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Domination and Independence in Fuzzy Semigraphs

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Abstract

Objectives: To identify dominating and independent sets in fuzzy semigraphs by developing suitable methods. To examine dominating and independent sets in fuzzy semigraphs for their characteristics and behaviors. The goal is to examine the use of the concepts of domination and independence in the fields of analysis of social networks, security of networks, and allocation of resources. **Methods:** This study introduces the concept of strong arc fuzzy semigraph and explores various parameters such as dominating and independent sets. The Minimum Adjacent Dominating Number (ad-number) $\gamma_a(\mathcal{G})$ and Maximum Adjacent Dominating Number (ad-number) $\Gamma_a(\mathcal{G})$, Minimum End Node Adjacent Dominating Number (ead-number) $\gamma_{ea}(\mathcal{G})$ and Maximum End Node Adjacent Dominating Number (ead-number) $\Gamma_{ae}(\mathcal{G})$ as well as Minimum Consecutive Adjacent Dominating Number (cad-number) $\gamma_{ca}(\mathcal{G})$ and Maximum Consecutive Adjacent Dominating Number (cad-number) $\Gamma_{ca}(\mathcal{G})$. These parameters represent the respective minimum and maximum cardinalities taken over all minimal dominating sets of nodes of $\mathcal{G}$. Additionally, it explores the e-independence dominating number i_e and e-independence number $\beta_e(\mathcal{G})$, strongly independence dominating number i_s and strongly independence number $\beta_s(\mathcal{G})$ and consecutive adjacent independence dominating number i_{ca} and consecutive adjacent independence number $\beta_{ca}(\mathcal{G})$. These parameters represent the respective minimum and maximum cardinalities derived from all maximal independent sets of nodes in $\mathcal{G}$. **Findings:** The study develops a thorough correlation between fuzzy dominant sets and independent sets in the fuzzy semigraph $\mathcal{G}$. The statement illustrates the way in which the introduced factors interact and impact the structure and behaviour of the fuzzy semigraph. **Novelty:** The introduction of strong arc fuzzy semigraphs is a novel notion in the examination of fuzzy graphs. The study involves the creation and examination of novel parameters associated with dominant and independent sets in fuzzy semigraphs. This study aims to establish novel theoretical linkages between fuzzy domination and independent sets, therefore expanding our knowledge of the structural features of fuzzy semigraphs.

Keywords: Domination in fuzzy semigraphs; Independent in fuzzy semigraphs; Adjacent-domination; ea-domination; ca-domination; e-independent; s-independent; ca-independent

1 Introduction

The concept of domination has been thoroughly examined in the field of semigraphs, with pioneering research[1]. Studying the concept of independent dominance in fuzzy graphs[2]. Further research[3] has explored various aspects of semigraphs, with a particular focus on a-Domination. Recent developments in this area involve studying the domination number in new graph structures[4]. The study in the area of semigraph domination has been ongoing, but there is a noticeable lack of attention given to fuzzy semigraphs. Specifically, there is a need for more investigation into dominating sets and their independence in this field. There is limited research comparing fuzzy and standard semigraphs, and even less exploring the unique characteristics and possible applications of dominating sets in fuzzy semigraphs. There is a need to conduct research on dominating sets in fuzzy semigraphs, using strong arcs, in order to fill these gaps. Optimization, resource allocation, and network analysis applications can greatly benefit from the use of fuzzy semigraphs because of their enhanced generalizability and flexibility. Using these concepts to solve problems in the real world, developing new algorithms to study independent dominating sets, and then analyzing the results will assist in extending the objective[5].

2 Methodology

This study introduces strong arc fuzzy semigraphs and analyzes parameters relating to dominating and independent sets. It defines minimum and maximum adjacent dominating numbers, end node, and consecutive dominating numbers, and analyzes end node, strongly, and consecutive independent dominating numbers. These parameters show minimum and maximum cardinalities across dominating sets.

2.1. Domination in fuzzy semigraphs

2.2. Definition

Let $\mathcal{G} : (\sigma, \mu, \eta)$ be a fuzzy semigraph with nodes u and v in $\mathcal{G}$, if there exists a strong arc from u *to* v, we define that u dominates v.

2.3. Definition

An adjacent dominating set (ad-set) in fuzzy semigraph $\mathcal{G}$, denoted as $D_a \subseteq V$ is defined as a set where for every $v \epsilon V - D_a$, there exists a node $u \epsilon D_a$ such that u is adjacent to v.

The minimum cardinality of an (ad-set) is $\gamma_a(\mathcal{G})$.

The maximum cardinality of an (ad-set) is $\Gamma_a(\mathcal{G})$.

2.4. Definition

An end node adjacent dominating set (ead-set) in fuzzy semigraph $\mathcal{G}$, denoted as $D_e \subseteq V$ satisfying the following condition (i) D is an ad- set and (ii) Each end node $v \epsilon V - D_e$, is e- adjacent to some end node $u \epsilon D_e$ in $\mathcal{G}$.

The minimum cardinality of an (ead-set) is $\gamma_{ea}(\mathcal{G})$.

The maximum cardinality of an (ead-set) is $\Gamma_{ea}(\mathcal{G})$.

2.5. Definition

A consecutive adjacent dominating set (cad-set) in fuzzy semigraph $\mathcal{G}$, denoted as $D_{ca} \subseteq V$ is defined as a set where for every $v \epsilon V - D_{ca}$, there exists a $u \epsilon D_{ca}$ such that u is consecutive adjacent to $u \epsilon \mathcal{G}$.

The minimum cardinality of a (cad-set) is $\gamma_{ca}(\mathcal{G})$.

The maximum cardinality of a (cad-set) is $\Gamma_{ca}(\mathcal{G})$.

2.6. Definition

Let $\mathcal{G} : (\sigma, \mu, \eta)$ be a fuzzy semigraph. The adjacent degree of a node v is defined as follows $d_a(u) = \sum\sum_{e\in E}\mu(e)$. The minimum degree of $\mathcal{G}$ is $\delta_a = \wedge\{d_a/v \in V\}$ and the maximum degree of $\mathcal{G}$ is $\triangle_a = \vee\{d_a/v \in V\}$.

2.7. Definition

Let $\mathcal{G} : (\sigma, \mu, \eta)$ be a fuzzy semigraph. The end adjacent degree of a node v is defined as follows $d_{ea}(u) = \sum\sum_{e\in E}\mu(e)$. The minimum degree of $\mathcal{G}$ is $\delta_{ea} = \wedge\{d_{ea}/v \in V\}$ and the maximum degree of $\mathcal{G}$ is $\triangle_{ea} = \vee\{d_{ea}/v \in V\}$.

2.8. Definition

Let $\mathcal{G} : (\sigma, \mu, \eta)$ be a fuzzy semigraph. The Consecutive adjacent degree of a node v is defined as follows $d_{ca}(u) = \sum\mu(uv)$. The minimum degree of $\mathcal{G}$ is $\delta_{ca} = \wedge\{d_{ca}/v \in V\}$ and the maximum degree of $\mathcal{G}$ is $\triangle_{ca} = \vee\{d_{ca}/v \in V\}$.

3 Independent Domination in fuzzy semigraph

3.1. Definition

Let $\mathcal{G} : (\sigma, \mu, \eta)$ be a fuzzy semigraph, a set $S \subseteq V$ is said to be fuzzy independent if there is no strong arc on S. The set S_e is end-independent if no two end nodes of an edge in $S_e \subseteq V$. The set S_s is strongly independent if no two adjacent nodes are in $S_s \subseteq V$. The set S_{ca} is ca-independent if no two nodes are consecutively adjacent in $S_{ca} \subseteq V$.

3.2. Definition

The minimum cardinality of an e-independent dominating set is called e-independence domination number which is noted as $i_e(\mathcal{G})$. Similarly, we can define $i_s(\mathcal{G})$ and $i_{ca}(\mathcal{G})$.

3.3. Definition

The maximum cardinality of an e-independent dominating set is called e-independence domination number which is noted as $\beta_e(\mathcal{G})$. Similarly, we can define $\beta_s(\mathcal{G})$ and $\beta_{ca}(\mathcal{G})$.

4 Results and Discussion

4.1. Example

Let $\mathcal{G} : (\sigma, \mu, \eta)$ be a fuzzy semigraph as shown in Figure 1.

Minimal Adjacent dominating set $D_a = \{P, S\}$.

Minimum Adjacent domination number $\gamma_a = \{0.5 + 0.5\} = 1$.

Maximal Adjacent dominating set $D_a = \{Q, U\}$.

Maximum Adjacent domination number $\Gamma_a = \{0.7 + 0.6\} = 1.3$.

Minimum Adjacent degree $\delta = \{P\} = 0.5$.

Maximum Adjacent degree $\Delta = \{S\} = 0.9$.

4.2. Example

Let $\mathcal{G} : (\sigma, \mu, \eta)$ be a fuzzy semigraph as shown in Figure 2.

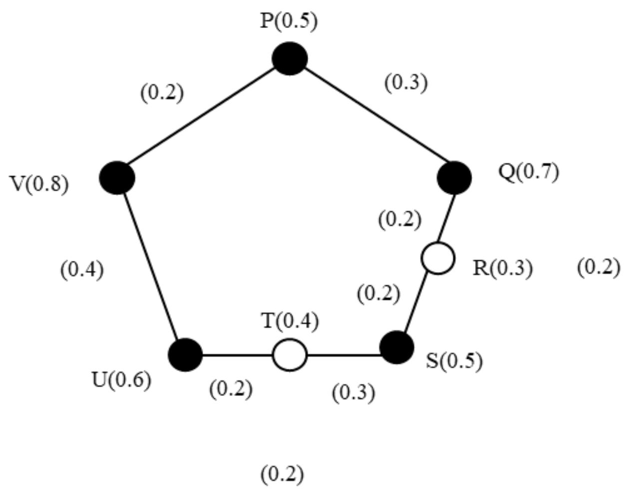

Fig 1. Adjacent dominating set

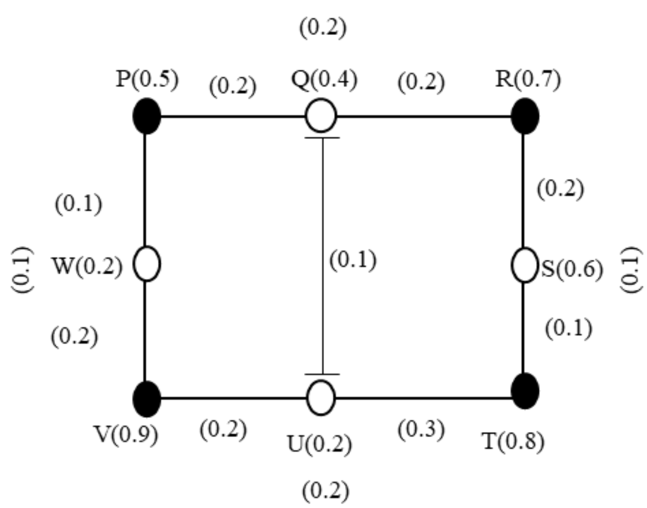

Fig 2. End adjacent dominating set

Minimal End adjacent dominating set $\mathrm{D}_{\mathrm{ea}} = \{\mathrm{P}, \mathrm{T}\}$.
Minimum End adjacent domination number $\gamma_{ea} = \{0.5 + 0.8\} = 1.3$.
Maximal End adjacent dominating set $\mathrm{D}_{\mathrm{ea}} = \{\mathrm{P}, \mathrm{R}, \mathrm{T}, \mathrm{V}\}$.
Maximum End adjacent domination number $\Gamma_{ea} = \{0.5 + 0.6 + 0.8 + 0.9\} = 2.8$.
Minimum End adjacent degree $\delta = \{Q\} = 0.4$.
Maximum End adjacent degree $\triangle = \{T\} = 0.8$.

4.3. Example

Let $\mathcal{G} : (\sigma, \mu, \eta)$ be a fuzzy semigraph as shown in Figure 3.
Minimal Consecutive adjacent dominating set $\mathrm{D}_{\mathrm{ca}} = \{\mathrm{Q}, \mathrm{S}, \mathrm{U}\}$.
Minimum Consecutive adjacent domination number $\gamma_{ca} = \{0.2 + 0.3 + 0.4\} = 0.9$.
Maximal Consecutive adjacent dominating set $\mathrm{D}_{\mathrm{ca}} = \{\mathrm{P}, \mathrm{S}, \mathrm{V}\}$.
Maximum Consecutive adjacent domination number $\Gamma_{ca} = \{0.4 + 0.3 + 0.8\} = 1.5$.
Minimum Consecutive adjacent degree $\delta = \{P\} = 0.2$.

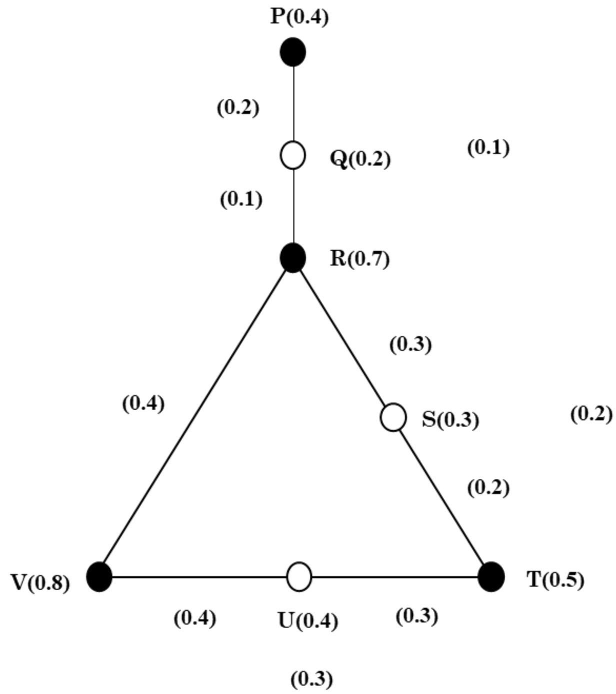

Fig 3. Consecutive adjacent dominating set

Maximum Consecutive adjacent degree $\Delta = \{V\} = 0.8$.

4.4. Definition

Let $\mathcal{G} : (\sigma, \mu, \eta)$ be a fuzzy semigraph. A node $u \epsilon D$ is said to be a-isolates in D, if no v in D such that u and v are adjacent.

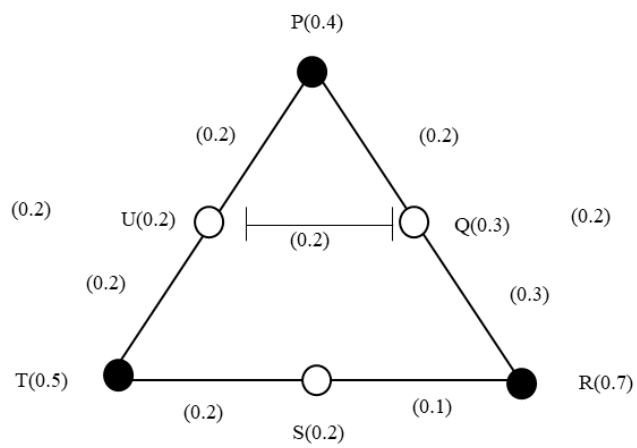

Fig 4.

Here $D = \{P, Q, S\}$, S is an a-isolate, but P & Q are not a-isolates.

4.5. Observation

For any given fuzzy semigraph $\mathcal{G}$,

i. $\gamma_a(\mathcal{G}) \leq \gamma_{ea}(\mathcal{G})$
ii. $\gamma_a(\mathcal{G}) \leq \gamma_{ca}(\mathcal{G})$
iii. $\gamma_{ea}(\mathcal{G}) \leq \gamma_{ca}(\mathcal{G})$

4.6. Observation

Let P_k as a path fuzzy semigraph and C_k as a cycle fuzzy semigraph with k nodes containing n end nodes and m middle nodes such that m+n =k and let $\mathcal{G}_k = P_k\ or\ C_k$. Then,
iv. $\gamma_a(\mathcal{G}) \leq \gamma_{ea}(\mathcal{G}) \leq [\frac{k}{2}]$
v. $\gamma_{ca}(\mathcal{G}) \leq [\frac{k}{2}]$

4.7. Theorem

An adjacent dominating set (ad-set) D_a of fuzzy semigraph $\mathcal{G}$ is a minimal if and only if for each node $u\ \epsilon D_a$ one of the following properties holds true.

(i) u is an adjacent- isolate node of D_a

(ii) There exists a node $v\ \epsilon\ V - D_a$ such that $N_a(v) \cap D_a = \{u\}$.

Proof:

If D_a is minimal (ad-set) of $\mathcal{G}$. For every node u in D_a, let's consider the set

$D_a^{'} = D_a - \{u\}$ which does not remain an (ad-set). As a result, there must exist a node $v\ \epsilon\ V - D_a^{'}$ which is not adjacent dominated to at least one node in $D_a^{'}$.

Now, consider two cases: either $v, u\ \epsilon\ D_a$ or $v\ \epsilon\ V - D_a$.

If $v, u\ \epsilon\ D_a$, then u is an adjacent-isolate node of D_a.

If $v\ \epsilon\ V - D_a$ and v is not an adjacent dominated by $D_a^{'}$, but is adjacent dominated by D_a, then $u\ \epsilon\ D_a$ is the only strong neighbour of $v\ \epsilon\ V - D_a$, i.e.,$N_a(v) \cap D_a = \{u\}$.

Conversely, assume that D_a is an (ad-set), and for each node $u\ \epsilon\ D_a$, one of the two stated conditions holds. Our goal is to show that D_a is minimal (ad-set).

Suppose D_a is not a minimal (ad-set). And there exists a node $u\ \epsilon\ D_a$ which means $D_a^{'}$ is an (ad-set). Consequently, u is a strong neighbour to at least one node in $D_a^{'}$. Thus condition (1) is not satisfied.

Furthermore, if $D_a^{'}$ is an (ad-set), then each node in $v - D_a$ is a strong neighbour to at least one node in $D_a^{'}$. Thus condition (2) is not satisfied.

Therefore, it is contradiction because neither condition (1) nor condition (2) hold.

4.8. Example

In Figure 4, the set {P,Q,S} is identified as a minimal ad-set where its nodes satisfy conditions (i) and (ii). Within this set, the nodes {S} meet condition (i), whereas the nodes {P,Q} satisfy condition (ii) of theorem 4.7 given above.

4.9. Theorem

For any fuzzy semigraph $\mathcal{G}$, $\gamma_a(\mathcal{G}) \leq k - \triangle_a(\mathcal{G})$

Proof:

Let v be a node of maximum adjacent degree $\triangle_a(\mathcal{G})$. Then v is adjacent to $N_a(v)$ node such that $\triangle_a(\mathcal{G}) \leq N_a(v)$. Hence, $V - N_a(v)$ is an (ad-set). Therefore, $\gamma_a(\mathcal{G}) \leq |V - N_a(v)|$. Thus, $\gamma_a(\mathcal{G}) \leq k - \triangle_a(\mathcal{G})$.

4.10. Theorem

If $\mathcal{G}$ is a fuzzy semigraph and D_a is a minimal (ad-set), then the complement of D_a, denoted $V - D_a$, is an (ad-set).

Proof:

Assuming $V - D_a$ is not an adjacent dominating set. This implies that there exists a node u in $V - D_a$ that isn't adjacent to any node in $V - D_a$. Since D_a is a minimal (ad-set), every node in $V - D_a$ must be adjacent to any one node in D_a. This leads to a contradiction, as it implies that the node u is not adjacent to any node in $V - D_a$, but each node in $V - D_a$ must be adjacent to some node in D_a.

Therefore, our assumption is that $V - D_a$ is not an (ad-set) is false. Consequently, $V - D_a$ is an (ad-set).

4.11. Example

Let $\mathcal{G}:(\sigma,\mu,\eta)$ be a fuzzy semigraph as shown in Figure 5.

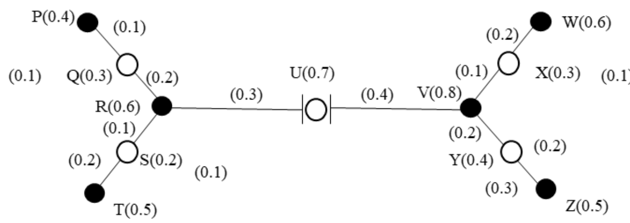

Fig 5. e-independent set

Maximal e-independent set $S_e = \{P,T,U,W,Z\}$.
Maximum e-independence number $\beta_e = \{0.4+0.5+0.7+0.6+0.5\} = \{2.7\}$.
Minimal e-independent set $S_e = \{R,V\}$.
Minimum e-independence number $i_e = \{0.6+0.8\} = \{1.4\}$.
Minimum e-independent degree $\delta = \{R\} = \{0.2\}$.
Maximum e-independent degree $\Delta = \{U\} = \{0.7\}$.

4.12. Example

Let $\mathcal{G}:(\sigma,\mu,\eta)$ be a fuzzy semigraph as shown in Figure 6.

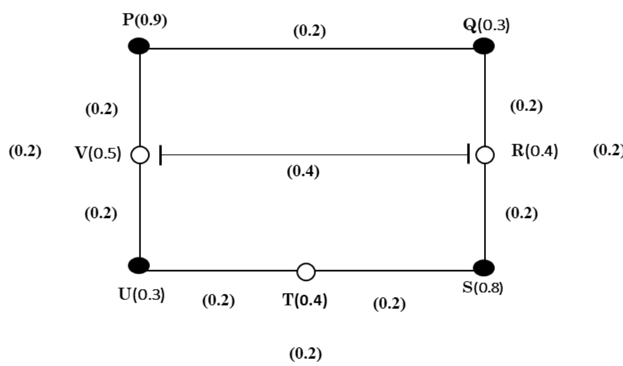

Fig 6. Strongly independent set Maximal

Strongly independent set $S_S = \{P,R,T\}$.
Maximum strongly independence number $\beta_S = \{0.9+0.4+0.4\} = 1.7$.
Minimal strongly independent set $S_S = \{Q,U\}$.
Minimum strongly independence number $i_s = \{0.3+0.3\} = 0.6$.
Minimum strongly independent degree $\delta = \{P\} = 0.4$.
Maximum strongly independent degree $\Delta = \{R\} = 0.6$.

4.13. Example

Let $\mathcal{G}:(\sigma,\mu,\eta)$ be a fuzzy semigraph as shown in Figure 7.
Maximal Consecutive adjacent independent set $S_{ca} = \{P,V,T,R\}$.
Maximum Consecutive adjacent independence number $\beta_{ca} = \{0.5+0.2+0.8+0.7\} = \{2.2\}$.
Minimal Consecutive adjacent independent set $S_{ca} = \{P,V,S\}$.
Minimum Consecutive adjacent independence number $i_{ca} = \{0.5+0.2+0.5\} = \{1.2\}$.
Minimum Consecutive adjacent independent degree $\delta = \{V\} = \{0.2\}$.
Maximum Consecutive adjacent independent degree $\Delta = \{S\} = \{0.7\}$.

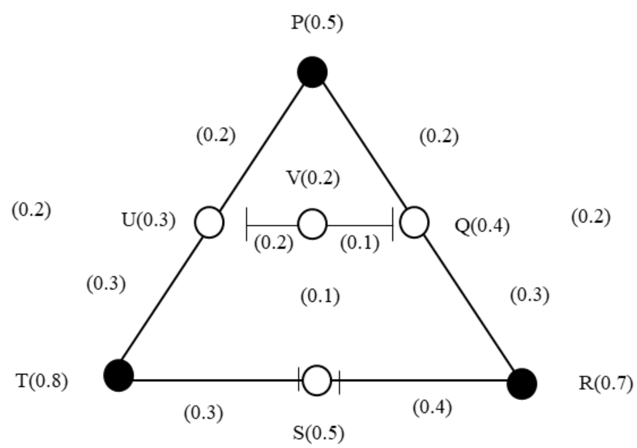

Fig 7. Consecutive adjacent independent set

4.14. Observation

For any given fuzzy semigraph $\mathcal{G}$,

i. $\beta_e(\mathcal{G}) \leq \beta_s(\mathcal{G})$
ii. $\beta_s(\mathcal{G}) \leq \beta_{ca}(\mathcal{G})$
iii. $\beta_e(\mathcal{G}) \leq \beta_{ca}(\mathcal{G})$

4.15. Observation

Let P_k as a path fuzzy semigraph and C_k as a cycle fuzzy semigraph with k nodes containing n end nodes and m middle nodes such that m+n =k and let $\mathcal{G}_k = P_k \; or \; C_k$. Then,

iv. $\beta_e(\mathcal{G}) \leq \beta_s(\mathcal{G}) \leq \left[\frac{k}{2}\right]$
v. $\beta_{ca}(\mathcal{G}) \leq \left[\frac{k}{2}\right]$

4.16. Theorem

A path fuzzy semigraph $\mathcal{G}$ is considered a maximal fuzzy e-independent set if and only if it satisfies the condition of being both a fuzzy e-independent set and a fuzzy ea-dominating set.

Proof:

Let us consider a path in fuzzy semigraph $\mathcal{G}$, and We define S_e as a maximal fuzzy e-independent set. For every node $u \in v{-}S_e$, the set $S_e \cup \{u\}$ is not fuzzy e-independent. This implies that for every node $u \in v{-}S_e$, there is node $v{\in}\, S_e$ such that (u, v) is strong neighbour. Therefore, S_e is an ea-dominating set. Hence S_e is both fuzzy e-independent and ea-dominating.

On the other hand, let's suppose that a set S_e is both fuzzy e-independent and ea-dominating. We show that it is maximal fuzzy e-independent set. Let's assume that S is not a maximal fuzzy e-independent set. There exists a node $u \in V - S_e$ such that $S_e \cup \{u\}$ is a fuzzy e-independent set. But, if $S_e \cup \{u\}$ is a fuzzy e-independent set then no node in S_e is a strong neighbour of u. Thus, S_e is not an ea-dominating set, which is a contradiction. Hence, S_e is maximal fuzzy e-independent set.

4.17. Theorem

In a fuzzy semigraph $\mathcal{G}$, each maximal strongly- independent set is a minimal (ad-set) in $\mathcal{G}$.

Proof:

The proof follows a similarly to the proof of theorem 4.16.

4.18. Observation

For any fuzzy semigraph $\mathcal{G}$, $\gamma_a \leq \gamma_{ea} \leq \gamma_{ca} \leq \beta_e \leq \beta_s \leq \beta_{ca}$.

Proof:

Refer the examples [4.1, 4.2, 4.3, 4.11, 4.12, 4.13].

5 Conclusion

This study has identified three distinct types of dominating sets and thoroughly examined the properties related to the domination number and independence domination number. In conclusion, we conducted a thorough examination of the properties of domination numbers in relation to particular fuzzy semigraphs. The findings might be used in social network analysis, network security, resource allocation, optimization, telecommunications, biological networks, supply chain management, and urban planning.

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