INDIAN JOURNAL OF SCIENCE AND TECHNOLOGY

RESEARCH ARTICLE

Intuitionistic Fuzzy Binary Soft Sets and Its Properties

H Sivasankari[1]*, J Subhashini[2]

1 Research Scholar, PG and Research Department of Mathematics, St.John's College, Palayamkottai, Affiliated to Manonmaniam Sundaranar University, Tirunelveli, Tamil Nadu, India
2 Assistant Professor, PG and Research Department of Mathematics,, St.John's College, Palayamkottai, Affiliated to Manonmaniam Sundaranar University, Tirunelveli, 627012,, Tamil Nadu, India

OPEN ACCESS
Received: 08-04-2024
Accepted: 10-08-2024
Published: 28-08-2024

Citation: Sivasankari H, Subhashini J (2024) Intuitionistic Fuzzy Binary Soft Sets and Its Properties. Indian Journal of Science and Technology 17(35): 3636-3642. https://doi.org/10.17485/IJST/v17i35.1161

* **Corresponding author**.
h.sankari1998@gmail.com

Funding: None

Competing Interests: None

Published By Indian Society for Education and Environment (iSee)

ISSN
Print: 0974-6846
Electronic: 0974-5645

Abstract

Objective: The objective of Intuitionistic fuzzy binary soft sets is to extend the existing frameworks of binary soft sets and Intuitionistic fuzzy sets to accommodate both uncertainty and ambiguity in decision-making processes. This extension aims to provide a more flexible, representation of real-world data and phenomena, allowing for analysis and reasoning. Specifically, the objective involves defining the fundamental structures of Intuitionistic fuzzy binary soft sets over two initial universal sets, U_1 and U_2. **Method:** Reviewing the existing literature on binary soft sets, Intuitionistic fuzzy sets, and their extensions. Defining the fundamental structures of Intuitionistic fuzzy binary soft sets over two initial universal sets, U_1 and U_2. The next step is to propose the operations on Intuitionistic fuzzy binary soft sets and analyze their characteristics. **Findings:** The extension of binary soft sets and Intuitionistic fuzzy sets to Intuitionistic fuzzy binary soft sets is feasible. Intuitionistic fuzzy binary soft sets possess distinct properties worthy of exploration. Operations such as union, intersection, difference, AND, Or of Intuitionistic fuzzy binary soft sets exhibit specific behaviors elucidated through our findings. **Novelty:** The development of operations on Intuitionistic fuzzy binary soft sets provides a comprehensive framework and analysis. Exploration of properties unique to Intuitionistic fuzzy binary soft sets is contributing to the advancement of soft computing and set theory.
Mathematics Classification code: 03D45, 03F55, 03E72

Keywords: Fuzzy; Soft; Intuitionistic; Binary soft; Fuzzy binary soft

1 Introduction

Real - world issues that are addressed in many disciplines, including social science, engineering, medicine, and economics, often involve imprecise data, leading to the need for mathematical principles that accommodate uncertainty and imprecision. The concept of intuitionistic fuzzy sets introduces a novel hesitancy function, while soft set theory provides parameterization tools for modeling vagueness and uncertainties.

Soft set theory, renowned for its flexibility in approximate description without strict restrictions, finds wide application in complex systems, medical science, engineering, management, economics, and clustering. This theory has been extended with new concepts and applications.

The human mind, with its inherently non-crisp reasoning that allows for degrees of certainty, serves as a model for such reasoning processes. While fuzzy logic and fuzzy set theory, pioneered by Zadeh, have played a significant role in computing, they have limitations such as the ambiguity in assigning membership values and the lack of parameterization techniques. The intuitionistic fuzzy set is making strides across various fields[1–6]. Its development is expanding rapidly, finding applications in diverse areas. This growth highlights its increasing significance and utility. J. Subhashini and P. Gino Metilda[7] introduced the idea of fuzzy binary soft sets in 2020 by extending the ideas from binary soft sets[8]. Some of the results are identified in the fuzzy binary soft sets[9–12].

In the pursuit of advancing fuzzy sets and soft sets, a new model, namely intuitionistic fuzzy binary soft sets, is proposed in this paper. By combining the advantages of intuitionistic fuzzy sets and binary soft sets, this model aims to offer a comprehensive approach to handling uncertain information. In our manuscript, we denoted the Intuitionistic fuzzy binary soft sets as IFBSS.

To facilitate our discussion, Section 2.1 presents the concept of intuitionistic fuzzy binary soft sets, along with definitions of complement, AND, OR, intersection, and union operations on these sets. In Section 2.2, we outline some properties of these operations. Finally, we conclude the paper with a summary and discuss potential avenues for further research.

The aim of Intuitionistic fuzzy binary soft sets is to provide a more flexible and comprehensive framework for decision-making by degrees of membership, non-membership, and hesitation, thereby enhancing the ability to handle uncertainty and imprecision in complex systems.

2 Methodology

2.1. Intuitionistic Fuzzy Binary Soft Set

Definition 2.1.1 Let U_1, U_2 denotes the two initial universal sets. Let E represents a set of parameters. Let F(U_1) and F(U_2) represents the set comprising all fuzzy sets of U_1 and U_2 respectively. Let A ⊂ E. A pair (F_{IFBSS}, A) is an Intuitionistic fuzzy binary soft set over U_1, U_2 where F_{IFBSS} is a mapping defined by,

$F_{IFBSS} : F(U_1) \times F(U_2) \to [0,1]$.

Then, Intuitionistic fuzzy binary soft sets can be written as,

$(F_{IFBSS}, A) = \{(x, \gamma_A(x), \delta_A(x)) : x \in E\}$

The value $\gamma_A(x)$ is an Intuitionistic fuzzy soft set called x-element of F_{IFBSS} for all x ∈ Eis characterized by,

$\gamma_A(x) = \{\left(u_1, \mu_{\gamma_A}(x)(u_1), \nu_{\gamma_A}(x)(u_1)\right) : u_1 \in E\}$

The value $\delta_A(x)$ is an Intuitionistic fuzzy soft set called x-element of F_{IFBSS} for all x ∈ Eis characterized by,

$\delta_A(x) = \{\left(u_2, \mu_{\delta_A}(x)(u_2), \nu_{\delta_A}(x)(u_2)\right) : u_2 \in E\}$

For all x ∈ E, Here the functions $\mu_{\gamma_A}, \mu_{\delta_A}$ and $\nu_{\gamma_A}, \nu_{\delta_A}$ are the membership and non-membership functions of U_1, U_2, to [0, 1] with the conditions, $0 \le \mu_{\gamma_A}(x) + \nu_{\gamma_A}(x) \le 1$ & $0 \le \mu_{\delta_A}(x) + \nu_{\delta_A}(x) \le 1$.

Example 2.1.2 Let $U_1 = \{S_1, S_2, S_3, S_4\}$ denotes the set of Shirts. Let $U_2 = \{T_1, T_2, T_3, T_4\}$ denotes the set of T – Shirts. Let $E = \{P_1, P_2, P_3, P_4, P_5\}$ denotes the set of Parameters, Where, P_1 - Expensive, P_2 - Modern, P_3 - Quality, P_4 - Cheap, P_5- Size. $A = \{P_1, P_2, P_3\} \subseteq E \gamma_A(P_1) = \{(S_1, 0.4, 0.6), (S_2, 0.2, 0.7), (S_3, 0.7, 0.2), (S_4, 0.4, 0.4)\}$

$\gamma_A(P_2) = ((S_1, 0.7, 0.3), (S_2, 0.3, 0.6), (S_3, 0.2, 0.8), (S_4, 0.5, 0.5)\}$

$\gamma_A(P_3) = \{(S_1, 0.4, 0.3), (S_2, 0.7, 0.2), (S_3, 0.8, 0.1), (S_4, 0.6, 0.3)\}$

$\delta_A(P_1) = \{(T_1, 0.2, 0.8), (T_2, 0.5, 0.5), (T_3, 0.4, 0.6), (T_4, 0.3, 0.6)\}$

$\delta_A(P_2) = \{(T_1, 0.3, 0.7), (T_2, 0.8, 0.1), (T_3, 0.6, 0.3), (T_4, 0.4, 0.4)\}$

$\delta_A(P_3) = \{(T_1, 0.5, 0.4), (T_2, 0.7, 0.2), (T_3, 0.9, 0.1), (T_4, 0.6, 0.3)\}$

Then an Intuitionistic fuzzy binary soft set is written as,

(F_{IFBSS}, A) = {(P_1,({(S_1, 0.4,0.6), (S_2, 0.2,0.7),(S_3, 0.7,0.2), (S_4, 0.4,0.4)}{(T_1, 0.2,0.8),
(T_2,0.5,0.5),(T_3, 0.4,0.6), (T_4, 0.3,0.6)})),(P_2,({(S_1, 0.7,0.3), (S_2, 0.3,0.6),
(S_3, 0.2,0.8), (S_4,0.5,0.5)}{(T_1, 0.3,0.7), (T_2, 0.8,0.1),(T_3, 0.6,0.3),
(T_4, 0.4,0.4)})),(P_3,({(S_1, 0.4,0.3), (S_2,0.7,0.2),(S_3, 0.8,0.1), (S_4, 0.6,0.3)}
{(T_1, 0.5,0.4), (T_2, 0.7,0.2),(T_3, 0.9,0.1), (T_4,0.6,0.3)}))}.

Definition 2.1.3. Let (F_{IFBSS}, A) and (G_{IFBSS}, B) be two Intuitionistic fuzzy binary soft sets over the common U_1, U_2. (F_{IFBSS}, A) is called an Intuitionistic fuzzy binary soft subset of (G_{IFBSS}, B) if, i) $A \subset B$ii) F_{IFBSS}(p) is the fuzzy subset of G_{IFBSS}(p) for eachP ∈ Aand is denoted by $(FIFBSS,\ A)\ \subset\ (GIFBSS,\ B)$. ($F_{IFBSS}$, A) is said to be an Intuitionistic fuzzy binary soft super set of (G_{IFBSS}, B) if (G_{IFBSS}, B) is an Intuitionistic fuzzy binary soft subset of (F_{IFBSS}, A). And we denoted by $(FIFBSS,\ A)\ \supset\ (GIFBSS,\ B)$.

Example 2.1.4. Let $U_1 = \{S_1, S_2, S_3, S_4\}$ denotes the set of Shirts. Let $U_2 = \{T_1, T_2, T_3, T_4\}$ denotes the set of T - Shirts. Let $E = \{P_1, P_2, P_3, P_4,\ P_5\}$ denotes the set of Parameters, $A = \{P_1, P_2\} \subseteq\ E\ and\ B = \{P_1, P_2,\ P_3\}\ \subseteq\ E$.. Now, $(F_{IFBSS},\ A)$, (G_{IFBSS}, B), are the Intuitionistic fuzzy binary soft sets over U_1, U_2 described in the following manner:

(F_{IFBSS}, A) = {(P_1,({(S_1, 0.4,0.6), (S_2, 0.2,0.7),(S_3, 0.7,0.2), (S_4, 0.4,0.4)}{(T_1, 0.2,0.8), (T_2, 0.5, 0.5),(T_3, 0.4,0.6), (T_4, 0.3,0.6)})),(P_2,({(S_1, 0.7,0.3), (S_2, 0.3,0.6), (S_3, 0.2,0.8), (S_4,0..5,0.5)}{(T_1, 0.3,0.7), (T_2, 0.8,0.1),(T_3, 0.6,0.3), (T_4, 0.4,0.4)}))}.

(G_{IFBSS}, B) = {(P_1, ({(S_1, 0.4, 0.6), (S_2, 0.2, 0.7),(S_3, 0.7,0.2), (S_4, 0.4,0.4)}{(T_1, 0.2,0.8), (T_2, 0.5, 0.5), (T_3, 0.4, 0.6), (T_4, 0.3, 0.6)})), (P_2, ({(S_1, 0.7,0.3), (S_2, 0.3,0.6), (S_3, 0.2, 0.8), (S_4, 0.5, 0.5)} {(T_1, 0.3, 0.7), (T_2, 0.8, 0.1),(T_3, 0.6,0.3), (T_4, 0.4, 0.4)})), (P_3,({(S_1, 0.4,0.3), (S_2,0.7,0.2),(S_3, 0.8,0.1), (S_4, 0.6,0.3)} {(T_1, 0.5, 0.4), (T_2, 0.7, 0.2),(T_3, 0.9,0.1), (T_4,0.6,0.3)}))}.

$\therefore (\mathbf{F_{IFBSS}}, \mathbf{A}) \subseteq (\mathbf{G_{IFBSS}}, \mathbf{B})$.

Definition 2.1.5. Let $(F_{IFBSS},\ A)\ and\ (G_{IFBSS},\ B)$ be two Intuitionistic fuzzy binary soft sets of common U_1, U_2,. $(F_{IFBSS},\ A)$ is called an Intuitionistic fuzzy binary soft equal set of (G_{IFBSS}, B) if (F_{IFBSS}, A) is an Intuitionistic fuzzy binary soft subset of (G_{IFBSS}, B) and (G_{IFBSS}, B) is an Intuitionistic fuzzy binary soft subset of (F_{IFBSS}, A) and we denoted by $\left(\mathbf{F_{IFBSS}}, \mathbf{A}\right) = (\mathbf{G_{IFBSS}}, \mathbf{B})$

Definition 2.1.6. The complement of an Intuitionistic fuzzy binary soft set (F_{IFBSS}, A) is symbolized by $(F_{IFBSS}, A)^C$ and it is defined by $(F_{IFBSS}, A)^C = (F^C_{IFBSS}, \neg A)$

where $F^C_{IFBSS} : \neg A \to F(U_2) \times F(U_2)$ is given by $F^C_{IFBSS}(p)\ =\ (F^C_{IFBSS}(\neg p))^C$ (p) = $(F_{IFBSS}(\neg p))^C$ for all $\neg p \in \neg A$.

Example 2.1 .7 $(F_{IFBSS}, A)^C$ = {(not expensive,({(S_1, 0.4,0.6), (S_2, 0.2,0.7),(S_3, 0.7,0.2), (S_4,0.4,0.4)} {(T_1, 0.2,0.8), (T_2, 0.5,0.5),(T_3, 0.4,0.6), (T_4, 0.3,0.6)})),(not modern, ({(S_1, 0.7,0.3), (S_2, 0.3,0.6),(S_3, 0.2,0.8), (S_4, 0.5,0.5)}{(T_1, 0.3,0.7), (T_2, 0.8,0.1),(T_3, 0.6,0.3), (T_4,0.4,0.4)})),(not quality,({(S_1, 0.4,0.3), (S_2, 0.7,0.2),(S_3, 0.8,0.1), (S_4, 0.6,0.3)}{(T_1, 0.5,0.4), (T_2, 0.7,0.2), (T_3, 0.9,0.1), (T_4, 0.6,0.3)}))}.

Definition 2.1.8. An Intuitionistic fuzzy binary soft set $\left(F_{IBSS}, A\right)$ over the common U_1, U_2, is said to be an Intuitionistic fuzzy binary null soft set if for all $p \in A$, $F_{IFBSS}(p)$ is the null fuzzy set and it is denoted by $\emptyset$ over U_1, U_2.

Example 2.1.9. Let $U_1 = \{S_1, S_2, S_3, S_4\}$ denotes the set of Shirts. Let $U_2 = \{T_1, T_2, T_3, T_4\}$ denotes the set of T – Shirts, $A = \{P_1, P_2, P_3\}$where A is the set of Parameters.

A pair (F_{IFBSS}, A) is an Intuitionistic fuzzy binary soft set described in the following manner:

(F_{IFBSS}, A)= {(P_1,({(S_1, 0, 0), (S_2, 0, 0),(S_3, 0, 0), (S_4, 0, 0)}{(T_1, 0, 0), (T_2, 0, 0),(T_3, 0, 0), (T_4,0, 0)})),(P_2,({(S_1, 0, 0), (S_2, 0, 0),(S_3, 0, 0), (S_4, 0, 0)}{(T_1, 0, 0), (T_2, 0, 0),(T_3, 0, 0), (T_4, 0, 0)})),(P_3,({(S_1, 0, 0), (S_2, 0, 0),(S_3, 0, 0), (S_4, 0, 0)}{(T_1, 0, 0), (T_2, 0, 0),(T_3, 0, 0), (T_4, 0, 0)}))}.

$\therefore (\mathbf{F_{IFBSS}}, \mathbf{A})$ **is an Intuitionistic fuzzy binary null soft sets.**

Definition 2.1.10. An Intuitionistic fuzzy binary soft set $(F_{IFBSS},\ A)$ of common U_1, U_2, is an Intuitionistic fuzzy binary absolute soft set if for all $p \in A$, $F_{IFBS}(p)$ is the absolute fuzzy set, it is denoted by $\widetilde{A}$ over U_1, U_2.

Example 2.1.11. Let $U_1 = \{S_1, S_2, S_3, S_4\}$denotes the set of Shirts. Let $U_2 = \{T_1, T_2, T_3, T_4\}$ denotes the set of T – Shirts, $A = \{P_1, P_2, P_3\}$ where A is the set of Parameters.

A pair (F_{IFBSS}, A) is the Intuitionistic fuzzy binary soft set described in the following manner:

(F_{IFBSS}, A) = {(P_1, ({(S_1, 1, 0), (S_2, 1, 0),(S_3, 1, 0), (S_4, 1, 0)}{(T_1, 1, 0), (T_2, 1, 0),(T_3, 1, 0), (T_4,1, 0)})),(P_2,({(S_1, 1, 0), (S_2, 1, 0),(S_3, 1, 0), (S_4, 1, 0)}{(T_1, 1, 0), (T_2, 1, 0),(T_3, 1, 0), (T_4, 1, 0)})),(P_3, ({(S_1, 1, 0), (S_2, 1, 0),(S_3, 1, 0), (S_4, 1, 0)}{(T_1, 1, 0), (T_2, 1, 0), (T_3, 1, 0), (T_4, 1, 0)}))}.

$\therefore (\mathbf{F_{IFBSS}}, \mathbf{A})$**is an Intuitionistic fuzzy binary absolute soft sets.**

Definition 2.1.12. Union of Intuitionistic fuzzy binary soft sets (F_{IFBSS}, A) and (G_{IFBSS}, B) of the common U_1, U_2 is the intuitionistic fuzzy binary soft set (H_{IFBSS}, C) where $C = A \cup B$ for each p $p \in C$.

$$(H_{IFBSS}, C) = \begin{cases} \mathrm{F_{IFBSS}(P)} & \mathrm{p} \in \mathrm{A-B} \\ \mathrm{G_{IFBSS}(P)} & \mathrm{p} \in \mathrm{B-A} \\ \mathrm{F_{IFBSS}(P)} \cup \mathrm{G_{IFBSS}(P)} & \mathrm{p} \in \mathrm{A \cap B} \end{cases}$$

We write, $(H_{IFBSS},\, C) = (F_{IFBSS},\, A) \cup (G_{IFBSS}, B)$.

Example 2.1.13. Let $U_1 = \{S_1, S_2, S_3, S_4\}$denotes the set of Shirts. Let$U_2 = \{T_1, T_2, T_3, T_4\}$ denotes the set of T – Shirts. Let $E = \{P_1, P_2, P_3, P_4, P_5\}$ denotes the set of Parameters, Where, P_1 - Expensive, P_2 - Modern, P_3 - Quality, P_4 - Cheap, P_5- Size. .

Consider an Intuitionistic fuzzy binary soft sets (F_{IFBSS}, A), (G_{IFBSS}, B) is described in the following manner:

(F_{IFBSS}, A)= {(P$_1$,({(S$_1$, 0.4,0.6), (S$_2$, 0.5,0.5),(S$_3$, 0.2,0.8), (S$_4$, 0.3,0.7)}{(T$_1$, 0.3,0.7), (T$_2$,0.1,0.9), (T$_3$, 0.4,0.6), (T$_4$, 0.3,0.6)})),(P$_2$,({(S$_1$, 0.5,0.5), (S$_2$, 0.3,0.6),(S$_3$, 0.4,0.4), (S$_4$,0.8,0.2)} {(T$_1$, 0.4,0.6), (T$_2$, 0.3,0.7),(T$_3$, 0.6,0.4), (T$_4$, 0.4,0.4)})),(P$_3$,({(S$_1$, 0.4,0.3), (S$_2$,0.7,0.1), (S$_3$, 0.8, 0.1), (S$_4$, 0.6, 0.3)}{(T$_1$, 0.5,0.4), (T$_2$, 0.7,0.2),(T$_3$, 0.5,0.1), (T$_4$,0.6,0.3)}))}.

(G_{IFBSS}, B)= {(P$_3$,({(S$_1$, 0.5,0.3), (S$_2$, 0.9,0.1),(S$_3$, 0.7,0.2), (S$_4$, 0.6,0.4)}{(T$_1$, 0.4,0.4), (T$_2$,0.8,0.1), (T$_3$, 0.6,0.3), (T$_4$, 0.5,0.4)})),(P$_4$,({(S$_1$, 0.6,0.3), (S$_2$, 0.2,0.7),(S$_3$, 0.7,0.3), (S$_4$,0.1,0.9)} {(T$_1$, 0.4, 0.6), (T$_2$, 0.4, 0.6), (T$_3$, 0.3,0.7), (T$_4$, 0.5,0.5)}))}.

Then, $(H_{IFBSS},\, C) = \ (F_{IFBSS},\, A) \ \cup \ (G_{IFBSS}, B)$ is an Intuitionistic fuzzy binary soft set as below such that $C = \ A \cup B$

$(H_{IFBSS},\, C)$= {(P$_1$,({(S$_1$, 0.4,0.6), (S$_2$, 0.5,0.5),(S$_3$, 0.2,0.8), (S$_4$, 0.3,0.7)}{(T$_1$, 0.3,0.7), (T$_2$,0.1,0.9), (T$_3$, 0.4,0.6), (T$_4$, 0.3,0.6)})),(P$_2$, ({(S$_1$, 0.5,0.5), (S$_2$, 0.3,0.6),(S$_3$, 0.4,0.4), (S$_4$,0.8,0.2)} {(T$_1$, 0.4,0.6), (T$_2$, 0.3,0.7),(T$_3$, 0.6,0.4), (T$_4$, 0.4,0.4)})),(P$_3$,({(S$_1$, 0.5,0.3), (S$_2$,0.9,0.1), (S$_3$, 0.8,0.2), (S$_4$, 0.6,0.4)}{(T$_1$, 0.5,0.4), (T$_2$, 0.8,0.2),(T$_3$, 0.6,0.3), (T$_4$,0.6,0.4)})) (P$_4$,({(S$_1$, 0.6, 0.3), (S$_2$, 0.2, 0.7), (S$_3$, 0.7, 0.3),(S4, 0.1, 0.9)}{(T$_1$, 0.4, 0.6),(T$_2$, 0.4, 0.6), (T$_3$, 0.3, 0.7),(T$_4$, 0.5,0.5)}))}.

Definition 2.1.14. Intersection of Intuitionistic fuzzy binary soft sets (F_{IFBS}, A) and $(G_{IFBS}, B)$of the common U_1, U_2 is an Intuitionistic fuzzy binary soft set(H_{IFBSS}, C), where $C = A \cap B$ for each $p \in C$ and

$H_{IFBSS}(p) = F_{IFBSS}(P) \cap G_{IFBSS}(P)$ for each $p \in C$. We denoted it by, $((H_{IFBSS}, C) = \ (F_{IFBSS,A}) \cap (G_{IFBSS}, B)$.

Example 2.1.15. In example 2.1.13, the Intersection of Intuitionistic fuzzy binary soft sets (F_{IFBSS}, A) and (G_{IFBSS}, B) is an Intuitionistic fuzzy binary soft set (H_{IFBSS}, C) where C= A $C = A \cap B = \{P_3\}$

(H_{IFBSS}, C)= {(P$_3$,({(S$_1$, 0.4,0.3), (S$_2$, 0.7,0.1),(S$_3$, 0.7,0.1), (S$_4$, 0.6,0.3)}{(T$_1$, 0.4,0.4), (T$_2$,0.7,0.1), (T$_3$, 0.5, 0.1), (T$_4$, 0.5, 0.3)}))}.

Definition 2.1.16. The Difference of Intuitionistic fuzzy binary soft sets (F_{IFBSS}, E) and (G_{IFBSS}, E) over the common U_1, U_2 is denoted by $(F_{IFBSS}, E) \setminus (G_{IFBSS}, E)$ it is defined as $(F_{IFBSS}, E) \setminus (G_{IFBSS}, E) = (F_{IFBSS}, E) \cap (G_{IFBSS}, E)^C$.

Example 2.1.17. Let$U_1 = \{S_1, S_2, S_3, S_4\}$, $U_2 = \{T_1, T_2, T_3, T_4\}$ and $E = \{P_1, P_2, P_3\}$. Let (F_{IFBSS}, E) and (G_{IFBSS}, E) be Intuitionistic fuzzy binary soft set described in the following manner:

(F_{IFBSS}, E) = {(P$_1$, ({(S$_1$, 0.5, 0.5), (S$_2$, 0.8, 0.2),(S$_3$, 0.6,0.4)}{(T$_1$, 0.4,0.6), (T$_2$, 0.6,0.3), (T$_3$, 0.9,0.1)})),(P$_2$,({(S$_1$, 0.4,0.5), (S$_2$, 0.5,0.5),(S$_3$, 0.8,0.2)}{(T$_1$, 0.7,0.3), (T$_2$, 0.6,0.4), (T$_3$, 0.4,0.5)})),(P$_3$,({(S$_1$, 0.6,0.3), (S$_2$, 0.9,0.1),(S$_3$, 0.5,0.5)}{(T$_1$, 0.2,0.8), (T$_2$, 0.4,0.5), (T$_3$, 0.1, 0.9)}))}.

(G_{IFBSS}, E) = {(P$_1$, ({(S$_1$, 0.3, 0.7), (S$_2$, 0.6,0.4),(S$_3$, 0.4,0.6)}{(T$_1$, 0.4,0.6), (T$_2$, 0.3,0.7), (T$_3$, 0.5,0.5)})),(P$_2$,({(S$_1$, 0.4,0.5), (S$_2$, 0.3,0.7),)(S$_3$, 0.6,0.4)}{(T$_1$, 0.5,0.5), (T$_2$, 0.6,0.4), (T$_3$, 0.4,0.5)}),(P$_3$,({(S$_1$, 0.5,0.5), (S$_2$, 0.7,0.3),(S$_3$, 0.5,0.5)}{(T$_1$, 0.8,0.2), (T$_2$, 0.5,0.5), (T$_3$, 0.2, 0.8)}))}.

Then $(F_{IFBSS}, E) \setminus (G_{IFBSS}, E)$= {(P$_1$,({(S$_1$, 0.5,0.3), (S$_2$, 0.4,0.2), (S$_3$, 0.6,0.4)}{(T$_1$, 0.4,0.4), (T$_2$, 0.6, 0.3), (T$_3$, 0.5, 0.5)})), (P$_2$, ({(S$_1$, 0.4, 0.5), (S$_2$, 0.5,0.3), (S$_3$, 0.4, 0.2)}{(T$_1$, 0.5, 0.5), (T$_2$, 0.4, 0.4), (T$_3$, 0.4, 0.5)})), (P$_3$, ({(S$_1$, 0.5, 0.3), (S$_2$, 0.3, 0.1), (S$_3$, 0.5, 0.5)} {(T$_1$, 0.2,0.8), (T$_2$, 0.4,0.5), (T$_3$, 0.1, 0.2)}))}.

Definition 2.1.18. If (F_{IFBSS}, A) and (G_{IFBSS}, B) be an Intuitionistic fuzzy binary soft sets then, (F_{IFBSS}, A) AND $(G_{IFBSS},\ B)$ denoted by $(\mathbf{F_{IFBSS}},\mathbf{A})\,\widetilde{\wedge}\,(\mathbf{G_{IFBSS}},\mathbf{B})$. It is defined by,

$(\mathrm{F_{IFBSS}},\mathrm{A})\,\widetilde{\wedge}\,(\mathrm{G_{IFBSS}},\mathrm{B}) = (H_{\mathrm{IFBSS}}, A\times \mathrm{B})$

where, H_{IFBSS} (e, f) = F_{IFBSS} (e) $\cap G_{IFBSS}$ (f) for each (e, f) ∈ A × B.

Example 2.1.19. Let $U_1 = \{S_1, S_2\}$, $U_2 = \{T_1, T_2\}$ and $E = \{P_1, P_2, P_3, P_4, P_5\}$ where, $A = \{P_1, P_2\} \subset E$, $B = \{P_3, P_4\} \subset E$.

(F_{IFBSS}, A)= {(P_1,({(S_1, 0.4, 0.6), (S_2, 0.5, 0.5)}{(T_1, 0.3, 0.7), (T_2, 0.1, 0.9)})),(P_2,({(S_1, 0.5, 0.5), (S_2, 0.3, 0.6)}{(T_1, 0.4, 0.6), (T_2, 0.3, 0.7)}))}.

(G_{IFBSS}, B)= {(P_3,({(S_1, 0.5, 0.3), (S_2, 0.9, 0.1)}{(T_1, 0.4, 0.4), (T_2, 0.8, 0.1)})),(P_4,({(S_1, 0.6, 0.3), (S_2, 0.2, 0.7)}{(T_1, 0.4, 0.6), (T_2, 0.4, 0.6)}))}.

$(\mathbf{F_{IFBSS}},\mathbf{A})\,\widetilde{\wedge}\,(\mathbf{G_{IFBSS}},\mathbf{B})$= {(($P_1$, P_3),({(S_1, 0.4, 0.3),(S_2, 0.5, 0.5)}{(T_1,0.3,0.4), (T_2,0.1, 0.7)})
(P_1, P_4)({(S_1, 0.4, 0.3), (S_2, 0.2, 0.5)}{(T_1,0.3, 0.6), (T_2, 0.1, 0.6)})
(P_2, P_3),({(S_1,0.5, 0.3), (S_2,0.3, 0.1)}, {(T_1, 0.4, 0.4), (T_2, 0.3,0.1)})
(P_2, P_4)({(S_1, 0.5, 0.3), (S_2, 0.2, 0.6)}, {(T_1, 0.4, 0.6), (T_2, 0.3, 0.6)}))}.

Definition 2.1.20. If (F_{IFBSS}, A) and (G_{IFBSS}, B) be an Intuitionistic fuzzy binary soft sets then, (F_{IFBSS}, A) OR (G_{IFBSS}, B) denoted by $(\mathbf{F_{IFBSS}},\mathbf{A})\,\widetilde{\vee}\ (\mathbf{G_{IFBSS}},\mathbf{B})$. It is defined by,

$(\mathrm{F_{IFBSS}},\mathrm{A})\,\widetilde{\vee}\ (\mathrm{G_{IFBSS}},\mathrm{B}) = \ (I_{IFBSS},\ A\times B)$

where, I_{IFBSS} (e, f) = F_{IFBSS} (e) $\cup G_{IFBSS}$(f) for each (e, f) ∈ A × B.

Example 2.1.21. Let $U_1 = \{S_1, S_2\}$, $U_2 = \{T_1, T_2\}$ and $E = \{P_1, P_2, P_3, P_4, P_5\}$where, $A = \{P_1, P_2\} \subset E$, $B = \{P_3, P_4\} \subset E$.

(F_{IFBSS}, A)= {(P_1,({(S_1, 0.1, 0.5), (S_2, 0.3, 0.2)}{(T_1, 0.3, 0.4), (T_2, 0.4, 0.4)})),(P_2,({(S_1, 0.2, 0.3), (S_2, 0.4, 0.4)}{(T_1, 0.6, 0.1), (T_2, 0.5, 0.5)}))}.

(G_{IFBSS}, B)= {(P_3,({(S_1, 0.4, 0.3), (S_2, 0.3, 0.4)}{(T_1, 0.2, 0.4), (T_2, 0.4, 0.4)})),(P_4,({(S_1, 0.5, 0.5), (S_2, 0.3, 0.2)}{(T_1, 0.4, 0.3), (T_2, 0.3, 0.4)}))}.

$(F_{IFBSS}, A)\,\widetilde{\vee}\,(G_{IFBSS}, B)$= {(($P_1$, P_3),({(S_1, 0.4, 0.5),(S_2, 0.3, 0.4)}{(T_1,0.3,0.4), (T_2,0.4, 0.4)})
(P_1, P_4)({(S_1, 0.5, 0.5), (S_2, 0.3, 0.2)}{(T_1,0.4, 0.4), (T_2, 0.4, 0.4)})
(P_2, P_3),({(S_1,0.4, 0.3), (S_2,0.4, 0.4)}, {(T_1, 0.6, 0.4), (T_2, 0.5,0.5)})
(P_2, P_4)({(S_1, 0.5, 0.5), (S_2, 0.4, 0.4)}, {(T_1, 0.6, 0.3), (T_2, 0.5, 0.5)}))}.

2.2. Properties of Intuitionistic Fuzzy Binary Soft Set

Theorem 2.2.1. Let (F_{IFBSS}, A), (G_{IFBSS}, B) and (H_{IFBSS}, C) be three Intuitionistic fuzzy binary soft sets. Next we outline the following findings:

i) $(F_{IFBSS}, A)\widetilde{\cup}(G_{IFBSS}, B) \cong (G_{IFBSS}, B)\widetilde{\cup}(F_{IFBSS}, A)$

ii) $(F_{IFBSS}, A)\widetilde{\cup}\left[(G_{IFBSS}, B)\widetilde{\cup}(H_{IFBSS}, C)\right] \cong \left[(F_{IFBSS}, A)\widetilde{\cup}(G_{IFBSS}, B)\right]\widetilde{\cup}(H_{IFBSS}, C)$iii) $(F_{IFBSS}, A)\widetilde{\cup}\emptyset \cong (F_{IFBSS}, A)$

iv) $(F_{IFBSS}, A)\widetilde{\cup}\widetilde{A} \cong \widetilde{A}$

v) $(F_{IFBSS}, A)\widetilde{\subseteq}(F_{IFBSS}, A)\widetilde{\cup}(G_{IFBSS}, B)\ and\ (G_{IFBSS}, B)\widetilde{\subseteq}(F_{IFBSS}, A)\widetilde{\cup}(G_{IFBSS}, B)$

Proof: It is evident from the provided definition 2.1.12.

Theorem 2.2.2 Let (F_{IFBSS}, A), (G_{IFBSS}, B) and $(H_{IFBSS},\ C)$ be three Intuitionistic fuzzy binary soft sets. Next we outline the following findings:

i) $(F_{IFBSS}, A)\widetilde{\cap}(G_{IFBSS}, B) \cong (G_{IFBSS}, B)\widetilde{\cap}(F_{IFBSS}, A)$

ii) $(F_{IFBSS}, A)\widetilde{\cap}\left[(G_{IFBSS}, B)\widetilde{\cap}(H_{IFBSS}, C)\right] \cong \left[(F_{IFBSS}, A)\widetilde{\cap}(G_{IFBSS}, B)\right]\widetilde{\cap}(H_{IFBSS}, C)$iii) $(F_{IFBSS}, A)\widetilde{\cap}\emptyset \cong \widetilde{\emptyset}$

iv)$(F_{IFBSS}, A)\widetilde{\cap}\widetilde{A} \cong (F_{IFBSS}, A)$

v) $(F_{IFBSS}, A)\widetilde{\cap}(G_{IFBSS}, B)\widetilde{\subset}(F_{IFBSS}, A)\ and\ (F_{IFBSS}, A)\widetilde{\cap}(G_{IFBSS}, B)\widetilde{\subset}(G_{IFBSS}, B)$

Proof: It is evident from the provided definition 2.1.14.

Theorem 2.2.3. Let $(F_{IFBSS},\ A)$, $(G_{IFBSS},$ B) and be two Intuitionistic fuzzy binary soft sets. Then we outline the following results:

i) $(F_{IFBSS},$ A) $\widetilde{\cup}(F_{IFBSS}, A)^c \widetilde{\cong} \widetilde{A}$

ii) $(F_{IFBSS},$ A) $\widetilde{\cap}(F_{IFBSS}, A)^c \widetilde{\cong} \widetilde{\emptyset}$

iii) $\left((F_{IFBSS}, A)\widetilde{\cup}(G_{IFBSS}, B)\right]^c \widetilde{\subseteq} (F_{IFBSS}, A)^c \widetilde{\cup} (G_{IFBSS}, B)^c$

iv)$(F_{IFBSS},A)^{c}\widetilde{\cap}(G_{IFBSS},B)^{c}\widetilde{\subseteq}\left((F_{IFBSS},A)\widetilde{\cap}(G_{IFBSS},B)\right]^{c}$
v) $\left((F_{IFBSS},A)\widetilde{\cup}(G_{IFBSS},B)\right]^{c}\widetilde{=}(F_{IFBSS},A)^{c}\widetilde{\cap}(G_{IFBSS},B)^{c}$
vi) $\left((F_{IFBSS},A)\widetilde{\cap}(G_{IFBSS},B)\right]^{c}\widetilde{=}(F_{IFBSS},A)^{c}\widetilde{\cup}(G_{IFBSS},B)^{c}$

Proof: i) and ii) are obviously true from the definitions 2.6, 2.12, 2.14. iii) Imagine that (F_{IFBSS},A) $\widetilde{\cup}$ (G_{IFBSS},B) = (H_{IFBSS}, A ∪ B).

Then,

$$\begin{aligned}\left[(F_{IFBSS},A)\widetilde{\cup}(G_{IFBSS},B)\right]^{c} &= (H_{IFBSS},\, A\cup B)^{C}\\ &= (H^{c}_{IFBSS},\neg\, A\cap\neg\, B\,)\end{aligned} \tag{1}$$

For, ¬ p ∈ A ∩¬ B, $(H^{c}_{IFBSS}(\neg p))$ = $(H_{IFBSS}(p))^{c}$

$$=\begin{cases}[F_{IFBSS}(p)]^{c} & \text{if}\neg\text{ p } \in \neg\text{ A - }\neg\text{ B}\\ [G_{IFBSS}(p)]^{c} & \text{if}\neg\text{ p } \in \neg\text{B - }\neg\text{ A}\\ [F_{IFBSS}(p)\cup G_{IFBSS}(p)]^{c} & \text{if}\neg\text{ p } \in \neg\text{ A } \cap \neg\text{B}\end{cases}$$

$$H^{c}_{IFBS}(\neg p)=\begin{cases}F^{c}_{IFBSS}(\neg p) & \text{if}\neg\text{ p } \in \neg\text{ A - }\neg\text{ B}\\ G^{c}_{IFBSS}(\neg p) & \text{if}\neg\text{ p } \in \neg\text{B - }\neg\text{ A}\\ F^{c}_{IFBSS}(\neg p)\cup G^{c}_{IFBSS}(\neg p) & \text{if}\neg\text{ p } \in \neg\text{ A } \cap \neg\text{B}\end{cases} \tag{2}$$

Now consider,

$$\begin{aligned}\left[(F_{IFBSS},A)\widetilde{\cup}(G_{IFBSS},B)\right]^{c} &= (F^{c}_{IFBSS},\neg\text{ A})\widetilde{\cup}(G^{c}_{IFBSS},\neg\text{ B})\\ &= (K_{IFBSS}\,,\neg\text{ A}\widetilde{\cup}B)\end{aligned} \tag{3}$$

Where,

$$K_{IFBSS}(\neg p)=\begin{cases}F^{c}_{IFBSS}(\neg p) & \text{if}\neg\text{ p } \in \neg\text{ A - }\neg\text{ B}\\ G^{c}_{IFBSS}(\neg p) & \text{if}\neg\text{ p } \in \neg\text{B - }\neg\text{ A}\\ F^{c}_{IFBSS}(\neg p)\cup G^{c}_{IFBSS}(\neg p) & \text{if}\neg\text{ p } \in \neg\text{ A } \cap\neg\text{B}\end{cases} \tag{4}$$

Therefore, from Equation (2) and Equation (4) we get,
$\left[(F_{IFBSS},A)\widetilde{\cup}(G_{IFBSS},B)\right]^{c}\ \widetilde{\subseteq}\ (F_{IFBSS},A)^{c}\ \widetilde{\cup}\ (G_{IFBSS},B)^{c}$
iv) Consider,
$(F_{IFBSS},A)^{c}\widetilde{\cap}(G_{IFBSS},B)^{c}$ = $(F^{c}_{IFBSS},\neg A)\widetilde{\cap}(G^{c}_{IFBSS},\neg B)$= $(H_{IFBSS},\neg A\cap\neg B)\,(say)$
where,

$$H_{IFBSS}(\neg p)=F^{c}_{IFBSS}(\neg p)\cap G^{c}_{IFBSS}(\neg p)\,,\ for\ all\neg p\in\neg A\cap\neg B. \tag{5}$$

Consider,
$\left[(F_{IFBSS},A)\widetilde{\cap}(G_{IFBSS},B)\right]^{c}$ = $(K_{IFBSS},A\cap B)^{c}$
= $(K^{c}_{IFBSS},\neg A\cup\neg B)$, by De Morgan's Law

Now for,

$$\begin{aligned}\neg p\in\neg A\cap\neg B, K^{c}_{IFBSS}(p) &= (K_{IFBSS}(p))^{c}=[F_{IFBSS}(p)\cap G_{IFBSS}(p)]^{c}\\ &= F^{c}_{IFBSS}(\neg p)\cup G^{c}_{IFBSS}(\neg p)\end{aligned} \tag{6}$$

Therefore, from Equation (5) and Equation (6) we get,
$(F_{IFBSS},A)^{c}\widetilde{\cap}(G_{IFBSS},B)^{c}\widetilde{\subseteq}\left[(F_{IFBSS},A)\widetilde{\cap}(G_{IFBSS},B)\right]^{c}$
Also from Equations (2) and (5) we get,
$\left[(F_{IFBSS},A)\widetilde{\cup}(G_{IFBSS},B)\right]^{c}\widetilde{=}(F_{IFBSS},A)^{c}\widetilde{\cap}(G_{IFBSS},B)^{c}$
Similarly from Equations (4) and (6) we get,
$\left[(F_{IFBSS},A)\widetilde{\cup}(G_{IFBSS},B)\right]^{c}\widetilde{=}(F_{IFBSS},A)^{c}\widetilde{\cup}(G_{IFBSS},B)^{c}$

Theorem 2.2.4. Let (F_{IFBSS}, A), (G_{IFBSS}, B) and (H_{IFBSS}, C) be three Intuitionistic fuzzy binary soft sets. Subsequently, these are the outcomes we obtain:

i) $\left[(F_{IFBSS},A)\widetilde{\vee}(G_{IFBSS},B)\right]^{C}\cong(F_{IFBSS},A)^{C}\widetilde{\wedge}(G_{IFBSS},B)^{C}$
ii) $\left[(F_{IFBSS},A)\widetilde{\wedge}(G_{IFBSS},B)\right]^{C}\cong(F_{IFBSS},A)^{C}\widetilde{\vee}(G_{IFBSS},B)^{C}$
iii) $[(F_{IFBSS}, \mathrm{A}) \widetilde{\vee} (G_{IFBSS}, \mathrm{B})] \widetilde{\vee} (H_{IFBSS}, \mathrm{C}) \cong (F_{IFBSS}, \mathrm{A}) \widetilde{\vee} [(G_{IFBSS}, \mathrm{B}) \widetilde{\vee} (H_{IFBSS}, \mathrm{C})]$
iv) $[(F_{IFBSS}, \mathrm{A}) \widetilde{\wedge} (G_{IFBSS}, \mathrm{B})] \widetilde{\wedge} (H_{IFBSS}, \mathrm{C}) \cong (F_{IFBSS}, \mathrm{A}) \widetilde{\wedge} [(G_{IFBSS}, \mathrm{B}) \widetilde{\wedge} (H_{IFBSS}, \mathrm{C})]$
Proof: The implication is clear from the definitions 2.1.18 and 2.1.20.

3 Results and Discussion

In this manuscript, we defined a new definition of Intuitionistic fuzzy binary soft sets. Also, we define Intuitionistic fuzzy binary soft set, Intuitionistic fuzzy binary super soft set, Intuitionistic fuzzy binary soft equal set, Intuitionistic fuzzy binary null soft set, Intuitionistic fuzzy binary absolute soft set, Union and Intersection of Intuitionistic fuzzy binary soft sets, difference of Intuitionistic fuzzy binary soft sets, AND operations, OR operations on Intuitionistic fuzzy binary soft sets. Also, we investigate some properties of Intuitionistic fuzzy binary soft sets with defined operations. Comparing the work of P. Gino Metilda and J. Subhashini [(7)], who introduced the fuzzy binary soft set with only membership values. Our newly defined definition of IFBSS extends this by offering both membership and non-membership values. This effectively addresses uncertainty and vagueness. Exploration of properties unique to Intuitionistic fuzzy binary soft sets, contributing to the advancement of soft computing and set theory, is much better than fuzzy binary soft sets. We strongly believe IFBSS will offer new and improved ideas compared to fuzzy binary soft sets, opening a new chapter in day-to-day applications.

4 Conclusion

This study investigated utilizing intuitionistic fuzzy sets for target-type recognition problems and identified the necessity for an enhanced modeling tool capable of managing multiple parameters, particularly recognition features. By integrating binary soft sets with intuitionistic fuzzy sets, we've developed a modeling approach for recognition knowledge. Intuitionistic fuzzy binary soft sets can be employed in pattern recognition tasks, especially in situations where the data may not be fully reliable or where there is a need to handle uncertainty in the recognition process. Overall, the new or recent studies on Intuitionistic fuzzy binary soft sets are likely to contribute to both theoretical advancements and practical applications in handling uncertainty and vagueness in decision-making and data analysis. We don't know precisely how these Intuitionistic fuzzy binary soft sets do it currently, but further investigation into it will undoubtedly offer opportunities for applications in everyday life.

References

1) Razzaque A, Masmali I, Latif L. On t-intuitionistic fuzzy graphs: a comprehensive analysis and application in poverty reduction. *Scientific Report-Nature*. 2023;13:17027–17027. Available from: https://doi.org/10.1038/s41598-023-43922-0.
2) Ma X, Qin H, Abawajy JH. Interval-Valued Intuitionistic Fuzzy Soft Sets Based Decision-Making and Parameter Reduction. *IEEE Transactions on Fuzzy Systems*. 2022;30(2):357–369. Available from: https://doi.org/10.1109/TFUZZ.2020.3039335.
3) Perveen PAF, John SJ. A similarity measure of spherical fuzzy soft sets and its application. *AIP Conference Proceedings*. 2021;2336. Available from: https://doi.org/10.1063/5.0045743.
4) Tiwari AK, Saini R, Nath A. Hybrid similarity relation based mutual information for feature selection in intuitionistic fuzzy rough framework and its applications. . *Scientific Report-Nature*;14. Available from: http://dx.doi.org/10.1038/s41598-024-55902-z.
5) Alreshidi N, Aedh Z, Khan MJS. Similarity and entropy measures for circular intuitionistic fuzzy sets. *Engineering Applications of Artificial Intelligence*. 2024;131:107786–107786. Available from: https://doi.org/10.1016/j.engappai.2023.107786.
6) Jebadass JR, Balasubramaniam P. Color image enhancement technique based on interval-valued intuitionistic fuzzy set. *Information Sciences*. 2024;653:119811–119811. Available from: https://doi.org/10.1016/j.ins.2023.119811.
7) Metilda PG, Subhashini J. Remarks on Fuzzy binary soft set and its characters. *proceedings of International conference on Materials and Mathematical sciences*. Available from: https://doi.org/10.1088/1742-6596/1947/1/012020.
8) Acikogz A, Tas N. Binary soft set theory. *European journal of pure and applied mathematics*. 2016;9(4):452–463. Available from: https://ejpam.com/index.php/ejpam/article/view/2148.
9) Metilda PG, Subhashini J. An application on fuzzy binary soft set in decision making problem. *Webology*. 2021;18:3672–3780. Available from: https://www.webology.org/abstract.php?id=2427.
10) Metilda PG, Subhashini J. Fuzzy binary soft separation. *Journal of physics: Conference series 1947 (2021) 012020 Available from*. Available from: https://doi.org/10.1088/1742-6596/1947/1/012020.
11) Metilda PG, Subhashini J. Some results on fuzzy binary soft point. *Malaya Journal of Mathematik*. 2021;1:589–592. Available from: https://doi.org/10.26637/MJMS2101/0134.
12) Metilda PG, Subhashini J. A note on fuzzy binary soft separation axioms. *AIP Conference Proceedings*. 2022;2385:140012–140012. Available from: https://doi.org/10.1063/5.0070744.