2". *International Journal for Research in Applied Science & Engineering Technology (IJRASET)*. 2023;11:726–729. Available from: https://www.ijraset.com/best-journal/on-the-ternary-quadratic-diophantine-equation.
4) Shanthi J, Mahalakshmi T, Vidhyalakshmi S, Gopalan MA. A search on Integer solutions to the Homogeneous Quadratic Equation with Three Unknowns x^2+17y^2=21z^(2,). *International Journal of Research Publication and Reviews*. 2023;14(5):3610–3619. Available from: https://ijrpr.com/uploads/V4ISSUE5/IJRPR13203.pdf.
5) Mahalakshmi T, Shalini E. On Finding Integer Solutions to the Homogeneous Ternary Quadratic Diophantine Equation 3(x^2+[y]^2)-5xy=15z^2, . *International Journal of Research publication and Reviews*. 2023;4(5):452–462. Available from: https://ijrpr.com/uploads/V4ISSUE5/IJRPR12677.pdf.
6) Shanthi J, Gopalan MA. On The Ternary Quadratic Diophantine equation 19z2= 3y2 +x2. *International Journal of Research Publication and Reviews*. 2021;2(8):69–76. Available from: https://www.ijrpr.com/uploads/V2ISSUE8/IJRPR915.pdf.
7) Gopalan MA, Vidhyalakshmi S, Shanthi J, Anbuvalli V. On Finding The Integer Solutions of Ternary Quadratic Diophantine EquationDOCX-UPLOAD:INLINE:66afcf4bb1dd4c18991a941257dfab88:ENDEQNBLOCK. *International Journal of Precious Engineering Research and Applications (IJPERA)*. 2022;7(4):44–47. Available from: https://www.allmultidisciplinaryjournal.com/uploads/archives/20230503103028_B-23-123.1.pdf.
8) Dhanalakshmi G, Gopalan MA. Homogeneous Quadratic Equation With Three Unknowns z2 =11 x2 -7 y2. *International Journal of Engineering And Science*. 2022;12(1):24–32. Available from: https://www.researchinventy.com/papers/v12i1/G12012432.pdf.
9) Thiruniraiselvi N, Gopalan MA. Delineation of Integer Solutions to Homogeneous Bi-quadratic Equation with Four Unknowns. *journal of Liaoning technical university*. 2023;17(10). Available from: https://lgjdxcn.asia/public_article.php?article=147.
10) Adiga S. On bi-quadratic equation with four unknowns 7xy+3z2 =3w4. *AIP Conf Proc*. 2020;2261(1):030–087. Available from: https://doi.org/10.1063/5.0016866.