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Fuzzy Range Labeling of Fuzzy Star Graph and Helm Graph

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Abstract

Objectives: To find a new labeling technique in Fuzzy Graph Theory called Fuzzy Range Labeling. This concept helps to handle the uncertainty in graph labeling areas. **Methods:** Fuzzy graph labeling condition - $\mu(u,v) < \sigma(u) \wedge \sigma(v)$ and Range labeling condition - $f^*(E) = Maximum\ value\ (v_k, v_{k+1}) - Minimum\ value\ (v_k, v_{k+1})$ (that is, the edge membership value is calculated as the difference between the maximum and minimum values of the vertex incident to that edge) are used. **Findings:** In this study, we have found that the fuzzy star graph and helm graph are fuzzy range graphs. **Novelty:** The fuzzy range labeling technique was not yet studied. Thus, this new notion will motivate many researchers to study further.

Keywords: Fuzzy graph labeling; Range labeling; Fuzzy range labeling; Fuzzy range graph; Fuzzy star graph; Helm graph

1 Introduction

Graph Labeling concept was first introduced by Rosa in 1967. It is one of the most interesting topics in Graph Theory [1]. Range labeling was first introduced and developed by R. Jahir Hussain and J. Senthamizh Selvan [2] who applied this notion to some graphs.

Due to the natural existence of uncertainty, Fuzzy Graph definition was first introduced by Kaufmann (1973) based on Zadeh's [3] fuzzy relations (1965). In 1975, Azriel Rosenfeld developed the Fuzzy Graph Theory independently. A fuzzy graph is based on the crisp graph. Hence, it is normal that numerous characteristics are comparable to crisp graph while yet variating in several places. Fuzzy Labeling Graphs has been introduced and studied by A. Nagoor Gani and D. Rajalaxmi [4] who discussed about the properties of fuzzy labeling graphs. A research on fuzzy labeling has tremendous growth in recent days especially remarkable progress have happened in 20^{th} century. Fuzzy labeling has so many applications in various fields of Mathematics and Engineering especially in Science and Technology, Information Theory, Neural Networks, Cluster Analysis, Medical Diagnosis, Control Theory, etc.

S. Vimala and R. Jebesty Shajila [5] discussed about fuzzy edge-vertex graceful labelling of Star graph and Helm graph. Their work motivated us to introduce the new concept of fuzzy graph labeling technique called Fuzzy Range Labeling and we are illustrating for fuzzy star graph and helm graph here.

In addition to that, this study is the further contribution on fuzzy graph labeling. "Fuzzy Range Labeling of Fuzzy Star Graph and Helm Graph".

This study uses only simple and finite graphs.

2 Methodology

The methodology of this study follows definitions.

Definition 2.1 [2]: Range Labeling

Let $G=(V,E)$ be a graph with n vertices. A bijection on $f: V \to \{1,2,\ldots,n\}$ is called a Range Labeling if for each edge E is distinct and E is defined by

$f^*(E) = Maximum\ value\ (v_k, v_{k+1}) - Minimum\ value\ (v_k, v_{k+1})$.

Definition 2.2: Range Graph

If a graph G admits range labeling, we say G is a range graph.

Definition 2.3 [3]: Fuzzy Labeling Graph

The bijective functions $\sigma: V \to [0,1]$ and $\mu: V \times V \to [0,1]$ such that the membership values of edges and vertices are distinct and $\mu(u,v) < \sigma(u) \wedge \sigma(v)$ for all $u,v \in V$, is called fuzzy labeling. A graph which admits fuzzy labeling is called a fuzzy labeling graph.

Definition 2.4 [5] : Fuzzy Star Graph

A fuzzy star graph $S_{1,n}$ consists of two vertex sets V and U with $|V|=1$ and $|U|>1$ such that $\mu(v,u_i)>0$ and $\mu(u_i,u_{i+1})=0, 1 \le i \le n$.

Definition 2.5 [6] : Fuzzy Relation

Let σ be a fuzzy subset of a set S and μ a fuzzy relation on S. In symbols, $\mu: S \times S \to [0,1]$ such that $0 \le \mu(x,y) \le 1$ for all $(x,y) \in S \times S$. Then μ is called a fuzzy relation on σ if $\mu(x,y) \le \sigma(x) \wedge \sigma(y)$ for all $x,y \in S$. Here $\wedge$ stands for minimum.

Definition 2.6 [6] : Fuzzy Graph

A fuzzy graph $G=(V,\sigma,\mu)$ is a triple consisting of a non empty set V together with a pair of functions $\sigma: V \to [0,1]$ and $\mu: E \to [0,1]$ such that for all $x,y \in V$, $\mu(xy) \le \sigma(x) \wedge \sigma(y)$ where σ denotes the fuzzy vertex set of G and μ denotes the fuzzy edge set of G.

Definition 2.7 [7]: Graph Labeling

A graph labeling is an assignment of integers to the vertices or edges or both, subject to certain conditions.

Definition 2.8: Fuzzy Range Graph

If fuzzy labeling exists along with range labeling condition, in which all the vertex or edge or both values are distinct, we say it is called Fuzzy Range Labeling. A fuzzy graph which admits fuzzy range labeling is called a Fuzzy Range Graph.

Definition 2.9 [8] : Star Graph

The star graph S_n is a tree on n vertices with one vertex having degree $n-1$ and the other $n-1$ vertices having degree 1. S_n is therefore isomorphic to the complete bipartite graph $K_{1,n-1}$.

Definition 2.10: Fuzzy Range Star Graph

In a fuzzy star graph, if all the edge and vertex values are distinctly labelled and it admits range labeling then it is called fuzzy range star graph.

Definition 2.11 [9] : Helm Graph

The helm graph H_n is the graph obtained from an $n-$wheel graph by adjoining a pendent edge at each vertex of the cycle.

3 Main Results

Theorem: 3.1

Every fuzzy star graph with at most 49 edges $(x_i - vertices)$ are fuzzy range graph.

Proof:

Let $S_{1,n}$ be a fuzzy star graph consists of two vertex sets Y and X with $|Y|=1$ and $|X|>1$ such that $\beta(y,x_i)>0$ and $\beta(x_i,x_{i+1})=0,\ 1 \le i \le n$..

Let $\alpha(y) \in (0,1]$ and $\alpha(y_j) = \frac{j}{10}$, where $j=1,2,\ldots,10$.

Also $\alpha(x_i) \in (0,1]$ and $\alpha(x_i) = \alpha(y) - \beta(y,x_i)$, $i=1,2,\ldots,n$.

$\Rightarrow \beta(y,x_i) = \alpha(y) - \alpha(x_i)$, where $i=1,2,\ldots,n$.

$i.e.,\ \beta(y,x_i) = max(\alpha(y),\alpha(x_i)) - min(\alpha(y),\alpha(x_i)), i = 1,2,...,n.$

Here $\beta(y,x_i) = \frac{i}{100}$, where $i = 1,2,...,n$ and

$n = 4j+t$, where $j = 1,2,...,10;\ t = 0\ to\ 9$ respectively.

When $j = 1$, $\alpha(y_j) = 0.1$ then the number of edges $n(x_i - vertices)$ satisfying the fuzzy range labeling conditions are only four. $i.e.,$ All the edges and vertices are distinct only when $n = 4(1)+0$ where $j = 1;\ t = 0$ respectively.

When $j = 2$, $\alpha(y_j) = 0.2$ then $n = 4(2)+1 = 9$ where $j = 2;\ t = 1$ respectively.

Continuing in this same way,

When $j = 10$, $\alpha(y_j) = 1$ then the number of edges $n(x_i - vertices)$ satisfying the fuzzy range labeling conditions are at most forty nine. $i.e.,$$n = 4(10)+9 = 49$ where $j = 10;\ t = 9$ respectively.

Hence $S_{1,n}$ will be fuzzy range graph when $n = 4j+t$, where $j = 1,2,...,10;\ t = 0\ to\ 9$ respectively.

Illustration: 3.1.1

Case (i):

When $j = 1$, $\alpha(y) = \frac{1}{10} = 0.1$

$$\begin{aligned}
\beta(y,x_1) &= \tfrac{1}{100} = 0.01 = 0.1 - 0.09 = \alpha(y) - \alpha(x_1)\\
\beta(y,x_2) &= \tfrac{2}{100} = 0.02 = 0.1 - 0.08 = \alpha(y) - \alpha(x_2)\\
\beta(y,x_3) &= \tfrac{3}{100} = 0.03 = 0.1 - 0.07 = \alpha(y) - \alpha(x_3)\\
\beta(y,x_4) &= \tfrac{4}{100} = 0.04 = 0.1 - 0.06 = \alpha(y) - \alpha(x_4)
\end{aligned}$$

Since all the membership values of edges and vertices are distinct along with the conditions

$\beta(y,x_i) < \alpha(y) \wedge \alpha(x_i);\ \forall i = 1,2,3,4$ and

$\beta(y,x_i) = max(\alpha(y),\alpha(x_i)) - min(\alpha(y),\alpha(x_i))$ for all $i = 1\ to\ 4$.

Here $n = 4j+t = 4(1)+0 = 4$ where $j = 1;\ t = 0$.

$S_{1,4}$ is a fuzzy range graph.

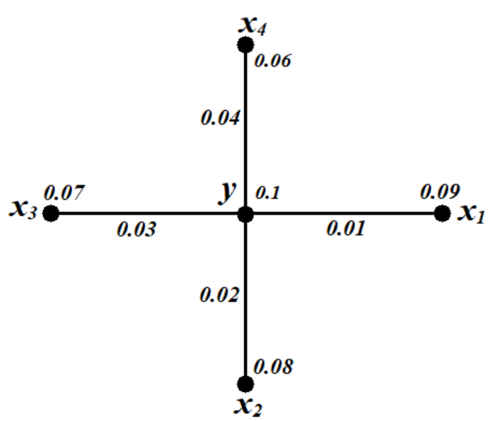

Fig 1. Fuzzy Range Labeling for $S_{1,4}$

Case (ii):

When $j = 2$, $\alpha(y) = \frac{2}{10} = 0.2$

$$\begin{aligned}
\beta(y,x_1) &= \tfrac{1}{100} = 0.01 = 0.2 - 0.19 = \alpha(y) - \alpha(x_1)\\
\beta(y,x_2) &= \tfrac{2}{100} = 0.02 = 0.2 - 0.18 = \alpha(y) - \alpha(x_2)\\
\beta(y,x_3) &= \tfrac{3}{100} = 0.03 = 0.2 - 0.17 = \alpha(y) - \alpha(x_3)\\
\beta(y,x_4) &= \tfrac{4}{100} = 0.04 = 0.2 - 0.16 = \alpha(y) - \alpha(x_4)\\
\beta(y,x_5) &= \tfrac{5}{100} = 0.05 = 0.2 - 0.15 = \alpha(y) - \alpha(x_5)\\
\beta(y,x_6) &= \tfrac{6}{100} = 0.06 = 0.2 - 0.14 = \alpha(y) - \alpha(x_6)\\
\beta(y,x_7) &= \tfrac{7}{100} = 0.07 = 0.2 - 0.13 = \alpha(y) - \alpha(x_7)\\
\beta(y,x_8) &= \tfrac{8}{100} = 0.08 = 0.2 - 0.12 = \alpha(y) - \alpha(x_8)\\
\beta(y,x_9) &= \tfrac{9}{100} = 0.09 = 0.2 - 0.11 = \alpha(y) - \alpha(x_9)
\end{aligned}$$

Here $n = 4j+t = 4(2)+1 = 9$ where $j = 2;\ t = 1$.

Since all the fuzzy range labeling conditions satisfied

$S_{1,9}$ is a fuzzy range graph.

Similarly,

When $\alpha(y) = 0.3, n = 4(3) + 2 = 14$ here $j = 3;\ t = 2$ then $S_{1,14}$ is a fuzzy range graph.

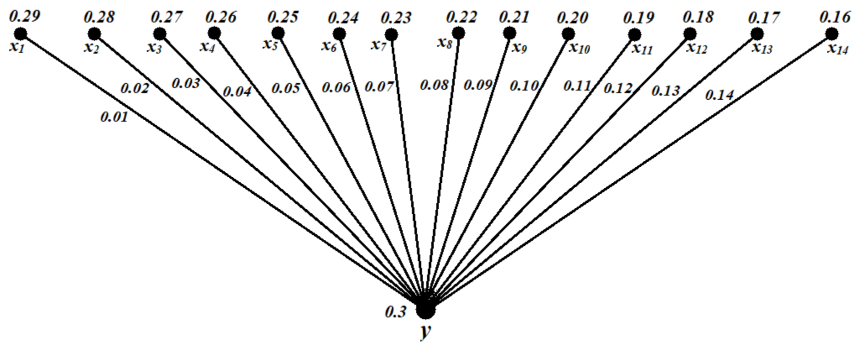

Fig 2. Fuzzy Range Labeling for $S_{1,14}$

When $\alpha(y) = 0.4, n = 4(4) + 3 = 19$ here $j = 4;\ t = 3$ then $S_{1,19}$ is a fuzzy range graph.
When $\alpha(y) = 0.5, n = 4(5) + 4 = 24$ here $j = 5;\ t = 4$ then $S_{1,24}$ is a fuzzy range graph.
When $\alpha(y) = 0.6, n = 4(6) + 5 = 29$ here $j = 6;\ t = 5$ then $S_{1,29}$ is a fuzzy range graph.
When $\alpha(y) = 0.7, n = 4(7) + 6 = 34$ here $j = 7;\ t = 6$ then $S_{1,34}$ is a fuzzy range graph.
When $\alpha(y) = 0.8, n = 4(8) + 7 = 39$ here $j = 8;\ t = 7$ then $S_{1,39}$ is a fuzzy range graph.
When $\alpha(y) = 0.9, n = 4(9) + 8 = 44$ here $j = 9;\ t = 8$ then $S_{1,44}$ is a fuzzy range graph.
When $\alpha(y) = 1, n = 4(10) + 9 = 49$ here $j = 10;\ t = 9$ then $S_{1,49}$ is a fuzzy range graph.

Hence, all $S_{1,n}$ with at most 49 edges (x_i - vertices) are fuzzy range graph.

Theorem: 3.2

The helm graph H_3 is a fuzzy range graph.

Proof:

Case (i):

In H_3, when y is to be assigned values from 0.4 to 1 $i.e.$,$0.4 \leq y \leq 1$, all the edge values in the outer cycle are multiples of 2 and the graph is fuzzy range labeling helm graph.

Let the helm graph H_3's centre vertex, other vertices in the outer cycle and the pendent vertices be $y,\ a_i\ (i = 1,2,3)$ and $b_i\ (i = 4,5,6)$ respectively.

In this cycle, Let $\alpha(y) = 0.4\ and\ \beta(y, a_1) = 0.01$

$\beta(a_i, a_{i+1}) = i \times 0.02$ and

$\beta(b_i, a_i) = i \times 0.02$, where $b_i\ (i = 4,5,6)$ and $a_i\ (i = 1,2,3)$.

Illustration: 3.2.1

$\beta(a_1, a_2) = 1 \times 0.02 = 0.02$		$\beta(b_4, a_1) = 4 \times 0.02 = 0.08$
$\beta(a_2, a_3) = 2 \times 0.02 = 0.04$	and	$\beta(b_5, a_2) = 5 \times 0.02 = 0.10$
$\beta(a_3, a_1) = 3 \times 0.02 = 0.06$		$\beta(b_6, a_3) = 6 \times 0.02 = 0.12$

Hence, all the edge and vertex values are distinct and the edges are multiples of 2.

Case (ii):

In H_3, when y is to be assigned values from 0.5 to 1 $i.e.$,$0.5 \leq y \leq 1$, all the edge values in the outer cycle are multiples of 3 and the graph is fuzzy range labeling helm graph.

In this cycle, Let $\alpha(y) = 0.9\ and\ \beta(y, a_1) = 0.01$

$\beta(a_i, a_{i+1}) = i \times 0.03$ and

$\beta(b_i, a_i) = i \times 0.03$, where $b_i\ (i = 4,5,6)$ and $a_i\ (i = 1,2,3)$.

Illustration: 3.2.2

$\beta(a_1, a_2) = 1 \times 0.03 = 0.03$		$\beta(b_4, a_1) = 4 \times 0.03 = 0.12$
$\beta(a_2, a_3) = 2 \times 0.03 = 0.06$	and	$\beta(b_5, a_2) = 5 \times 0.03 = 0.15$
$\beta(a_3, a_1) = 3 \times 0.03 = 0.09$		$\beta(b_6, a_3) = 6 \times 0.03 = 0.18$

Hence, all the edge and vertex values are distinct and the edges are multiples of 3.

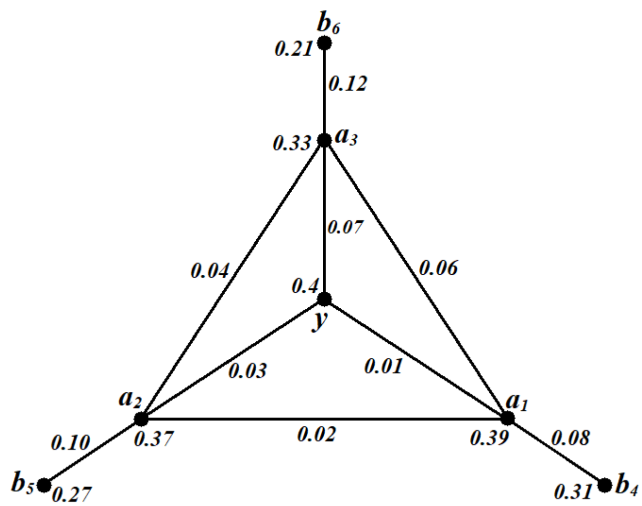

Fig 3. Fuzzy Range Labeling for H_3 (Edge values are multiples of 2)

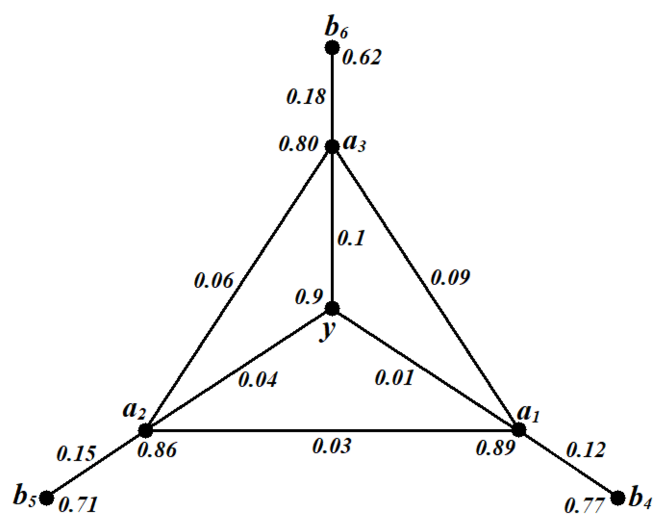

Fig 4. Fuzzy Range Labeling for H_3 (Edge values are multiples of 3)

Since all the fuzzy range labeling conditions satisfied H_3 is a fuzzy range graph.
Remark: 3.3
For $n \geq 4$, the Helm graph H_n will not satisfy the above conditions (Case (ii)).

4 Conclusion

The new notion of fuzzy graph labeling technique called fuzzy range labeling has been introduced in this article. Also, in this study, discussion was made on fuzzy range labeling for Fuzzy Star Graph and Helm Graph and proved that $S_{1,n}$ and H_3 are fuzzy range graph. In future this research work will be extended for some more graphs.

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