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Reliability Estimation of Scale Parameter in Type II Generalized Half-Logistic Distribution from Doubly Censored Samples Based on Monte Carlo Simulation

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Abstract

Objectives: This article deals with estimation of the reliability function for one-parameter Type II Generalized Half- Logistic distribution (Type II GHLD). **Methods:** The Maximum likelihood (ML) and Approximate Maximum likelihood (AML) methods provided by Jilani et.al.[1]. **Findings:** Comparisons between the estimators were made using Monte Carlo Simulation based on statistical indicter bias and mean squared error (MSE). We have generated 3000 random samples of size n=10(5)20 from a standard one-parameter Type II GHLD with θ = 0.5, 1.5 and 2.0. Each sample is then ordered and from this sample a doubly (including left and right) censored sample is considered, by censoring r smallest observations and s largest observations with all possible choices of r and s corresponding to this resultant sample we compute MLEs $\widehat{\sigma}$ (using Newton-Raphson method), the LAMLE $\widehat{\sigma}$ and unbiased LAMLE $\widetilde{\sigma}^*$. Finally, the results are explained with tables. **Novelty:** This study has empirically proves that all the three reliability estimators have the same asymptotic variance. Hence, in this case, any one of the three estimators can be used but preferable estimator based unbiased estimator.

Keywords: Maximum likelihood estimation; Approximate Maximum likelihood estimation; Bias; Mean squared error; Variances

1 Introduction

Reliability studies deal with life testing experiments designed to observe the life times of items that have life and eventual death. It is therefore, natural that the observations in a life test data originate in ascending order (the weakest item fails first, next weakest fails next and so on) thus giving rise to an ordered sample, either complete or censored. The aim of this research is to estimate the reliability function in one-parameter Type II Generalized Half- Logistic distribution (Type II GHLD), using Maximum likelihood (ML) and Approximate Maximum likelihood (AML) methods, where scale parameter is unknown and shape parameter is known.

Kantam et al.[2] proposed the probability density function (pdf), cumulative distribution function (cdf) of Type II generalized half logistic distribution (Type II GHLD) are respectively given by

$$f(x,\, \theta) = \frac{2^{\theta}\theta e^{x}}{(1+e^{x})^{\theta+1}}\ ;\ 0 \le x < \infty,\ \theta > 0 \tag{1.1}$$

$$F(x,\, \theta) = 1 - \frac{2^{\theta}}{(1+e^{x})^{\theta}}; \tag{1.2}$$

$$h(x,\, \theta) = 1 - \frac{\theta}{1+e^{-x}}; \tag{1.3}$$

where θ is the shape parameter. Here, it may be noted that the above standard Type II GHLD is same as the Type I GHLD proposed by Olapade[3].

The pdf and cdf of two-parameter Type II GHLD are respectively given by-

$$p(x;\theta,\sigma) = \frac{2^{\theta}\theta e^{\frac{x}{\sigma}}}{\sigma\left(1+e^{\frac{x}{\sigma}}\right)^{\theta+1}};\ 0 \le x < \infty,\ \theta > 0\ and\ \sigma > 0 \tag{1.4}$$

$$P(x;\theta,\sigma) = 1 - \frac{2^{\theta}}{\left(1+e^{\frac{x}{\sigma}}\right)^{\theta}} \tag{1.5}$$

Where σ is the scale parameter. Kantam et al.[2] estimated parameters using ML methods, while Rosaiah et al.[4] developed a reliability test plan for Type II GHLD. Kumar et al.[5] calculated moments and MLEs for a three-parameter Type II GHLD model. Bagheri et al.[6] analyzed the PDF and CDF in the Weibull extension model, assessing bias and mean squared error. Awodutire et al.[7] used a three-parameter Type II GHLD for breast cancer survival analysis, estimating parameters using ML.

In general, ML method is a viable estimation method for estimation of the parameters of a distribution. But, this method does not yield explicit estimator to σ of Type II GHLD even when θ is known. For this reason, assuming θ is known, Jilani et.al.[1] have constructed a linear estimator of σ called as linear approximate ML estimator (LAMLE) and based on a Monte Carlo simulation study, they have empirically shown that LAMLE is almost as efficient as MLE, even though it is biased than MLE.

Hassan et al.[8] developed a stress–strength modeling using median-ranked set sampling: Estimation, simulation and application. Malkewitz et al.[9] proposed the estimating reliability: A comparison of Cronbach's α, McDonald's ωt and the greatest lower bound. Philipp et al.[10] proposed the Beta-Binomial Meta-Analysis of Individual Differences Based on Sample Means and Standard Deviations: Studying Reliability of Sum Scores of Binary Items. Kumari et al.[11] developed a Reliability estimation for Kumaraswamy distribution under block progressive type-II censoring. Pawe[12] proposed the A new method for multi-state flow networks reliability estimation based on a Monte Carlo simulation and intersections of sets.

2 Methodology

An attempt has been made to use the estimates of one-parameter Type II GHLD, namely MLEs, and AMLEs to compute the reliability estimates from doubly censored samples (including left and right censored samples).

We have provided the ML estimation, AML estimation of σ based on a Type-II doubly censored sample respectively. Provide the necessary theory for reliability estimation of Scale parameter in Type II GHLD. We present the Monte-Carlo simulation study for comparing the performance of different reliability estimators based on the empirical sampling characteristics bias, mean square error and variances in scale parameter Type II GHLD and conclusion of the study is presented.

2.1. Maximum Likelihood Estimation

We consider a life time experiment, where n components are put to test simultaneously at time x=0. Then failure times of these components are recorded. If some initial observations are censored (possibly due to failure during the time when checks and adjustments are being made on the devices) and some final observations are also censored (possibly due to termination of the experiment, in order to save time and cost, before all components have failed) then, the resultant sample is called as doubly type-II censored sample. Let $X_{r+1:n} \leq X_{r+2:n} \leq \ldots\ldots \leq X_{n-s:n}$ be the available doubly censored sample from the Type II GHLD given in Equation (1.4) where the first r and last s observations are censored without watching the state of any component.

The ML equation of the scale parameter σ (assuming shape θ is known) is given by

$$\frac{\partial logL}{\partial \sigma} = \frac{-1}{\sigma}\left[n-r-s+r\theta\, G\left(z_{r+1:n}\right)+\sum_{i=r+1}^{n-s} z_{i:n}-(\theta+1)\sum_{i=r+1}^{n-s} H(z_{i:n})-s\theta H(z_{n-s:n})\right] \tag{2.1}$$

where

$$\begin{aligned} &G\left(z_{r+1:n}\right)=H(z_{r+1:n})\left[1-F\left(z_{r+1:n}\right)\right]/F\left(z_{r+1:n}\right); \\ &H(z_{i:n})=(z_{i:n})/(1+e^{-z_{i:n}}) \\ &z_{i:n=}\frac{X_{i:n}}{\sigma} \quad for\ r+1 \leq i \leq n-s \end{aligned} \tag{2.2}$$

The ML Equation (2.1) does not yield explicit solution for σ because of the presence of the intractable terms $G\left(z_{r+1:n}\right)$ and $H\left(z_{i:n}\right)$. Hence, we have to employ some iterative method, such as Newton- Raphson method, to obtain the MLE of σ which requires a starting solution near the global maximum.

2.2. Approximate Maximum Likelihood Estimation

Even when θ is known, the ML Equation (2.1) does not yield explicit solution for σ because of the intractable terms $G\left(z_{r+1:n}\right)$ and $H\left(z_{i:n}\right) r+1 \leq i \leq n-s$. Therefore, with an aim of deriving an explicit estimator of σ, Jilani et al.[1] proposed the following linear approximations to the intractable terms

$$G\left(z_{r+1:n}\right) \cong \alpha_{r+1}+\beta_{r+1} z_{r+1:n}\ and\ H\left(z_{i:n}\right) \cong \gamma_i+\delta_i z_{i:n},\ \ i=1,\ldots,n-s \tag{3.1}$$

Upon using the above linear approximations along with Equation (2.1) yields a linear estimator of$\sigma\tilde{\sigma}$ given by

$$\tilde{\sigma}=\sum_{i=r+1}^{n-s} l_i x_{i:n} \tag{3.2}$$

$$l_i=\begin{cases} \dfrac{-r\theta\beta_i+(\theta+1)\,\delta_i-1}{\triangle} & if\ i=r+1 \\ \dfrac{(\theta+1)\,\delta_i-1}{\triangle} & if\ r+1<i<n-s \\ \dfrac{(s\theta+\theta+1)\,\delta_i-1}{\triangle} & if\ i=n-s \end{cases} \tag{3.3}$$

where

$\triangle=n-r-s+r\theta\alpha_{r+1}-(\theta+1)\sum_{i=r+1}^{n-s}\gamma_i-s\theta\gamma_{n-s}$

We call the above estimator $\tilde{\sigma}^*$as linear approximate ML estimator (LAMLE) of σ.The mean and variance of the LAMLE $\tilde{\sigma}^*$ are respectively given by

$$E\left(\tilde{\sigma}^*\right)=\sigma\sum_{i=1}^{n-s} l_i a_{i:n}\ and\ V\left(\tilde{\sigma}^*\right)=\sigma^2\sum_{i=1}^{n-s}\sum_{j=1}^{n-s} l_i l_j b_{ij:n} \tag{3.4}$$

where $a_{i:n}=E\left(Z_{i:n}\right)\ and\ b_{ij:n}=Cov\left(Z_{i:n},Z_{j:n}\right)$.

By applying least squares method (for details of method please see Vasudeva Rao et al.[13]) to Equation (3.1), we get

$$\beta_{r+1}=\left[\sum_{j=1}^{2m+1} z_{r+1,j} G\left(z_{r+1,j}\right)-\tfrac{1}{2m+1}\sum_{j=1}^{2m+1} z_{r+1,j}\sum_{j=1}^{2m+1} G\left(z_{r+1,j}\right)\right]\ /\left[\sum_{j=1}^{2m+1} z_{r+1}^2-\tfrac{1}{2m+1}\left(\sum_{j=1}^{2m+1} z_{r+1,j}\right)^2\right]$$

$$and\ \alpha_{r+1=\frac{1}{2m+1}} \sum_{j=1}^{2m+1} G\left(z_{r+1,j}\right) - \beta_{r+1\,\frac{1}{2m+1}} \sum_{j=1}^{2m+1} z_{r+1,j}$$

$$\delta_i = \frac{\left[\sum_{j=1}^{2m+1} z_{ij} H\left(z_{ij}\right) - \frac{1}{2m+1} \sum_{j=1}^{2m+1} z_{ij} \sum_{j=1}^{2m+1} H\left(z_{ij}\right)\right]}{\left[\sum_{j=1}^{2m+1} z_{ij}^2 - \frac{1}{2m+1} \left(\sum_{j=1}^{2m+1} z_{ij}\right)^2\right]}, \tag{3.5}$$

$and\ \ \gamma_i = \left[\frac{1}{2m+1} \sum_{j=1}^{2m+1} H\left(z_{ij}\right)\right] - \delta_i \frac{1}{2m+1} \sum_{j=1}^{2m+1} z_{ij}, \quad for\, i = r+1, \ldots\ldots, n-s$ where

$z_{ij} = F^{-1}\left(p_{ij}\right) = log_e \left[2\left(1-p_{ij}\right)^{\frac{-1}{\theta}} - 1\right], p_{ij} = p_i + \frac{j-m-1}{2m(hn+0.4)}, j = 1, 2, \ldots\ldots, 2m+1$

From Equation (3.4), we may write

$$E\left(\tilde{\sigma}^*\right) = \sigma,\ where\ \ \tilde{\sigma}^* = \frac{\tilde{\sigma}}{\sum_{i=r+1}^{n-s} l_i a_{i:n}} \tag{3.6}$$

So that $\tilde{\sigma}^*$ is a linear unbiased estimator. Since we are using simulated $a'_{i:n}s$ values in place of actual $a'_{i:n}s$ values, we call $\tilde{\sigma}^*$ as unbiased LAMLE. The variance of σ^* is given by:

$$V\left(\sigma^*\right) = \left[\sum_{i=r+1}^{n-s} l_i a_{i:n}\right]^{-2} V\left(\tilde{\sigma}^*\right) \tag{3.7}$$

where $V\left(\tilde{\sigma}^*\right)$ can be obtained from Equation (3.4) .

2.3. Reliability Estimation from Doubly Censored Samples in one-parameter Type II GHLD

The reliability function of one-parameter Type II GHLD is given by

$$R(x) = 1 - P(x) = \left(\frac{2}{1+e^{\frac{x}{\sigma}}}\right)^{\theta} \tag{4.1}$$

Since namely based on on least square method and unbiased LAMLEs constructed based on a doubly censored sample in the above equation to estimate. Now the reliability estimators based on the estimators and are respectively given by

$$\widehat{R}(x) = \left(\frac{2}{1+e^{\frac{x}{\widehat{\sigma}}}}\right)^{\theta};\ \widetilde{R}(x) = \left(\frac{2}{1+e^{\frac{x}{\widetilde{\sigma}}}}\right)^{\theta};\ \widetilde{R}^*(x) = \left(\frac{2}{1+e^{\frac{x}{\widetilde{\sigma}^*}}}\right)^{\theta} \tag{4.2}$$

The analytical variances of the three reliability estimators $\widehat{R}(x)$, $\widetilde{R}(x)$ $and\ \widetilde{R}^*(x)$ cannot be mathematically tractable. Hence, we compare the relative performance of these three estimators in small samples through Monte Carlo simulation based on the empirical sampling characteristics bias, Mean square error and Variances. The results of the simulation are presented in the following section.

3 Results and discussion

3.1. Comparison of reliability estimators in small samples

We have generated 3000 random samples of size n=10(5)20 from a standard one-parameter Type II GHLD with θ = 0.5, 1.5 and 2.0. After ordering and doubly censoring each sample, we computed MLE (using the Newton-Raphson method), LAMLE, and unbiased LAMLE for all possible combinations of r and s. These estimates were used to calculate reliability functions. We determined reliabilities by substituting values of x where the true population reliability R(x) = 0.5, 0.7 and 0.9 for each θ.

For each θ and for each choice of x, we simulated reliability estimates for each method using 3000 doubly censored samples. We computed bias and mean square error for the three reliability estimators. The simulated biases and mean square errors for R(x) = 0.5, 0.7 and 0.9 for sample size n=10(5)20, for θ = 0.5, 1.5 and 2.0 and for some choices of r and s are tabulated in Table 1 and **Supplementary Table 2**.

Table 1. Table of x values

θ	R(x)=0.5	R(x)=0.7	R(x)=0.9
0.5	1.9459	1.1255	0.3847
1.5	0.7769	0.4297	0.1359
2.0	0.6035	0.3296	0.1027
2.5	0.4941	0.2675	0.0826
3.0	0.4186	0.2251	0.0690

4 Conclusions

Table 1, **Supplementary Table 2** and **Supplementary Table 3** reveal the following observations and conclusions:

- The simulated sampling characteristics bias of the reliability estimators are generally increasing with respect to sample size in small samples and the number of observations censored and decreasing with respect to sample size in moderate sample. Further, they are not influenced by the true value of the reliability R.
- The simulated sampling characteristics mean square error of the reliability estimators are generally decreasing with respect to sample size and increasing with respect to the number of observations censored. Further, they are reducing with respect to the true value of the reliability R.
- With respect to bias, $\widehat{R}(x)$performing better than (i.e. less biased than) both $\widehat{R}(x)$ and $\widetilde{R}(x)$. Here it may be noted, the biasness carries forward to the reliability estimators also - perhaps an invariance property empirically proved.
- With respect to mean square error, all reliability estimators are almost equal mean square errors for all values of n and whatever it may be the case of censoring, the order of the performance of the reliability estimators is given by $\widetilde{R}(x) \geq \widehat{R}(x) \geq \widetilde{R}*$,which empirically proves that all the three reliability estimators are have the same asymptotic variance.

References

1) Jilani SD, Rao AV, Prakash BS. Linear approximate ml estimation of scale parameter in type II generalized half-logistic distribution under Type-II censoring. *International Journal of Statistics and Applied Mathematics*. 2018;3(2):446–455. Available from: https://www.mathsjournal.com/archives/2018/vol3/issue2/PartF/3-1-74.
2) Kantam RRL, Ramakrishna V, Ravikumar M. Estimation and testing in Type -II generalized half logistic distribution. *Modern applied statistical methods*. 2014;12(1):198–206. Available from: http://dx.doi.org/10.22237/jmasm/1367382060.
3) Olapade AK. The type I generalized half logistic distribution. *Journal of the Iranian Statistical Society*. 2014;13(1):69–82. Available from: https://doi.org/10.1155/2015/393608.
4) Rosaiah K, Kantam RRL, Krishna R, V. Type- II Generalized Half Logistic Distribution An Economic Reliability Test Plan. *Journal of Chemical, Biological and Physical Sciences*. 2014;4(1):501–508. Available from: https://www.researchgate.net/publication/341431955.
5) Kumar D, Jain N, Gupta S. The Type I Generalized Half-Logistic Distribution Based on Upper Record Values. *Journal of Probability and Statistics*. 2015;p. 1–11. Available from: https://doi.org/10.1155/2015/393608.
6) Bagheri S, Alizadeh M, Nadarajah S, Deiri E. Efficient Estimation of the PDF and the CDF of the Weibull extension model. . *Communications in Statistics - Simulation and Computations*. 2016;45(06):2191–2207. Available from: https://doi.org/10.1080/03610918.2014.894059.
7) Awodutire PO, Olapade AK, Kolawole OA. The Type I Generalized Half Logistic Survival Model. *International Journal of Theoretical and Applied Mathematics*. 2016;2(2):74–78. Available from: https://doi.org/10.11648/j.ijtam.20160202.17.
8) Hassan A, Ibrahim M, Al-Omari A, Loai A, Heba F. stress-strength modeling using median-ranked set sampling: Estimation. Simulation and application. *Mathematic*. 2023;11:1–22. Available from: .https://doi.org/10.3390/math11020318.
9) Malkewitz CP, Schwall P, Meesters C, Hardt J. Estimating reliability: A comparison of Cronbach's α, McDonald's ωt and the greatest lower bound. *Social Sciences & Humanities*. 2023;7(1):1–10. Available from: https://doi.org/10.1016/j.ssaho.2022.100368.
10) Philipp D, Susanne F, D A. Beta-Binomial Meta-Analysis of Individual Differences Based on Sample Means and Standard Deviations: Studying Reliability of Sum Scores of Binary Items. *Psychological Methods*. 2024;14:1–10. Available from: https://doi.org/10.1037/met0000649.
11) Kumari R, Tripathi YM, L W, Sinha RK. Reliability estimation for Kumaraswamy distribution under block progressive type-II censoring. *Statistics*. 2024;58(1):142–175. Available from: https://doi.org/10.1080/02331888.2024.2301736.
12) Paweł MK. A new method for multi-state flow networks reliability estimation based on a Monte Carlo simulation and intersections of sets. *Simulation Modelling Practice and Theory* . 2024;130:1–12. Available from: https://doi.org/10.1016/j.simpat.2023.102846.
13) Rao VA, Prakash BS, Jilani SD. Best Linear Unbiased Estimation of Location and Scale Parameters in Generalized Exponential Distribution under Type-II Censoring. *International Journal of Scientific and Research Publications*. 2017;7(8):38–53. Available from: https://doi.org/ijsrp-p6807.pdf.