INDIAN JOURNAL OF SCIENCE AND TECHNOLOGY

RESEARCH ARTICLE

A Study of Multiple Comparison Tests with Imprecise and Uncertain Information

Nabanita Kakati[1]*, **Bipin Gogoi**[2]

1 Research Scholar, Department of Statistics, Dibrugarh University, Dibrugarh, 786004, Assam, India
2 Rtd. Professor, Department of Statistics, Dibrugarh University, Dibrugarh, Assam, India

OPEN ACCESS

Received: 08-05-2024
Accepted: 25-08-2024
Published: 05-09-2024

Citation: Kakati N, Gogoi B (2024) A Study of Multiple Comparison Tests with Imprecise and Uncertain Information. Indian Journal of Science and Technology 17(35): 3669-3677. https://doi.org/10.17485/IJST/v17i35.1547

* **Corresponding author**.

nabanitakakati3@gmail.com

Funding: None

Competing Interests: None

Published By Indian Society for Education and Environment (iSee)

ISSN
Print: 0974-6846
Electronic: 0974-5645

Abstract

Objectives: This study aims to demonstrate the application of multiple comparison tests in situations involving imprecise, incomplete, and uncertain observations. Such data arises in many practical situations. **Methods:** Five post hoc multiple comparison tests viz. Fisher's LSD test, Tukey's HSD, Bonferroni (BT_N), Scheffe, and Hochberg (HB_N) method is utilized within a one-way design framework, with a significance level of 5%. Simulation studies are applied to the empirical level and power calculation. **Findings:** The study shows that Tukey's HSD (TK_N) shows the highest power in the determinant part, not the indeterminant part, compared to other tests. For, e.g., For (5, 5, 5, 5) with location parameter (0, .5, 1, 0), the power interval shows [.3953, .3584] where $I_N \in [0,\ .05]$. Overall, BT_N and HB_N shows a prominent result and are preferable compared to other tests. From application, NANOVA provided the P-value is [<.001, <.001], i.e., tests are statistically significant. The P-value of the tests for pairs [1, 1] and [1, 1] accepted (greater than .05) means the average daily ICU occupancy of corona patients of more than 55 years and between 35 and 55 age groups are not equal. Similarly, other tests also show the same. **Novelty:** Neutrosophic Multiple comparisons are still developing, and computer programs are still unavailable to support them. We have applied a simulation study to check the performance of the tests.

Keywords: Multiple comparisons; NANOVA; Neutrosophic Statistics; Indeterminacy; Covid-19

1 Introduction

Multiple comparison analyses help compare pairs of experimental and control groups to get ANOVA results when ANOVA cannot provide detailed information on differences among groups or their combinations in classical statistics alone. Multiple comparison analysis is widely employed in many fields, including medical research, agriculture, engineering market research, etc. Tukey's HSD, LSD, Bonferroni, Scheffe, Dunnet's, Sidak-Holm, etc., are some commonly employed post hoc tests for multiple comparisons using ANOVA available in the literature. However, the existing ANOVA tool can be used if selected sample data are uncertain [(1)]. But the practical world

sometimes exhibits entirely different situations. The real-world scenarios often include incomplete and non-precise data, making the current hypothesis testing procedures based on traditional test statistics impractical. Neutrosophic statistics is a generalization of classical statistics that considers the indeterminate, unclear, incomplete, and contradictory data in statistics [2,3]. It helps the experiment to control the situations where the data is not precisely known. The Neutrosophic statistics can be expressed as N = d + I, where d represents the determinant part and I represents the indeterminant part. For example, if a statistician is unsure about an experiment's sample size, which can be a = [15, 15.5] where neutrosophic observation a = 15 + I where I ∈ [0, 0.5]. In this case, data is recorded in the indeterminacy interval and can be analyzed using neutrosophic statistics, a criterion of classical statistics. The classical and Neutrosophic statistics coincide when indeterminacy is absent [4–6]. It is well known in classical statistics that binary true-or-false logic is handled, in which the ambiguous part is constantly disregarded. Still, neutrosophic statistics provide a framework for evaluating data that includes true, false, and ambiguous elements, promoting a more thorough inquiry than traditional binary logic. Traditional statistics often use probability distribution to model uncertainty, and decision-making is based on precise probabilities and confidence intervals [7]. However, neutrosophic insights can improve decision-making by consolidating indeterminacy, particularly in fields where decisions often need to be made under instability, such as the social sciences, design, and medical sciences. When comparing multiple groups or treatments, it is critical to consider data uncertainty and indeterminacy. In medical, engineering, and social sciences, neutrosophic multiple comparison techniques provide a more nuanced understanding that helps decision-makers make more informed choices [8].

Several authors have worked in different fields of statistics using neutrosophic statistics. [9]. The concept of neutrosophic statistics (NS) was first presented by Smarandache (1995) for neutrosophic probability, which consists of truth probability, indeterminacy probability, and falsity probability [10]. Muhammad Aslam introduced the neutrosophic analysis of variance (NANOVA) for uncertain and indeterminant observations in 2019. Muhammad Aslam and Mohammed Albassam first applied multiple comparisons in NS using real data collected from university students [11]. However, computational implications for checking the performance of the tests are still not available in the literature review. In this paper, we have applied multiple comparison tests viz. Fisher's LSD, Scheffe, Tukey's HSD, Bonferroni, and Horchberg test under the NS. Simulation work is applied to check the empirical level and power of the tests. Also, the study comprises multiple comparison analyses using NANOVA in a health-related application with a Covid-19 dataset [12].

2 Test Procedures

Let $X_N \in [X_L, X_U]$ be a neutrosophic random variable (nrv) follows the neutrosophic normal distribution with mean, say $\mu_N \in [\mu_L, \mu_U]$ and neutrosophic standard deviation $\sigma_N \in [\sigma_L, \sigma_U]$ respectively. Where X_L and X_U are the lower and upper values of the indeterminacy interval. Suppose the neutrosophic form of nrv can be written in the form $X_N = X_L + X_U I_N$, where X_L be the determinant part and $X_U I_N$; $I_N \in [I_L, I_U]$ is the indeterminant part with an indeterminacy interval $I_N \in [I_L, I_U]$. Suppose $n_N \in [n_L, n_U]$ be the neutrosophic random sample taken from the N_N, the population size. Therefore, the neutrosophic sample mean and standard deviation can be written as

$$\bar{X}_N \in \left[\frac{\sum_{i=1}^{n_L} \bar{X}_L}{n_L}, \frac{\sum_{i=1}^{n_U} \bar{X}_U}{n_U}\right]; \bar{X}_N \in \left[\bar{X}_L, \bar{X}_U\right] \text{ and} \tag{1}$$

$$S_N \in \left[\sqrt{\frac{\sum_{i=1}^{n_L} (X_L - \bar{X}_L)}{n_L - 1}}, \sqrt{\frac{\sum_{i=1}^{n_U} (X_U - \bar{X}_U)}{n_U - 1}}\right], S_N \in [S_L, S_U] \tag{2}$$

Also, the population mean and standard deviation can be written as

$$\mu_N \in \left[\frac{\sum_{i=1}^{N_L} X_L}{N_L}, \frac{\sum_{i=1}^{N_U} X_U}{N_U}\right]; \overline{X}_N \in \left[\overline{X_L}, \overline{X_u}\right] \text{ and} \tag{3}$$

$$\sigma_N \in \left[\sqrt{\frac{\sum_{i=1}^{N_L} (X_L - \mu_L)}{N_L - 1}}, \sqrt{\frac{\sum_{i=1}^{N_U} (X_L - \mu_U)}{N_U - 1}}\right], \sigma_N \in [\sigma_L, \sigma_U] \tag{4}$$

2.1. Neutrosophic Fisher's LSD test

When observations contain imprecise and indeterminant information, the traditional Fisher's LSD test is used as a Neutrosophic LSD (NLSD) test to determine the significance of the difference for pairwise comparison. The Neutrosophic LSD test for the significance of the difference in means is

$$LSD_{(iN-jN)N} = \frac{\hat{\mu}_{iN} - \hat{\mu}_{jN}}{S_{N_{(\hat{\mu}_{iN}-\hat{\mu}_{jN})}}} \tag{5}$$

Where, $LSD_{(iN-jN)N} \in [LSD_{(iL-jL)L}, LSD_{(iU-jU)U}]$ follows the neutrosophic t-distribution. $\hat{\mu}_{iN}$ and $\hat{\mu}_{jN}$ are the ith and jth groups of neutrosophic means, respectively, and $S_{N_{(\hat{\mu}_{iN}-\hat{\mu}_{jN})}}$ be the neutrosophic standard error between the ith and jth groups.

The neutrosophic Fisher's LSD test can be written in a neutrosophic form, is

$$LSD_{(iN-jN)N} = A_N + BI_N; I_N \in [I_L, I_U] \tag{6}$$

Where A_N and BI_N are determinant and indeterminant part of the NLSD test. If I_N = 0, then the NLSD test should be considered a classical LSD test.

2.2. Neutrosophic Scheffe test (NST)

Scheffe is particularly useful for an exploratory study because it examines all possible comparisons. Based on the F-distribution, the Scheffe test enables researchers to determine which groups or combinations of groups are responsible for the significant differences that ANOVA first found. The Scheffe test (ST) is generally calculated with the help of LSD values. The Neutrosophic Scheffe test is

$$ST_N = \frac{\left(LSD_{(iN-jN)N}\right)^2}{j_N - 1} \; where, \; NST_N \in [NST_L, NST_u] \tag{7}$$

Here, $LSD_{(iN-jN)N} \in [LSD_{(iL-jL)L}, LSD_{(iU-jU)U}]$ and ST_N follows the neutrosophic F – distribution.

The neutrosophic form of the NST test is

$$ST_N = C_N + DI_N; I_N \in [I_L, I_U] \tag{8}$$

Where C_N and DI_N are determinant and indeterminant part of the NST test. I_N = 0, then the NST test reduces to the classical ST test.

2.3. Neutrosophic Bonferroni test

The neutrosophic Bonferroni test (NBT) extends the classical Bonferroni test for imprecise and indeterminacy datasets. For the test, the significance level is obtained by multiplication of $LSD_{(iN-jN)N}$ by the neutrosophic number of tests performed in the population. The neutrosophic Bonferroni test is

$$BT_N = \frac{\hat{\mu}_{iN} - \hat{\mu}_{jN}}{[S^2(\frac{1}{n_1} + \frac{1}{n_2})]^{1/2}}; \; \hat{\mu}_{iN} - \hat{\mu}_{jN} \in \left(\hat{\mu}_{iL} - \hat{\mu}_{jL}, \; \hat{\mu}_{iU} - \hat{\mu}_{jU}\right) \tag{9}$$

Where $BT_N \in [BT_L, BT_U]$ and follows t – distribution. Bailey's (1977) Bonferroni t-tables have been considered for table value.

The neutrosophic form of the test is defined as

$$BT_N = E_N + FI_N; I_N \in [I_L, I_U] \tag{10}$$

Where E_N and FI_N are determinant and indeterminant parts of the neutrosophic Bonferroni test. The test reduces to the classical Bonferroni test if I_N = 0.

2.4. NeutrosophicTukey's HSD Test

The neutrosophic Tukey test statistic is

$$TK_N = \mathrm{p2}LSD_{(iN-jN)N}; LSD_{(iN-jN)N} \in \left[LSD_{(iL-jL)L}, LSD_{(iU-jU)U}\right]; \\ TK_N \in [TK_L, TK_U] \tag{11}$$

Work for conducting multiple comparisons Tukey's statistics under neutrosophic obtained from Studentized range distribution. The neutrosophic form of the test is defined as

$TK_N = G_N + \mathrm{H}I_N; I_N \in [I_L, I_U]$

Where G_N and $\mathrm{H}I_N$ are determinant and indeterminant part of the NST test. If I_N = 0, then the NST test reduces to the classical Tukey test.

2.5. Neutrosophic Horchberg GT2 Test

The neutrosophic horchberg test statistic is

$$HB_N = \frac{\hat{\mu}_{iN} - \hat{\mu}_{jN}}{s[\frac{s_i^2}{n_i} + \frac{s_i^2}{n_j}]^{1/2}}; \ \hat{\mu}_{iN} - \hat{\mu}_{jN} \in \left(\hat{\mu}_{iL} - \hat{\mu}_{jL}, \hat{\mu}_{iU} - \hat{\mu}_{jU}\right) \tag{12}$$

$where, HB_N \in [HB_L, HB_U]$. The neutrosophic Horchberg test statistic follows the Studentized Maximum Modulus distribution. Now, the neutrosophic form of the Horchberg test is

$$HB_N = P + RI_N; I_N \in [I_L, I_U] \tag{13}$$

If indeterminant part I_N is zero then HB_N comes down to the original classical test only.

3 Simulation Results

A simulation study was conducted to compare the tests' empirical level and power and to know the statistical significance of the experimental design. In this study, 10,000 normal random samples are generated for T = 4 treatments where even observations are considered imprecise at a 5% significance level. The indeterminacy $I_N \in [0, .05]$ is taken for the simulation work. The simulation work summary is tabulated for equal and unequal sample sizes, variances, and for different location parameters. Some of the essential steps in designing the simulation process are

(i) The equal number of observations are (5, 5, 5, 5) and (10, 10, 10, 10).

(ii) The unequal number of observations are (5, 10, 10, 5), (5, 10, 15, 20), and (14, 12, 8, 6).

(iii) The location parameters are (0, 0, .5, 0), (0, 0, 1, 0), (0, .5, 1, 0), and (.5, 0, 1.5, 0).

(iv) The scale parameters are (1, 1, 1, 1), (.5, 1, 2, 2), (.5, 1.5, 3, 2), (1, 1, 2, 3) and (1, 3, 4,6).

The summary of Empirical Levels of selected multiple comparison tests for neutrosophic statistics under equal and unequal sample sizes and variances at 0.05 level tabulated in Table 1.

Table 1. Empirical Levels of some selected multiple comparison tests for neutrosophic statistics under equal and unequal sample sizes and variances at 0.05 level

Sample sizes n_i	Value of SD (σ_i)	LSD_N	ST_N	BT_N	TK_N	HB_N
	1 1 1 1	[.0334, .0318]	[.0426, .0495]	[.0514, .0526]	[.0490, .0540]	[.0564, .0564]
	.5 1 2 2	[.0617, .0609]	[.0762, .0789]	[.0918, .0887]	[.0710, .0800]	[.0985, .0963]
5 5 5 5	.5 1.5 3 2	[.0739, .0724]	[.0864, .0927]	[.0870, .0950]	[.1284, .1307]	[.1073, .1129]
	1 1 2 3	[.0734, .0736]	[.0889, .0927]	[.1067, .1030]	[.1010, .1010]	[.1145, .1103]
	1 3 4 6	[.0729, .0731]	[.0873, .0875]	[.1042, 1037]	[.0840, .0890]	[.1109, .1101]
	1 1 1 1	[.0391, .0390]	[.0377, .0491]	[.0788, .0806]	[.0670, .0681]	[.0531, .0540]
	.5 1 2 2	[.0668, .0690]	[.0657, .0726]	[.1175, .1170]	[.1044, .1046]	[.0868, .0886]
5 10 10 5	.5 1.5 3 2	[.0393, .0390]	[.0386, .0396]	[.0700, .0750]	[.0597, .0616]	[.0514, .0495]
	1 1 2 3	[.1524, .1457]	[.1504, .1515]	[.2272, .2196]	[.2071, .2015]	[.1819, .1762]

Continued on next page

Table 1 continued

	1 3 4 6	[.1225, .1246]	[.1208, .1245]	[.1901, .1902]	[.1714, .1735]	[.1505, .1515]
	1 1 1 1	[.0429, .0419]	[.0335, .0423]	[.0504, .0493]	[.0677, .0611]	[.0531, .0509]
	.5 1 2 2	[.0254, .0236]	[.0214, .0223]	[.0287, .0273]	[.0342, .0340]	[.0293, .0282]
5 10 15 20	.5 1.5 3 2	[.0395, .0404]	[.0341, .0373]	[.0420, .0450]	[.0539, .0536]	[.0466, .0477]
	1 1 2 3	[.0168, .0177]	[.0142, .0156]	[.0196, .0203]	[.0247, .0248]	[.0205, .0207]
	1 3 4 6	[.0179, .0186]	[.0148, .0147]	[.0218, .0217]	[.0269, .0272]	[.0229, .0225]
	1 1 1 1	[.0372, .0368]	[.0333, .0412]	[.0997, .1002]	[.0584, .0599]	[.0492, .0465]
	.5 1 2 2	[.0644, .0655]	[.0601, .0659]	[.0834, .0846]	[.0985, .0993]	[.0815, .0827]
10 10 10 10	.5 1.5 3 2	[.0754, .0751]	[.0697, .0743]	[.1210, .1150]	[1091, .1116]	[.0949, .0949]
	1 1 2 3	[.0806, .0801]	[.0751, .0798]	[.0997, .1002]	[.1120, .1125]	[.0988, .0989]
	1 3 4 6	[.0759, .0750]	[.0702, .0705]	[.0954, .0946]	[.1085, .1075]	[.0946, .0935]
	1 1 1 1	[.0392, .0391]	[.0293, .0376]	[.0510, .0580]	[.0627, .0626]	[.0439, .0441]
	.5 1 2 2	[.2032, .1954]	[.0436, .0482]	[.2350, .2520]	[.2742, .2627]	[.0618, .0623]
14 12 8 6	.5 1.5 3 2	[.1772, .1765]	[.0680, .0728]	[.1970, .2220]	[.2328, .2344]	[.0918, .0935]
	1 1 2 3	[.2545, .2470]	[.0395, .0419]	[.2880, .2910]	[.3171, .3310]	[.2964, .2856]
	1 3 4 6	[.2418, .2408]	[.2286, .2309]	[.2700, .2800]	[.3146, .3164]	[.2833, .2828]

Also, the summary of Empirical powers for considered tests for neutrosophic statistics under equal and unequal sample sizes and variances with location parameters at 0.05 level included in Table 2.

Table 2. Empirical power of the multiple comparison tests for neutrosophic statistics under equal and unequal sample sizes, variances, and means at 0.05 level

Sample sizes n_i	**Value of SD** (σ_i)	**Value of means** (μ_i)	LSD_N	ST_N	BT_N	TK_N	HB_N
5 5 5 5	1 1 1 1	0 0 .5 0	[.0730, .0667]	[.0919, .1033]	[.1120, .1360]	[.1440, .1330]	[.1124, .1194]
		0 0 1 0	[.2694, .2325]	[.3198, .3310]	[.3570, .3830]	[.4471, .3978]	[.3467, .3929]
		0 .5 1 0	[.2309, .2017]	[.2779, .2894]	[.3180, .3990]	[.3953, .3584]	[.3067, .3446]
		.5 0 1.5 0	[.6121, .5538]	[.7003, .7152]	[.7650, .7860]	[.9004, .8349]	[.8190, .7564]
5 5 5 5	.5 1 2 2	0 0 .5 0	[.0910, .0873]	[.1121, .1112]	[.1170, .1260]	[.1566, .1498]	[.1295, .1358]
		0 0 1 0	[.1781, .1613]	[.2076, .2059]	[.2290, .2350]	[.2819, .2627]	[.2338, .2512]
		0 .5 1 0	[.1595, .1492]	[.1871, .1853]	[.1960, .1950]	[.2627, .2819]	[.2110, .2264]
		.5 0 1.5 0	[.3021, .2980]	[.3471, .3460]	[.3710, .3730]	[.4520, .4455]	[.4014, .4110]
5 10 15 20	.5 1.5 3 2	0 0 .5 0	[.0683, .0670]	[.0598, .0605]	[.0810, .0850]	[.0917, .0901]	[.0770, .0786]
		0 0 1 0	[.1802, .1767]	[.1601, .1604]	[.1950, .2010]	[.2351, .2251]	[.1982, .2044]
		0 .5 1 0	[.1486, .1443]	[.1321, .1320]	[.1680, .1670]	[.1910, .1828]	[.1612, .1679]
		.5 0 1.5 0	[.4059, .4029]	[.3680, .3741]	[.4280, .4320]	[.4863, .4858]	[.4438, .4469]
5 10 15 20	1 1 2 3	0 0 .5 0	[.0342, .0362]	[.0559, .0596]	[.0490, .0540]	[.3469, .3416]	[.0425, .0427]
		0 0 1 0	[.1089, .1077]	[.2139, .2139]	[.1310, .1340]	[.1396, .1404]	[.1238, .1236]
		0 .5 1 0	[.1062, .1052]	[.1797, .1792]	[.1330, .1340]	[.1349, .1353]	[.1198, .1203]
		.5 0 1.5 0	[.2778, .2731]	[.5629, .5658]	[.3080, .3100]	[.3469, .3416]	[.3057, .3110]
14 12 8 6	1 1 1 1	0 0 .5 0	[.1311, .1437]	[.1324, .1450]	[.1600, .1860]	[.2107, .1908]	[.1644, .1744]
		0 0 1 0	[.6409, .7055]	[.6754, .6832]	[.7650, .7820]	[.8864, .8180]	[.7458, .8090]
		0 .5 1 0	[.5311, .5802]	[.5521, .5645]	[.6500, .6510]	[.7598, .6965]	[.6241, .6793]
		.5 0 1.5 0	[1.3765, 1.4768]	[1.4281, 1.4376]	[1.590, 1.573]	[1.7357, 1.6268]	[1.5242, 1.6274]
14 12 8 6	2 1 1 .5	0 0 .5 0	[.4809, .5343]	[.0487, .0524]	[.0750, .0790]	[.0779, .0789]	[.0660, .0663]
		0 0 1 0	[.1951, .2122]	[.1998, .2027]	[.2480, .2630]	[.2953, .2720]	[.2362, .2566]
		0 .5 1 0	[.1826, .1932]	[.1817, .1849]	[.2300, .2410]	[.2633, .2438]	[.2172, .2301]
		.5 0 1.5 0	[.4809, .5343]	[.5043, .4913]	[.5980, .6100]	[.7253, 6584]	[.5811, .6433]

4 Discussion

This section will discuss the results obtained from the simulation work under neutrosophic statistics based on the five multiple comparison tests. From Table1, it is seen that when sample sizes are equal and variances are equal, the NLSD test is slightly lower in (5, 5, 5, 5) than the nominal level and marginally lower for NLSD and more prominent for the Bonferroni test in (10, 10, 10, 10). For equal samples and unequal variances, Tukey's HSD and Bonferroni tests increased compared to others and were higher than the nominal level. Similarly, Table 1 shows the same outputs for unequal samples and equal variances. Increasing the

ordered sample group (5, 10, 15, 20) slightly shows lower outputs for all five tests, and decreasing the ordered sample group (14, 12, 8, 6) indicates higher values than the nominal level. Table 2 provides empirical power results for all five tests with equal and unequal sample sizes and variances for four location parameter groups. Table 2 depicts equal sample groups and equal variances then power for the observations, which includes indeterminacy $I_N \in [0,\ .05]$ decreases for LSD_N and TK_N and increases for other tests. Similarly, for equal sample sizes (5, 5, 5, 5) with unequal variances, power decreases for LSD_N and ST_N in indeterminacy upper interval. Again, Tukey's HSD showed comparatively small power values in the indeterminacy-included observation group for decreasing the ordered sample group (14, 12, 8, 6) with equal and unequal variances. Table 2 shows Tukey's HSD results with large power values in the determinant part but decreases in the indeterminant part. At the same time, Hochberg GT2 and Bonferroni multiple comparison tests maintain consistency and give a smooth result under Neutrosophic statistical analysis.

4.1. Illustration of the test

An illustration is applied to check the performance of the tests for multiple comparisons of the daily ICU occupancy of COVID-19 patients in Pakistan, carried out in December 2020 with a particular focus on age groups. For clarity, the study introduces uncertainty or neutrosophic elements into the dataset. The null hypothesis is that there is no significant difference in the daily ICU occupancy of COVID-19 patients according to their age groups. Table 3 shows the daily ICU occupancy of patients 55 and older, between 35 and 55, and 35 years and below with COVID-19 disease. For that case, the experimenter has chosen a Neutrosophic sample size N = 60 for the data given in the following table below:

Table 3. The Daily I CU occupancy of Corona patients in Pakistan in Dec, 2020

Day/Age	55 and above	35-55	35 and below
1	[443, 450]	[359, 361]	[460, 465]
2	[421, 426]	[352, 365]	[427, 429]
3	[436, 450]	[445, 455]	[407, 410]
4	[376, 385]	[410, 410]	[378, 380]
5	[458, 458]	[458, 464]	[364, 368]
6	[408, 420]	[410, 415]	[345, 349]
7	[422, 425]	[463, 470]	[342, 346]
8	[431, 440]	[580, 584]	[345, 345]
9	[459, 462]	[432, 440]	[313, 318]
10	[369, 369]	[379, 379]	[277, 280]
11	[360, 360]	[370, 370]	[268, 271]
12	[431, 445]	[584, 589]	[259, 262]
13	[403, 415]	[410, 416]	[256, 260]
14	[436, 445]	[587, 590]	[251, 251]
15	[376, 376]	[415, 415]	[249, 249]
16	[370, 370]	[419, 422]	[223, 227]
17	[443, 443]	[357, 357]	[209, 211]
18	[445, 445]	[467, 472]	[187, 191]
19	[355, 365]	[415, 418]	[173, 175]
20	[450, 450]	[358, 358]	[168, 168]

All five post hoc tests are applied to the dataset using neutrosophic statistics for multiple comparisons. Descriptive statistics of the observations in the indeterminacy intervals are shown in Table 4.

Table 4. The neutrosophic d escriptive statistics of the COVID-19 patient's data

Col.	N	Neutrosophic mean	Neutrosophic S.D.	Neutrosophic Std. Error	95% Neutrosophic CI for Mean	
					Lower Bound	Upper Bound
1	[20, 20]	[414.6, 421.2]	[34.7584, 35.9321]	[1.7379, 1.7966]	[413.5144, 420.1071]	[415.6856, 422.2929]
2	[20, 20]	[433.5, 437.5]	[73.8779, 74.5763]	[3.6939, 3.7288]	[432.4144, 436.4071]	[434.5856, 438.5929]
3	[20, 20]	[295.05, 297.75]	[85.7029, 86.2132]	[4.2851, 4.3107]	[293.9644, 296.1356]	[296.1356, 298.8429]
Total	[60, 60]	[381.05, 385.48]	[19.1868, 19.3139]	[2.4770, 2.4934]		

To test the hypothesis of equality of more than one population means, like ANOVA in classical statistics, the NANOVA is applied to the neutrosophic population means. The assumptions of NANOVA are considered as neutrosophic variance remains

constant across the neutrosophic population, and the neutrosophic sample is selected from the neutrosophic population. The null hypothesis for the NANOVA is

H_{N0}: $\mu_{N1} = \mu_{N2} = \mu_{N3}$, $H_{N0} \in [H_{L0}, H_{U0}]$

The alternative hypothesis is that the average daily occupancy differs in at least two age groups. The structure of the NANOVA for the application is given below in Table 5.

Table 5. The NANOVA table for the COVID-19 data

Source	ndf	NSS	NMSS	F_N	P_N-value
Between samples	[2, 2]	[225452.1, 233571.03]	[112726.05, 116785.515]	[24.1365, 24.5254]	[<.001, <.001]
Within samples	[57, 57]	[266210.75, 271423.95]	[271423.95, 4761.8237]		
Totals	[59, 59]	[491662.85, 504994.98]			

The neutrosophic null hypothesis is not rejected if the minimum P_N-value $\leq \alpha$, the level of significance (α=.05)[4]. But for the application, NANOVA reveals that P-values are statistically significant (α = .05) for the indeterminacy interval. Therefore, the null hypothesis is rejected for upper neutrosophic observations, meaning the daily average ICU occupancy is not the same among at least two age groups. However, NANOVA only reveals a significant difference between means or not. Then, let's execute post hoc multiple comparison tests to determine which population pairs exhibit different means using neutrosophic statistics. Results for the neutrosophic post hoc multiple comparison tests are shown in Table 6.

Table 6. Multiple comparisons for Daily ICU occupancy of Corona patients

Tests	Group (i)	Groups (j)	Neutrosophic Mean diff. (i-j)	Neutrosophic S.E.	Sig. values	95% Neutrosophic CI for Mean	
						Lower Bound	Upper Bound
LSD	[1, 1]	[2, 2]	[-18.9000, -16.30000]	[21.6110, 21.8216]	[<.385, .458]	[-62.1753, -59.9970]	[24.3753, 27.3970]
		[3, 3]	[119.5500, 123.4500]	[21.6110, 21.8216]	[<.001, <.001]	[76.2747, 79.7530]	[162.8253, 167.1470]
	[2, 2]	[1, ,1]	[18.9000, 16.30000]	[21.6110, 21.8216]	[.385, .458]	[-24.3753, -27.3970]	[62.1753, 59.9970]
		[3, 3]	[138.4500, 139.7500]	[21.6110, 21.8216]	[<.001, <.001]	[-95.1747, 96.0530]	[181.7253, 183.4470]
	[3, 3]	[1, 1]	[-119.5500, -123.4500]	[21.6110, 21.8216]	[<.001, <.001]	[-162.8253, -167.1470]	[-76.2747, -79.7530]
		[2, 2]	[-138.4500, -139.7500]	[21.6110, 21.8216]	[<.001, <.001]	[-181.7253, -183.4470]	[-95.1747, -96.0530]
Tukey HSD	[1, 1]	[2, 2]	[-18.9000, -16.30000]	[21.6110, 21.8216]	[.658, .737]	[-70.9052, -68.8119]	[33.1052, 36.2119]
		[3, 3]	[119.5500, 123.4500]	[21.6110, 21.8216]	[<.001, <.001]	[67.5448, 70.9380]	[171.5552, 175.9619]
	[2, 2]	[1, 1]	[18.9000, 16.30000]	[21.6110, 21.8216]	[.658, .737]	[-33.1052, -36.2119]	[70.9052, 68.8119]
		[3, 3]	[138.4500, 139.7500]	[21.6110, 21.8216]	[<.001, <.001]	[86.4448, 87.2381	[190.4552, 192.2619]
	[3, 3]	[1, 1]	[-119.5500, -123.4500]	[21.6110, 21.8216]	[<.001, <.001]	[-171.5552, -175.9619]	[-67.5448, -70.9381]
		[2, 2]	[-138.4500, -139.7500]	[21.6110, 21.8216]	[<.001, <.001]	[-190.4552, -192.2619]	[-86.4448, -87.2381]
Scheffe	[1, 1]	[2, 2]	[-18.9000, -16.30000]	[21.6110, 21.8216]	[.684, .758]	[-73.2193, -71.1486]	[35.4193, 38.5486]
		[3, 3]	[119.5500, 123.4500]	[21.6110, 21.8216]	[<.001, <.001]	[65.2307, 68.8119]	[173.8693, 178.2986]
	[2, 2]	[1, 1]	[18.9000, 16.30000]	[21.6110, 21.8216]	[.684, .758]	[-35.4193, -38.5486]	[73.2193, 71.1486]
		[3, 3]	[138.4500, 139.7500]	[21.6110, 21.8216]	[<.001, <.001]	[84.1307, 84.9014]	[192.7693, 194.5986]
	[3, 3]	[1, 1]	[-119.5500, -123.4500]	[21.6110, 21.8216]	[<.001, <.001]	[-173.8693, -178.2986]	[-65.2307, -68.6014]

Continued on next page

Table 6 continued

		[2, 2]	[-138.4500, -139.7500]	[21.6110, 21.8216]	[<.001, <.001]	[-192.7693, -194.5986]	[-84.1307, -84.9014]
	[1, 1]	[2, 2]	[-18.9000, -16.30000]	[21.6110, 21.8216]	[1.000, 1.000]	[-72.2076, -70.1271]	[34.4076, 37.5272]
		[3, 3]	[119.5500, 123.4500]	[21.6110, 21.8216]	[<.001, <.001]	[-66.2424, 69.6229]	[172.8576, 177.2771]
	[2, 2]	[1, 1]	[18.9000, 16.30000]	[21.6110, 21.8216]	[1.000, 1.000]	[-34.4076, -37.5271]	[72.2076, 70.1271]
		[3, 3]	[138.4500, 139.7500]	[21.6110, 21.8216]	[<.001, <.001]	[85.1424, 85.9229]	[191.7576, 193.5771]
	[3, 3]	[1, 1]	[-119.5500, -123.4500]	[21.6110, 21.8216]	[<.001, <.001]	[-172.8576, -177.2771]	[-66.2424, -69.6229]
		[2, 2]	[-138.4500, -139.7500]	[21.6110, 21.8216]	[<.001, <.001]	[-191.7576, -193.5771]	[-85.1424, -85.9229]
	[1, 1]	[2, 2]	[-18.9000, -16.30000]	[21.6110, 21.8216]	[.764, .838]	[-72.0013, -69.9187]	[34.2013, 37.5271]
		[3, 3]	[119.5500, 123.4500]	[21.6110, 21.8216]	[<.001, <.001]	[66.4487, 69.8313]	[172.6513, 177.2771]
	[2, 2]	[1, 1]	[18.9000, 16.30000]	[21.6110, 21.8216]	[.764, .838]	[-34.2013, -37.3187]	[72.0013, 70.1271]
		[3, 3]	[138.4500, 139.7500]	[21.6110, 21.8216]	[<.001, <.001	[85.3487, 86.1313]	[191.5513, 193.5771]
	[3, 3]	[1, 1]	[-119.5500, -123.4500]	[21.6110, 21.8216]	[<.001, <.001]	[-172.6513, -177.0687]	[-66.4487, -69.6229]
		[2, 2]	[-138.4500, -139.7500]	[21.6110, 21.8216]	[<.001, <.001]	[-191.5513, -1933687]	[-85.3487, -85.9229]

5 Conclusions

In this study, five multiple comparison tests are studied using neutrosophic statistics with the help of a simulation process. Based on five multiple comparison tests, the simulation work under neutrosophic statistics provides important insights into the ways that variances and sample sizes impact test performance. Tukey's HSD test maintains high power values in determinant sections but displays reduced power in groups that contain indeterminacy. Overall, the Hochberg GT2 and Bonferroni tests exhibit consistent performance and smooth results under neutrosophic statistical analysis, highlighting their robustness in handling indeterminate data. The study shows that more appropriate decisions can be made to acknowledge potentially uncertain information by applying neutrosophic statistics. So, it may help the researchers in such tests and imprecise data for proper conclusion.

Acknowledgments

The authors are grateful to the reviewers for their suggestions for improving the paper. The authors also gratefully thank the Associate Editors for their dedication and time in offering insightful criticism and recommendations that have significantly raised the caliber of the article.

References

1) Aslam M. Neutrosophic analysis of variance: application to university students. *Complex & Intelligent Systems*. 2019;5(4):403–407. Available from: https://doi.org/10.1007/s40747-019-0107-2.
2) Smarandache. Neutrosophic Statistics is a generalization of Classical and Interval Statistics. . Available from: https://fs.unm.edu/NS/NeutrosophicStatistics.htm.
3) Smarandache F. Neutrosophic Statistics is an extension of Interval Statistics, while Plithogenic Statistics is the most general form of statistics (second version). *International Journal of Neutrosophic Science*. 2022;19(1):148–165. Available from: https://doi.org/10.54216/IJNS.190111.
4) Smarandache F. Foundation of Appurtenance and Inclusion Equations for Constructing the Operations of Neutrosophic Numbers Needed in Neutrosophic Statistics (revised). *Neutrosophic Systems With Applications*. 2024;15:16–32. Available from: https://doi.org/10.61356/j.nswa.2024.1513856.
5) Smarandache F. Foundation of Appurtenance and Inclusion Equations for Constructing the Operations of Neutrosophic Numbers Needed in Neutrosophic Statistics (revised). *Prospects for Applied Mathematics and Data Analysis*. 2023;3(1):29–48. Available from: https://doi.org/10.54216/PAMDA.030103.
6) Lee S, Lee DK. What is the proper way to apply the multiple comparison test? *Korean Journal of Anesthesiology*. 2018;71(5):353–360. Available from: https://doi.org/10.4097/kja.d.18.00242.

7) Smarandache F. Indeterminacy in neutrosophic theories and their applications. *Infinite Study*. 2021. Available from: https://doi.org/10.54216/IJNS.150203.
8) Aslam M. A new failure-censored reliability test using neutrosophic statistical interval method. *International Journal of Fuzzy Systems*. 2019;21(4). Available from: https://doi.org/10.1007/s40815-018-0588-y.
9) Albassam M, Khan N, Aslam M. The W/S Test for Data Having Neutrosophic Numbers: An Application to USA Village Population. *Complexitydoi*. 2020. Available from: https://doi.org/10.1155/2020/3690879.
10) Smarandache F. Introduction to neutrosophic statistic. *Infinite Study*. 2014. Available from: https://doi.org/10.13140/2.1.2780.1289.
11) Aslam M, Albassam M. Presenting post hoc multiple comparison tests under neutrosophic statistics. *Journal of King Saud University - Science*. 2020;32:2728–2732. Available from: https://doi.org/10.1016/j.jksus.2020.06.008.
12) Sherwani RAK, Shakeel H, Awan WB, Faheem M, Aslam M. Analysis of COVID-19 data using neutrosophic Kruskal Wallis H test. *BMC Medical Research Methodology*. 2021;21:1–7. Available from: https://doi.org/10.1186/s12874-021-01410-x.