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Generalized Parametric Fuzzy Information Measure

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Abstract

Objective: To generalize an existing fuzzy information measure and verify its important properties. **Methods:** Both objective and descriptive methods have been adopted; problem-solving and problem-posing methods have been used. A collection of properties presented by De Luca and Termini is used as a standard for establishing new fuzzy information measures. **Findings:** This study has analysed the different features of the parametric measure of fuzzy information that we have presented. The suggested fuzzy entropy measure is a legitimate measure of fuzzy information since it meets all necessary requirements. Its monotonic behaviour has also been studied. Comparisons between the proposed measure of fuzzy information and the corresponding measure of probabilistic information have been discussed. **Novelty:** The unique quality of the suggested fuzzy information measure is found in its monotonic behaviour corresponding to parameter α. When conducting a multicriteria decision analysis and directing the decision-making process, monotonic fuzzy information measures are more useful. Because the suggested measure satisfies fundamental set features, it can be applied in soft computing for economics, decision-making, and data aggregation.
Mathematics Subject Classification: 94 D 05

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1 Introduction

Shannon [1] measured the degree of uncertainty in a probability distribution's randomness using a concept known as "entropy." Zadeh [2] established the concept of fuzzy sets as a framework for handling problems that contain inherent uncertainty. Fuzziness is the outcome of ambiguity and is defined as the lack of clear boundaries for set membership when an entity is neither clearly inside a set nor outside of it. De Luca and Termini [3] initially worked on fuzzy entropy. Bhagwan Dass et. al. [4] proposed the generalized exponential measure of fuzzy entropy. Dutta and Banik [5] suggested a new generalized similarity measure having applications to criminal investigations in intuitionistic fuzzy surroundings. Singh and Ganie [6] applied fuzzy information measures to multicriteria decision analysis. A. Munde [7,8] suggested that fuzzy information measures should be applied to decision-making. Gleb Beliakov, Jian-Zhang Wu, Weiping Ding[9] explained the representation, optimization, and generation of fuzzy measures. Hooda[10] introduced a new measure of fuzzy entropy that corresponds to the non-additive entropy of discrete probability distributions, having a parameter α, characterized by Sharma and Taneja[11]. This fuzzy entropy measure involves only one parameter. To enhance this paradigm, a new generalized parametric measure of fuzzy entropy with two parameters α and β has been introduced. Because of its parameters, the suggested measure has greater applicability in decision analysis. Compared to conventional entropy measures, the two-parameter fuzzy entropy offers a more nuanced assessment of uncertainty and complexity in each of these domains. The entropy measure's parameters generally provide the adjustment of its sensitivity to various elements of the data, hence augmenting its usefulness and efficacy in a wide range of intricate circumstances. In Section 2, we define some terms related to fuzzy set theory, probability theory, and the measurement of fuzzy data. In Section 3, this study introduces an innovative fuzzy entropy measure called "parametric measure of fuzzy entropy with parameter α" and verifies its axiomatic validity. The monotonic behaviour of the proposed measure is also presented. The proposed measure and corresponding probabilistic measure are compared in this section. Also some fundamental set properties have been discussed, and finally, the study concludes in Section 4.

2 Methodology

2.1. Measure of Information

Assume that in an experiment using the probability distribution $P=(P_1,P_2,....,P_n)$, X is a discrete random variable. The well-known Shannon[1] entropy,

$$\mathrm{H}(\mathrm{P})=-\sum_{\mathrm{i}=1}^{\mathrm{n}}\mathrm{p_i}\log\mathrm{p_i} \tag{2.1.1}$$

provides the information found in this experiment.

2.2. Fuzzy Set Theory

Recall that a membership function $\mu_{\mathrm{A}}:\mathrm{U}\to[0,1]$ characterizes a fuzzy subset A in U (collection of speech) and expresses the degree of membership of $\mathrm{x}\in\mathrm{U}$ as follows in A:

$\mu_{\mathrm{A}}(\mathrm{x})=0$ if $\mathrm{x}\notin\mathrm{A}$, and there is no uncertainty

$=1$ if $\mathrm{x}\in\mathrm{A}$ and there is no uncertainty

$=0.5$ if there is greatest uncertainty

In actuality, $\mu_A(x)$ is related to each $x\in U$ a classification of membership within set A. As the characteristic function of a crisp (i.e. non-fuzzy) set, $\mu_A(x)$ has a value in $\{0,1\}$ that is the characteristic function of a crisp (i.e. nonfuzzy) set.

2.3. Generalization of Fuzzy Entropy Measures

In 1968, Zadeh[2] introduced the concept of fuzzy entropy as a fuzziness measure. Since $\mu_A(x)$ and $1-\mu_A(x)$ yields the similar degree of fuzziness and thus approximates the entropy due to Shannon[1], De Luca and Termini[3] presented the fuzzy entropy measure listed below:

$$\mathrm{H}(\mathrm{A})=-\left[\sum_{\mathrm{i}=1}^{\mathrm{n}}\mu_{\mathrm{A}}(\mathrm{x_i}).\log\mu_{\mathrm{A}}(\mathrm{x_i})+\sum_{\mathrm{i}=1}^{\mathrm{n}}(1-\mu_{\mathrm{A}}(\mathrm{x_i})).\log(1-\mu_{\mathrm{A}}(\mathrm{x_i}))\right] \tag{2.3.1}$$

A collection of properties presented by De Luca and Termini[3] is commonly used as a standard for establishing new fuzzy entropies. Entropy, a measure of fuzziness in fuzzy set theory, reflects the average degree of ambiguity or having problems recognizing if an element is part of a set or not. Therefore, to be considered legitimate fuzzy entropy, a fuzzy set's average fuzziness measurement needs to satisfy the minimum requirements listed below:

i) $\mathrm{H}(\mathrm{A})=0$ when $\mu_A(x_i)=0\ or\ 1$

ii) $\mathrm{H}(\mathrm{A})$ increases as $\mu_{\mathrm{A}}(\mathrm{x_i})$ increases between 0 to 0.5.

iii) $\mathrm{H}(\mathrm{A})$ decreases as $\mu_A(x_i)$ increases between 0.5 to 1.

iv) $\mathrm{H}(\mathrm{A})=\mathrm{H}\left(\overline{A}\right)$,i.e. $\mu_{\mathrm{A}}(\mathrm{x_i})=1-\mu_{\mathrm{A}}(\mathrm{x_i})$

v) $\mathrm{H}(\mathrm{A})$ represents a concave function of $\mu_{\mathrm{A}}(\mathrm{x_i})$.

3 Result and Discussion

Sharma and Taneja[11] characterize the entropies of discrete probability distributions that are not additive, as follows:

$$H^{\beta}(P) = \frac{1}{2^{1-\beta}}\left[2^{1-\beta}.\sum_{i=1}^{n} p_i \log p_i\right], \beta > 0 \text{ and } \beta \neq 1 \quad (3.1)$$

and

$$H_{\alpha}^{\beta}(P) = \frac{1}{2^{1-\beta}}\left[\left(\sum_{i=1}^{n} p^{\alpha}{}_{i}\right)^{\frac{\beta-1}{\alpha-1}} - 1\right], \alpha \neq \beta,\ \alpha > 0,\ \beta > 0 \text{ and } \alpha \neq 1 \quad (3.2)$$

according to Equation (3.1) and Equation (3.2), Hooda[10] proposed the following measure of fuzzy entropy:

$$H_{\alpha}^{\beta}(A) = \frac{1}{1-\beta}\left[2^{(\beta-1)\sum_{i=1}^{n}\{\mu_A(x_i).\log\mu_A(x_i)+(1-\mu_A(x_i)).\log(1-\mu_A(x_i))\}} - 1\right], \beta > 0 \; and \; \beta \neq 1 \quad (3.3)$$

Corresponding to Equation (3.3), we suggest the fuzzy entropy measure listed below:

$$H_{\alpha}^{\beta}(A) = \frac{1}{1-\beta}\left[2^{(\beta-1)\sum_{i=1}^{n}\{\mu_A^{\alpha}(x_i).\log\mu_A^{\alpha}(x_i)+(1-\mu_A(x_i))^{\alpha}.\log(1-\mu_A(x_i))^{\alpha}\}} - 1\right], \beta > 0 \; and \; \beta \neq 1 \quad (3.4)$$

Theorem: The fuzzy entropy Equation (3.4) is a valid measure.

Proof: $H^{\beta}(A)$ is a suitable fuzzy entropy measure if all of the following conditions apply:

$H_{\alpha}^{\beta}(A) = 0$ if $\mu_A(x_i) = 0$ or $1, \forall x_i$ is trivially satisfied.

Also $H_{\alpha}^{\beta}(A) = H_{\alpha}^{\beta}(\overline{A})$ i.e. $\mu_A(x_i) = 1 - \mu_A(x_i)$

Now to show that $H_{\alpha}^{\beta}(A)$ is a concave function of $\mu_A(x_i)$

$$\frac{dH_{\alpha}^{\beta}(A)}{d\mu_A(x_i)} = \frac{(\beta-1)}{1-\beta}\left[2^{(\beta-1).\{\mu_A^{\alpha}(x_i).\log\mu_A^{\alpha}(x_i)+(1-\mu_A(x_i))^{\alpha}.\log(1-\mu_A(x_i))^{\alpha}\}} - 1\right]$$

$$\left[\alpha\mu_A^{\alpha-1}(x_i) + \alpha\mu_A^{\alpha-1}(x_i).\log\mu_A^{\alpha}(x_i) - \alpha(1-\mu_A(x_i))^{\alpha-1} - \alpha(1-\mu_A(x_i))^{\alpha-1}.\log(1-\mu_A(x_i))^{\alpha}\right]$$

Assume that $0 < \mu_A(x_i) < 0.5$,, then two different situations arise

Case 1: When $\alpha > 1$, then we have $\frac{dH_{\alpha}^{\beta}(A)}{d\mu_A(x_i)} \geq 0$

Case 2: When $\alpha < 1$, then we have $\frac{dH_{\alpha}^{\beta}(A)}{d\mu_A(x_i)} \geq 0$

Hence $H_{\alpha}^{\beta}(A)$ is a function that increases for $\mu_A(x_i)$ for $0 < \mu_A(x_i) < 0.5$

Similarly $H_{\alpha}^{\beta}(A)$ is a function that decreases for $\mu_A(x_i)$ for $0.5 < \mu_A(x_i) < 1$

Hence $H_{\alpha}^{\beta}(A)$ is a concave function. It satisfies each and every criterion for fuzzy entropy. Consequently, it is an accurate representation of fuzzy information.

3.1. Monotonic Behavior of Generalized Measure of Fuzzy Information

$$\frac{dH_{\alpha}^{\beta}(A)}{d\alpha} = -\left[2^{(\beta-1).(\mu_A^{\alpha}(x_i).\log\mu_A^{\alpha}(x_i)+(1-\mu_A(x_i))^{\alpha}.\log(1-\mu_A(x_i))^{\alpha}).\log 2}\right]$$

$$[\mu_A^{\alpha}(x_i).\log\mu_A^{\alpha}(x_i).\log\mu_A^{\alpha}(x_i) + \mu_A^{\alpha}(x_i).\log\mu_A(x_i).\alpha\log(1-\mu_A(x_i)).\log(1-\mu_A(x_i))^{\alpha}$$

$$+\alpha(1-\mu_A(x_i))^{\alpha}.\log(1-\mu_A(x_i))^{\alpha}]$$

So $\frac{dH_{\alpha}^{\beta}(A)}{d\alpha} < 0$and therefore we conclude that $H_{\alpha}^{\beta}(A)$ is monotonically decreasing function of α.

We plot the graph of $H_{\alpha}^{\beta}(A)$ w.r.t. $\mu_A(x_i)$ various values of α and for a constant value of β.

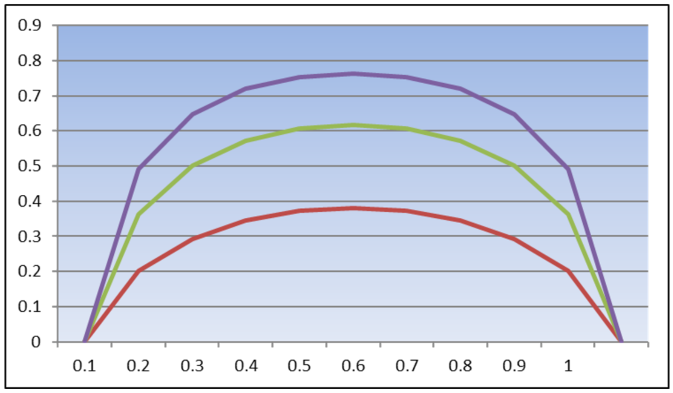

Fig 1. $\mathbf{H}_{\alpha}^{\beta}(\mathbf{A})$ **with respect to** $\mu_{\mathbf{A}}(\mathbf{x}_{\mathbf{i}})$

3.2 Comparison between $H_{\alpha}^{\beta}(P)$ and $H_{\alpha}^{\beta}(A)$

- If P is uniform distribution, then clearly, that $H_{\alpha}^{\beta}(P)$ does not depend on α. Similarly, if set A is the crisp or the fuzziest, then $H_{\alpha}^{\beta}(A)$ does not depend on α.
- Both are decreasing functions with a monotonicity of α
- When $\alpha \to \infty$.

$$H_{\alpha}^{\beta}(P) \to \frac{1}{2^{1-\beta}-1} \sum_{i=1}^{n} \left[2^{-(1-\beta).\log P_{\max}}\right]$$

$$H_{\alpha}^{\beta}(A) \to \frac{1}{1-\beta} \sum_{i=1}^{n} \left[2^{-(1-\beta).\max.\{\ln \mu_A(x_i);\ln(1-\mu_A(x_i))\}} - 1\right]$$

3.3. Some Properties of Entropy Measure

Definition: Let $FS(X)$ be the family of all fuzzy sets in the universe X and $R, S, T \in FS(X)$ is given by

$$R = [(x, \mu_R(x)) : x \in X]$$
$$S = [(x, \mu_S(x)) : x \in X]$$
$$T = [(x, \mu_T(x)) : x \in X]$$

then the following set operations are defined as:

a. $R \cup S = \{(x, \max(\mu_R(x), \mu_S(x))) : x \in X\}$
b. $R \cap S = \{(x, \min(\mu_R(x), \mu_S(x))) : x \in X\}$
c. $(R \cup S) \cup T = \{x, [\max\{max(\mu_R(x), \mu_S(x)), \mu_T(x)\}] : x \in X\}$
d. $(R \cap S) \cap T = \{x, [\min\{\min(\mu_R(x), \mu_S(x)), \mu_T(x)\}] : x \in X\}$
e. $R^C = \{(x, \mu_{R^C}(x) = 1 - \mu_R(x)) : x \in X\}$

Now we discuss on some important properties of the proposed exponential entropy measure.

Theorem: For fuzzy set $R, S, T \in FS(X)$ prove that

$$\text{(I)}\ H_{\alpha}^{\beta}(R \cup S) + H_{\alpha}^{\beta}(R \cap S) = H_{\alpha}^{\beta}(R) + H_{\alpha}^{\beta}(S)$$

$$\text{(II)}\ H_{\alpha}^{\beta}((R \cup S) \cup T) = H_{\alpha}^{\beta}(R \cup (S \cup T))$$

$$\text{(III)}\ H_{\alpha}^{\beta}((R \cap S) \cap T) = H_{\alpha}^{\beta}((R \cap (S \cap T)))$$

$$\text{(IV)}\quad \mathrm{H}_{\alpha}^{\beta}(\mathrm{R}) = \mathrm{H}_{\alpha}^{\beta}(\mathrm{R}\cap\mathrm{R}^{\mathrm{C}}) = \mathrm{H}_{\alpha}^{\beta}(\mathrm{R}\cup\mathrm{R}^{\mathrm{C}}) = \mathrm{H}_{\alpha}^{\beta}(\mathrm{R}^{\mathrm{C}})$$

Proof: We suppose that

$$\begin{aligned} \mathrm{X}_{\phi} &= [\mathrm{x_i}/\mathrm{x_i} \in \mathrm{X}, \mu_{\mathrm{R}}(\mathrm{x_i}) \geq \mu_{\mathrm{S}}(\mathrm{x_i}) \geq \mu_{\mathrm{T}}(\mathrm{x_i})] \\ \mathrm{X}_{\psi} &= [\mathrm{x_i}/\mathrm{x_i} \in \mathrm{X}, \mu_{\mathrm{R}}(\mathrm{x_i}) < \mu_{\mathrm{S}}(\mathrm{x_i}) < \mu_{\mathrm{T}}(\mathrm{x_i})] \end{aligned}$$

where $\mu_{\mathrm{R}}(\mathrm{x_i}), \mu_{\mathrm{S}}(\mathrm{x_i}), \mu_{\mathrm{T}}(\mathrm{x_i})$ be the fuzzy membership functions of R, S and T respectively.

(I) We know that

$$\mathrm{H}_{\alpha}^{\beta}(\mathrm{R}\cup\mathrm{S}) = \frac{1}{1-\beta}\sum_{\mathrm{i=1}}^{\mathrm{n}}\left[2^{(\beta-1).\sum_{\mathrm{i=1}}^{\mathrm{n}}\{\mu_{\mathrm{R}\cup\mathrm{S}}^{\alpha}(\mathrm{x_i}).\log\mu_{\mathrm{R}\cup\mathrm{S}}^{\alpha}(\mathrm{x_i})+(1-\mu_{\mathrm{R}\cup\mathrm{S}}(\mathrm{x_i}))^{\alpha}.\log(1-\mu_{\mathrm{R}\cup\mathrm{S}}(\mathrm{x_i}))^{\alpha}-1\}}\right]$$

$$\Rightarrow \mathrm{H}_{\alpha}^{\beta}(\mathrm{R}\cup\mathrm{S}) = \frac{1}{1-\beta}\left[\sum_{\mathrm{x_i}=\mathrm{X}_{\phi}}\left[2^{(\beta-1).\sum_{\mathrm{i=1}}^{\mathrm{n}}\{\mu_{\mathrm{R}}^{\alpha}(\mathrm{x_i}).\log\mu_{\mathrm{R}}^{\alpha}(\mathrm{x_i})+(1-\mu_{\mathrm{R}}(\mathrm{x_i}))^{\alpha}.\log(1-\mu_{\mathrm{R}}(\mathrm{x_i}))^{\alpha}\}-1}\right]\right]$$

$$\Rightarrow \frac{1}{1-\beta}\left[\sum_{\mathrm{x_i}=\mathrm{X}_{\psi}}\left[2^{(\beta-1).\sum_{\mathrm{i=1}}^{\mathrm{n}}\{\mu_{\mathrm{S}}(\mathrm{x_i})^{\alpha}.\log\mu_{\mathrm{S}}^{\alpha}(\mathrm{x_i})+(1-\mu_{\mathrm{S}}(\mathrm{x_i}))^{\alpha}.\log(1-\mu_{\mathrm{S}}(\mathrm{x_i}))^{\alpha}\}-1}\right]\right] \tag{3.3.1}$$

and

$$\mathrm{H}_{\alpha}^{\beta}(\mathrm{R}\cup\mathrm{S}) = \frac{1}{1-\beta}\left[\sum_{\mathrm{x_i}=\mathrm{X}_{\phi}}\left[2^{(\beta-1).\sum_{\mathrm{i=1}}^{\mathrm{n}}\{\mu_{\mathrm{S}}^{\alpha}(\mathrm{x_i}).\log\mu_{\mathrm{S}}^{\alpha}(\mathrm{x_i})+(1-\mu_{\mathrm{S}}(\mathrm{x_i}))^{\alpha}.\log(1-\mu_{\mathrm{S}}(\mathrm{x_i}))^{\alpha}\}-1}\right]\right]$$

$$+\frac{1}{1-\beta}\left[\sum_{\mathrm{x_i}=\mathrm{X}_{\psi}}\left[2^{(\beta-1).\sum_{\mathrm{i=1}}^{\mathrm{n}}\{\mu_{\mathrm{R}}^{\alpha}(\mathrm{x_i}).\log\mu_{\mathrm{R}}^{\alpha}(\mathrm{x_i})+(1-\mu_{\mathrm{R}}(\mathrm{x_i}))^{\alpha}.\log(1-\mu_{\mathrm{R}}(\mathrm{x_i}))^{\alpha}\}-1}\right]\right] \tag{3.3.2}$$

Adding Equation (3.3.1) & Equation (3.3.2) we obtain

$$\mathrm{H}_{\alpha}^{\beta}(\mathrm{R}\cup\mathrm{S}) + \mathrm{H}_{\alpha}^{\beta}(\mathrm{R}\cap\mathrm{S})$$

$$= \frac{1}{1-\beta}\left[\sum_{\mathrm{x_i}=\mathrm{X}_{\phi}}\left[2^{(\beta-1).\sum_{\mathrm{i=1}}^{\mathrm{n}}\{\mu_{\mathrm{R}}^{\alpha}(\mathrm{x_i}).\log\mu_{\mathrm{R}}^{\alpha}(\mathrm{x_i})+(1-\mu_{\mathrm{R}}(\mathrm{x_i}))^{\alpha}.\log(1-\mu_{\mathrm{R}}(\mathrm{x_i}))^{\alpha}\}-1}\right]\right]$$

$$+\frac{1}{1-\beta}\left[\sum_{\mathrm{x_i}=\mathrm{X}_{\psi}}\left[2^{(\beta-1).\sum_{\mathrm{i=1}}^{\mathrm{n}}\{\mu_{\mathrm{S}}^{\alpha}(\mathrm{x_i}).\log\mu_{\mathrm{S}}^{\alpha}(\mathrm{x_i})+(1-\mu_{\mathrm{S}}(\mathrm{x_i}))^{\alpha}.\log(1-\mu_{\mathrm{S}}(\mathrm{x_i}))^{\alpha}\}-1}\right]\right]$$

$$+\frac{1}{1-\beta}\left[\sum_{\mathrm{x_i}=\mathrm{X}_{\phi}}\left[2^{(\beta-1).\sum_{\mathrm{i=1}}^{\mathrm{n}}\{\mu_{\mathrm{S}}^{\alpha}(\mathrm{x_i}).\log\mu_{\mathrm{S}}^{\alpha}(\mathrm{x_i})+(1-\mu_{\mathrm{S}}(\mathrm{x_i}))^{\alpha}.\log(1-\mu_{\mathrm{S}}(\mathrm{x_i}))^{\alpha}\}-1}\right]\right]$$

$$+\frac{1}{1-\beta}\sum_{\mathrm{x_i}=\mathrm{X}_{\psi}}\left[2^{(\beta-1).\sum_{\mathrm{i=1}}^{\mathrm{n}}\{\mu_{\mathrm{R}}^{\alpha}(\mathrm{x_i}).\log\mu_{\mathrm{R}}^{\alpha}(\mathrm{x_i})+(1-\mu_{\mathrm{R}}(\mathrm{x_i}))^{\alpha}.\log(1-\mu_{\mathrm{R}}(\mathrm{x_i}))^{\alpha}\}-1}\right]$$

$$\Rightarrow \mathrm{H}_{\alpha}^{\beta}(\mathrm{R}\cup\mathrm{S}) + \mathrm{M_e}(\mathrm{R}\cap\mathrm{S}) = \mathrm{H}_{\alpha}^{\beta}(\mathrm{R}) + \mathrm{H}_{\alpha}^{\beta}(\mathrm{S})$$

Hence first property is satisfied.

$$\text{(II)}\quad \mathrm{H}_{\alpha}^{\beta}(\mathrm{R}\cup\mathrm{S}) = \frac{1}{1-\beta}\sum_{\mathrm{x_i}=\mathrm{X}_{\phi}}\left[2^{(\beta-1).\sum_{\mathrm{i=1}}^{\mathrm{n}}\{\mu_{\mathrm{R}}^{\alpha}(\mathrm{x_i}).\log\mu_{\mathrm{R}}^{\alpha}(\mathrm{x_i})+(1-\mu_{\mathrm{R}}(\mathrm{x_i}))^{\alpha}.\log(1-\mu_{\mathrm{R}}(\mathrm{x_i}))^{\alpha}\}-1}\right]$$

$$+\frac{1}{1-\beta}\sum_{x_i=X_\psi}\left[2^{(\beta-1).\sum_{i=1}^{n}\{\mu_S^\alpha(x_i).\log\mu_S^\alpha(x_i)+(1-\mu_S(x_i))^\alpha.\log(1-\mu_S(x_i))^\alpha\}-1}\right]$$

$$H_\alpha^\beta((R\cup S)\cup T)=\frac{1}{1-\beta}\sum_{x_i=X_\phi}\left[2^{(\beta-1).\sum_{i=1}^{n}\{\mu_R^\alpha(x_i).\log\mu_R^\alpha(x_i)+(1-\mu_R(x_i))^\alpha.\log(1-\mu_R(x_i))^\alpha\}-1}\right]$$

$$+\frac{1}{1-\beta}\sum_{x_i=X_\psi}\left[2^{(\beta-1).\sum_{i=1}^{n}\{\mu_S^\alpha(x_i).\log\mu_S^\alpha(x_i)+(1-\mu_S(x_i))^\alpha.\log(1-\mu_S(x_i))^\alpha\}-1}\right] \tag{3.3.3}$$

$$H_\alpha^\beta(S\cup T)=\frac{1}{1-\beta}\sum_{x_i=X_\phi}\left[2^{(\beta-1).\sum_{i=1}^{n}\{\mu_S^\alpha(x_i).\log\mu_S^\alpha(x_i)+(1-\mu_S(x_i))^\alpha.\log(1-\mu_S(x_i))^\alpha\}-1}\right]$$

$$+\frac{1}{1-\beta}\sum_{x_i=X_\phi}\left[2^{(\beta-1).\sum_{i=1}^{n}\{\mu_T^\alpha(x_i).\log\mu_T^\alpha(x_i)+(1-\mu_T(x_i))^\alpha.\log(1-\mu_T(x_i))^\alpha\}-1}\right]$$

$$H_\alpha^\beta(R\cup(S\cup T))=\frac{1}{1-\beta}\sum_{x_i=X_\phi}\left[2^{(\beta-1).\sum_{i=1}^{n}\{\mu_R^\alpha(x_i).\log\mu_R^\alpha(x_i)+(1-\mu_R(x_i))^\alpha.\log(1-\mu_R(x_i))^\alpha\}-1}\right]$$

$$+\frac{1}{1-\beta}\sum_{x_i=X_\psi}\left[2^{(\beta-1).\sum_{i=1}^{n}\left\{\mu_{R^C}^\alpha(x_i).\log\mu_{R^C}^\alpha(x_i)+(1-\mu_{R^C}(x_i))^\alpha.\log(1-\mu_{R^C}(x_i))^\alpha\right\}-1}\right] \tag{3.3.4}$$

By equation Equation (3.3.3) & Equation (3.3.4) we can easily show that

$$H_\alpha^\beta((R\cup S)\cup T)=H_\alpha^\beta(R\cup(S\cup T))$$

(III) Using above result we can easily show that

$$H_\alpha^\beta((R\cap S)\cap T)=H_\alpha^\beta(R\cap(S\cap T))$$

(IV) We know that

$$H_\alpha^\beta(R\cup R^C)=\frac{1}{1-\beta}\sum_{x_i=X_\phi}\left[2^{(\beta-1).\sum_{i=1}^{n}\{\mu_R^\alpha(x_i).\log\mu_R^\alpha(x_i)+(1-\mu_R(x_i))^\alpha.\log(1-\mu_R(x_i))^\alpha\}-1}\right]$$

$$+\frac{1}{1-\beta}\sum_{x_i=X_\psi}\left[2^{(\beta-1).\sum_{i=1}^{n}\left\{\mu_{R^C}^\alpha(x_i).\log\mu_{R^C}^\alpha(x_i)+(1-\mu_{R^C}(x_i))^\alpha.\log(1-\mu_{R^C}(x_i))^\alpha\right\}-1}\right] \tag{3.3.5}$$

or

$$H_\alpha^\beta(R\cup R^C)=\frac{1}{1-\beta}\sum_{x_i=X_\phi}\left[2^{(\beta-1).\sum_{i=1}^{n}\left\{\mu_{R^C}^\alpha(x_i).\log\mu_{R^C}^\alpha(x_i)+(1-\mu_{R^C}(x_i))^\alpha.\log(1-\mu_{R^C}(x_i))^\alpha\right\}-1}\right]$$

$$+\frac{1}{1-\beta}\sum_{x_i=X_\psi}\left[2^{(\beta-1).\sum_{i=1}^{n}\left\{\mu_{R^C}^\alpha(x_i).\log\mu_{R^C}^\alpha(x_i)+(1-\mu_{R^C}(x_i))^\alpha.\log(1-\mu_{R^C}(x_i))^\alpha\right\}-1}\right] \tag{3.3.6}$$

After solving both equations Equation (3.3.5) & Equation (3.3.6) we get

$$H_\alpha^\beta(R)=H_\alpha^\beta(R\cup R^C)$$

Using above result we can prove that

$$H_\alpha^\beta(R)=H_\alpha^\beta(R\cap R^C)=H_\alpha^\beta(R\cup R^C)=H_\alpha^\beta(R^C)$$

4 Conclusion

This study has proposed the fuzzy information's parametric measure in Section 3 and examined its various properties. Also, its monotonicity, comparison with the corresponding measure of probabilistic information and some useful basic set properties have been discussed. The newly introduced generalized fuzzy information measure is useful in decision-making processes. In these scenarios, decision-makers need to evaluate alternatives based on multiple criteria or attributes, often characterized by fuzzy or uncertain information. When measuring the importance of fuzzy information, monotonic measurements are critical to the relevance of these criteria and guiding the decision-making process. The proposed measure can be used in soft computing for data aggregation, decision-making, and economics because it satisfies basic set properties. Also, by introducing the degree of hesitation, the proposed measure can be generalized as the measure of intuitionistic fuzzy information; this is better suited to solving issues in the actual world.

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